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George Haskell

Book

January 2 the d 1836
George Haskell
Arithmetick

[*Ruger's Arithmetick, with Questions and Answers.*]

A

NEW SYSTEM

OF

ARITHMETICK;

IN WHICH

THE RULES ARE FAMILIARLY DEMONSTRATED,

AND THE

PRINCIPLES OF THE SCIENCE

CLEARLY AND FULLY EXPLAINED,

BY SHOWING THE RELATION WHICH THE SIMPLE RULES BEAR TO EACH OTHER,

AND REDUCING THE MORE DIFFICULT PARTS BACK TO THOSE RULES.

BY WILLIAM RUGER, A. B.

NEW YORK.

SOLD BY THE PRINCIPAL BOOKSELLERS.

Watertown, N. Y.

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1836.

For recommendations, see last page.

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INTRODUCTION.

AS perfection is seldom, if ever, attained in matters of science, there is still a chance for improvement. And, as reason, under the hand of refinement, acquires new powers, so almost every science for a century past has acquired new beauties, except that of Arithmetick, which has remained almost stationary. Even in our standard works, we find the Rules completely arbitrary, without a single reason given, why they should produce their results. Thus, a science which may properly be considered as the greatest work of human wisdom, has been left shrouded in mystery. Those who have written on the subject, have left the principles of the rules unexplained, as if they were beyond the reach of human investigation, or beneath the dignity of scientifick research. The object of the author, in this treatise, is to disperse the gloom, and represent the science in its true light, freed from all its obscurity. In this work, the principles of each rule will be found illustrated by plain demonstrations; a thing that is novel in the science of Arithmetick, yet, in the opinion of the author, it is the true and only mode by which the principles of the science can be communicated or studied to advantage. It is hardly possible for the student to retain arbitrary rules: WORDS are soon forgotten, while time is scarcely sufficient to efface PRINCIPLES from the memory.

After the Examples given under each rule, it was thought best to annex a list of questions and answers for the benefit of teachers and students, illustrating more fully the principles of the rules and the operations of the work. These are calculated to brighten the faculties of the student, and leave a lasting impression on the mind. This method of instructing by questions and answers, is considered by many to be so important, that some of the best institutions in the Union are now conducted solely on this plan.

The author has dwelt on the Simple Rules beyond what is usual, in order to make the student thoroughly understand the principles of numbers; the relation which these rules bear to each other, and their practical application in business. He believes it will generally be allowed by instructors, that most authors are so deficient in these rules, that the student passes to the more intricate parts, while he is yet *ignorant of the first rudiments of the science*; the consequence is, his ambition is dampened; he can see no beauty in a science in which obscurity is back of him, and impenetrable darkness before him.

Federal money being next in simplicity to whole numbers, is introduced immediately after each of the Simple Rules. This arrangement will be of great service to him who has but little time or opportunity to devote to the science, since an accurate knowledge of these rules and that Currency, will enable him to transact correctly most of the business of life. The teacher will discover that in this work, the labour of the student is greatly abridged in acquiring a knowledge of the Reduction of Currencies; for, on inspection, it will be found that many authors have given nearly or quite THIRTY RULES to perform what is here embraced in TWO. Most authors have given several different rules for casting INTEREST, which produce different results; neither giving the proper authority of our courts, nor deciding which rule should be followed by the student in practical business. The consequence has been, that even business men follow as many different modes of casting interest, as there are different rules in our books, the results of which are widely different; but in this work the author has observed a strict adherence to legal interest; quoting the decisions of our courts. As Commission or Factorage, Brokerage, Ensurance, and likewise Buying and Selling Stocks, are depending upon the same principles with Simple Interest, they are placed under the same running title, with demonstrations as they severally occur, showing that the principles involved are the same.

The Rule of Three has, in this treatise, been so illustrated by reducing its principles back to Multiplication and Division, as to render it almost as simple as those rules. Since the Double Rule of Three, Practice, Single and Double Fellowship, Tare and Tret, Barter, Loss and Gain, Alligation, Discount and Annuities, belong strictly to the Single Rule of Three, they are therefore placed under the running title of that rule, and shown to depend upon the same principles, so that the student, when he understands the one, may properly be said to understand the whole; because he will at once perceive that he is acquainted with all the principles on which they depend.

In short, all the rules throughout the work will be found demonstrated, not merely by examples worked out, dignified with the name of demonstrations, but by plain reasoning on numbers.

THE AUTHOR.

Watertown, N. Y. July 4, 1831.

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ARITHMETICK,

Is the art or science which treats of the nature and properties of numbers. *Unity* or *unit* is that by which every thing is called *one*, or the beginning of a number. An *integer*, or whole number, is some entire quantity, as one, ten, fifteen, twenty, &c.; so called in opposition to fractions, which are broken numbers, or parts of integers; as, one-half, two-thirds, or three-fourths.

We have two methods of expressing all numbers; the *Arabick* and the *Roman*. The Arabick method is by ten characters or figures, nine of which are significant of value, the tenth is insignificant, or of no value.

Notation and Numeration of Numbers.

The characters employed in the Arabick method, are expressed and written as follows:—

Unit, unity or one,	1	Six,	6
Two,	2	Seven,	7
Three,	3	Eight,	8
Four,	4	Nine,	9
Five,	5	And a cipher,	0

These figures are also called digits, from the Latin word *digitus*, a finger. The first nine figures are called significant, because each expresses value of its own; the cipher is called insignificant, because it expresses no value of itself, yet it alters the value of those at the left hand; thus, the number, 9, expresses nine, join a 0, it becomes ninety, 90. All numbers may be expressed by the repetition and different arrangement of these figures.

Note.—It is about a thousand years since the Arabick method of notation was introduced into Europe by the Arabs, when they established themselves in the southern provinces of Spain. Although they introduced the Arabick numeral figures and the principles of notation in Europe, yet they were not the original inventors: they derived their knowledge from India.

NUMERATION TABLES.—(*Table I.*)

Note.—The words at the head of the table should be committed to memory; they are applicable to other numbers as well as those in the table.

Hundreds of Millions.	Tens of Millions.	Millions.	Hundreds of Thousands.	Tens of Thousands.	Thousands.	Hundreds.	Tens.	Units.	
								1	One.
							2	1	Twenty-one.
						3	2	1	Three hundred and Twenty-one.
					4	3	2	1	4 Thousand 3 2 1.
				5	4	3	2	1	5 4 Thousand 3 2 1.
			6	5	4	3	2	1	6 5 4 Thousand 3 2 1
		7	6	5	4	3	2	1	7 Million 6 5 4 Thousand 3 2 1.
	8	7	6	5	4	3	2	1	8 7 Million 6 5 4 Thousand 3 2 1.
9	8	7	6	5	4	3	2	1	9 8 7 Million 6 5 4 Thousand 3 2 1.
Period of Millions.			Period of Thousands.			Period of Units.			

Note.—Large numbers are frequently separated by periods, that they may more readily be expressed in words.

The table already furnished is sufficient for all practical purposes, but as greater numbers sometimes occur, we subjoin the following

TABLE.—(*Table II.*)

of Quadrillions.	of Trillions.			of Billions.			of Millions.			of Thousands.			of Units.		
Units.	Hundreds.	Tens.	Units.	Hundreds.	Tens.	Units.	Hundreds.	Tens.	Units.	Hundreds.	Tens.	Units.	Hundreds.	Tens.	Units.
9,	5	7	6,	3	6	4,	8	6	5,	4	9	6,	3	7	4

---

* Figures expressing the same significant value, when standing alone, increase in a tenfold proportion when they are joined; thus in the number 66, the left hand figure expresses ten times more than the one at the right; but when the left hand figure expresses greater significant value than the one at the right, the proportion is greater than

ten-fold; thus in the number 91, the proportion is ninety-fold, that is, the left hand figure expresses ninety times as much as the one at the right; but when the left hand figure expresses less significant value than that at the right, the proportion is less than ten-fold, thus in the number 19, the left hand figure expresses only one more than that at the right.

We numerate whole numbers from the right hand to the left, as may be seen from the tables already given; but decimals which are parts of integers, must be numerated from the left to the right, as may be seen from the following

TABLE. (*Table III.*)

Hundreds of Millions.	Tens of Millions.	Millions.	Hundreds of Thousands.	Tens of Thousands.	Thousands.	Hundreds.	Tens.	Units.	Tenth parts.	Hundredth parts.	Thousandth parts.	Ten thousandth parts.	Hundred thousandth parts.	Millionth parts.	Ten Millionth parts.	Hundred Millionth parts.
9	8	7	6	5	4	3	2	1,	2	3	4	5	6	7	8	9

Integers. *Decimals.*

NOTE.—As integers increase in a ten-fold proportion counted from units to the left, so decimals decrease in the same proportion counted from the left to the right.

The comma (,) placed between whole numbers and decimals, is called SEPARATRIX OR DECIMAL POINT.—The figures at the left of the separatrix are whole numbers, those at the right are decimals.

Numeration is expressing in words what is written in figures; thus, 66, when expressed in words, reads, sixty-six.

Notation is writing in figures what is proposed in words, thus, sixty-six, in words, is written in figures, 66.

The Arabick method has an advantage over the Roman, on account of the figures expressing simple and local value. A figure standing alone expresses simple value; thus 3, simply expresses three, but when we write down more than one figure, all except the right hand figure, have a local value; thus, in the number 433, the left hand figure stands in the place of hundreds and expresses four hundred, according to the laws of notation, and the next figure at the right, stands in the place of tens, and from its local placing expresses 30, and the next at the right, standing in the place of units, expresses its simple value three.

The Roman Method is by letters; thus, I expresses one; V five, X ten, L fifty, C one hundred, D five hundred, M one thousand. By the different arrangement and repetition of these seven letters, all numbers may be expressed. These letters express only simple value. As often as the same letter is repeated, its value is repeated; thus, X represents ten, and XX twenty, the left hand letter expressing no more than that at the right. If a letter expressing less value, be placed after one expressing greater value, the effect is addition; thus, XI represents eleven. But if the letter expressing less value be placed at the left, the effect is subtraction, thus, IX represents nine.

Notation by Roman Letters.

I. Represents	1				
II.	2	XV.	15	C.	100
III.	3	XVI.	16	CC.	200
IV. or IIII.	4	XVII.	17	CCC.	300
V.	5	XVIII.	18	CCCC.	400
VI.	6	XIX.	19	D. or IↃ.*	500
VII.	7	XX.	20	DC.	600
VIII.	8	XXX.	30	DCC.	700
IX. or VIIII.	9	XL.	40	DCCC.	800
X.	10	L.	50	DCCCC.	900
XI.	11	LX.	60	M. or CIↃ.†	1000
XII.	12	LXX.	70	MD.	1500
XIII.	13	LXXX.	80	MDC.	1600
XIV.	14	XC.	90	MDCCCXXXI.	1831

* IↃ is sometimes used instead of D to represent five hundred, and every additional Ↄ placed at the right hand, increases the number ten times.

† CIↃ is sometimes used to express one thousand, and every C and Ↄ placed at each end, increases the number ten times; thus CIↃ expresses one thousand, and CCIↃↃ ten thousand.

¶ NOTE.—A line drawn over a number increases it a thousand times, thus, $\overline{X}$ expresses ten thousand, and $\overline{XX}$ twenty thousand.

It may now be seen from the two methods of notation, that every number, above unity, is formed from the continual addition of unity to itself; and it has justly been said that NATURE, in forming the human hand, furnished us with the first elements of calculation. For the simplest method of expressing one, is a single line (|) which according to the Roman notation, is written I, which was for one finger, and two by two lines, (| |) three by three lines, (| | |) and four by four lines, (| | | | .) As this exhausts all the fingers of one hand, the next number, five, would naturally be represented by a new character, as V, formed by the opening between the thumb and forefinger, then six would be by VI, and so on to ten, which might be expressed by two V's, but to express it in the least room possible, they must be written together, thus, X, ten. Thus they proceeded from the different arrangement and repetition of these characters to fifty, which they found necessary to represent by some new combination of lines.

Proceeding on the supposition of a quintuple scale, it would require three lines, or a new combination of two; they naturally concluded that it would be sufficiently distinct if the two lines formed a right angle, thus └ represents fifty; accordingly one hundred was written ⊏, which, for the ease of writing, was afterwards rounded off and became C. From the reasoning advanced, five hundred would require a new character, and as the next combination most simple is three lines forming a triangle, it was written thus, △, from the Greek Δ; and one angle afterwards being rounded off, it is now represented by the Roman D. And one thousand, which was represented by two D's, has since, by means of contraction, been represented by M.

To facilitate the progress of the young student in writing numbers according to the Arabick notation, where ciphers come between or after the significant figures, the following table is annexed.

Hundreds of Millions.	Tens of Millions.	Millions.	Hundreds of Thousands.	Tens of Thousands.	Thousands.	Hundreds.	Tens.	Units.	
							1	0	Ten.
						1	0	4	One hundred and four.
					1	0	0	1	One thousand and one.
				1	0	0	5	0	Ten thousand and 50.
			1	0	0	0	0	1	100 Thousand and 1.
		1	6	0	0	5	0	0	1 Million 600 thousand 500.
	1	0	8	0	0	1	0	1	10 Million 800 thousand 101.
1	0	1	0	0	9	0	0	0	101 Million 9 thousand.
6	0	6	3	0	3	3	0	3	606 Million 303 thousand 303

Let the student be required to express in words the following sums: 79, 65, 105, 1005, 1010, 100300, 9846, 3804, 64890.

Write in figures the following sums:

Seventy-nine, ninety-seven, one hundred and one, one thousand two hundred, one hundred thousand three hundred, ninety-nine thousand nine hundred and ninety-nine.

Let the student be required to express in words the following sums:

IV, IX, XX, XC, CCX, XXV, MDC, LVI, DLX, DCC, CVII.

Questions to be answered by the student.

NOTE.—It is the interest of every teacher to advance his pupils as fast as possible, and to do that he must teach thoroughly. He should class his students and question them, at least as often as once a day, on the sums, the rules and the principles on which they are founded; and the student should make this his maxim—never to pass on from rule to rule, while there remains any thing back of him unconquered.

QUESTIONS ON NUMERATION AND NOTATION.

What is Arithmetick? A. The art or science which treats of the nature and properties of numbers. What is unity or unit? A. That by which every thing is called one. What is an integer or whole number? A. Some entire quantity, as 6, 16, 20, &c. Why is an integer so called? A. In opposition to fractions, as one-half, two-thirds. How many methods have we of expressing numbers? A. Two, the Arabick and Roman. What is the Arabick method? A. By ten characters or figures. How many of them are significant of value? A. Nine, the tenth is of no value. Can all numbers be expressed by these ten characters? A. Yes. By whom was the Arabick notation introduced into Europe? A. By the Arabs. How long since? A. About one thousand years. Where did they obtain their knowledge? A. From India. What are the words at the head of the numeration table in numerating nine figures? A. Units, tens, hundreds, thousands, tens of thousands, hundreds of thousands, millions, tens of millions, hundreds of millions. In what direction are whole numbers numerated? A. From the right to the left. How are decimals numerated? A. From the left to the right. What is a decimal? A. Something less than a unit. What are the words made use of in numerating three figures in decimals? A. Tenth parts, hundredth parts, thousandth parts. In what proportion do figures increase from the right to the left? A. Ten-fold when they express the same significant value. If the left hand figure expresses greater simple value than that at the right, what is the proportion? A. Greater than ten-fold. If the left hand figure expresses less simple value than that at the right, what is the proportion? A. Less than ten-fold. What is numeration? A. Expressing figures in words. What is notation? A. Writing figures for words. What value do the Arabick characters express? A. Simple and local. What value does a figure express standing alone? A. Simple. When a number is expressed by more than one figure, what value does that at the right express? A. Simple. What do those at the left express? A. Local. What is the Roman method of notation? A. By seven letters. What are they? A. I, V, X, L, C, D, M. What value do they express? A. Simple.

SIMPLE ADDITION,

Is collecting *two* or more *numbers* of the same denomination in one *sum*. The *number* arising from the operation of the work, is called the *sum* or *amount*.

RULE.

I. Write the numbers to be added, so that units may stand under units, tens under tens, hundreds under hundreds, &c. and draw a line under the whole.

II. Then add the right hand column, and if the sum be less than ten, write it directly under the column added, if it exceed ten, write down the right hand figure, and add the left hand figure or figures, to the first figure of the next column. Observe the same rule in all the left hand columns, setting down the whole amount of the last column.

PROOF. Add the columns downwards, as you added upwards, and if the same amount is produced, the work is presumed to be right.

Two parallel lines = denote equality; thus, 6 and 2=8, that is 6 and 2 are equal to 8.

For the sake of brevity, addition is often denoted by the following character +, thus, 2+6 signifies the amount of 2 and 6, that is 2+6=8.

ADDITION TABLE.

2	and	1	are	3	3	and	9	are	12	5	and	5	are	10	7	and	1	are	8
2	+	2	=	4	3	+	10	=	13	5	+	6	=	11	7	+	2	=	9
2	„	3	„	5	3	„	11	„	14	5	„	7	„	12	7	„	3	„	10
2	„	4	„	6	3	„	12	„	15	5	„	8	„	13	7	„	4	„	11
2	„	5	„	7	4	„	1	„	5	5	„	9	„	14	7	„	5	„	12
2	„	6	„	8	4	„	2	„	6	5	„	10	„	15	7	„	6	„	13
2	„	7	„	9	4	„	3	„	7	5	„	11	„	16	7	„	7	„	14
2	„	8	„	10	4	„	4	„	8	5	„	12	„	17	7	„	8	„	15
2	„	9	„	11	4	„	5	„	9	6	„	1	„	7	7	„	9	„	16
2	„	10	„	12	4	„	6	„	10	6	„	2	„	8	7	„	10	„	17
2	„	11	„	13	4	„	7	„	11	6	„	3	„	9	7	„	11	„	18
2	„	12	„	14	4	„	8	„	12	6	„	4	„	10	7	„	12	„	19
3	„	1	„	4	4	„	9	„	13	6	„	5	„	11	8	„	1	„	9
3	„	2	„	5	4	„	10	„	14	6	„	6	„	12	8	„	2	„	10
3	„	3	„	6	4	„	11	„	15	6	„	7	„	13	8	„	3	„	11
3	„	4	„	7	4	„	12	„	16	6	„	8	„	14	8	„	4	„	12
3	„	5	„	8	5	„	1	„	6	6	„	9	„	15	8	„	5	„	13
3	„	6	„	9	5	„	2	„	7	6	„	10	„	16	8	„	6	„	14
3	„	7	„	10	5	„	3	„	8	6	„	11	„	17	8	„	7	„	15
3	„	8	„	11	5	„	4	„	9	6	„	12	„	18	8	„	8	„	16

Addition Table—Continued.

8 „ 9 „ 17	9 „ 1 „ 10	9 „ 5 „ 14	9 „ 9 „ 18
8 „ 10 „ 18	9 „ 2 „ 11	9 „ 6 „ 15	9 „ 10 „ 19
8 „ 11 „ 19	9 „ 3 „ 12	9 „ 7 „ 16	9 „ 11 „ 20
8 „ 12 „ 20	9 „ 4 „ 13	9 „ 8 „ 17	9 „ 12 „ 21

NOTE. 1st. To acquire a facility in adding, the student should carefully examine this table previous to his working sums. He will thereby render his task comparatively easy.

NOTE. 2d. To test the student's knowledge of the table, the teacher should question him on the table, occasionally varying the order, and sometimes mention the last of the two numbers to be added first.

FAMILIAR QUESTIONS.

NOTE. The student should be required mentally to answer each of these sums, directly after he has read the question.

1. John had 5 apples, and James gave him two more; how many had he after receiving the two from James?

2. Peter gave 4 shillings for a grammar, and 2 shillings for an English Reader; what did both cost him?

3. William has 5 peaches and Charles 4; how many have they both? How many then do five and four make?

4. George Washington served as President 8 years, and John Adams 4 years; how long did they both serve?

5. James Monroe served as President of the United States 8 years, James Madison 8 years, and J. Q. Adams 4 years; how long did the three serve? How many then do 8 and 8 and 4 make?

6. Joseph paid 3 shillings for a slate, 5 shillings for an arithmetick, and 2 shillings for paper; how much did he pay out?

7. Henry was two days in learning addition, he learned subtraction in one, multiplication in two, and division in three; how long was he going over the four rules? How many are two, one, two and three?

8. Charles has 3 notes, one of 6 dollars, one of 8 dollars and 1 of 4 dollars; what is the amount of the three?

9. A man bought 8 bushels of wheat for 12 dollars, and eight bushels of corn for 6 dollars; how much did he pay for both? How many are 12 and 6?

10. A farmer paid 10 dollars for a plough, and five for a harrow; what did he pay for both?

Exercises to be performed on the slate.

1. Find the amount of 6 and 8.

6 units
8 units
———
14 Amount.

Here we find the amount by saying 8 and 6 are 14. By adding 6 directly to eight we mount several steps at once, thereby saving the trouble of adding unity to 8 six times, or of adding unity to 6 eight times; for in both cases the amount is 14.

6
8
—
14 Amount.

Proof. In proving the work we commence adding at the top, by saying 6 and 8 are 14. Here we find the last amount to equal the first; consequently the work is right according to our method of proof; and it is evident that it can make no difference whether we say 8 and 6 are 14, or 6 and 8 are 14.

Addition is sometimes proved by crossing off the top line and adding the remaining lines; and if this last amount added to the top line, equal the first amount, the work is right; for this simple reason, that the sum of the parts must equal the whole, in whatever order they are put together.

2. Lent a friend at one time 14 dollars, at another 45, at another 50, and at another 66; how much is he in my debt?

tens. units.
1 4
4 5
5 0
6 6
———
1 7 5 D.
Amount.

We first say 6 and 5 are 11, and 4 are 15, which is the amount of the first column; we then set down 5, the right hand figure of the amount, directly under the column added; and add 1, the left hand figure, to the first figure of the next column; by saying 1 and 6 are 7, and 5 are 12, and 4 are 16, and 1 are 17, setting down the whole amount, it being the left hand column.

Demonstration.—Setting down the right hand figure, and adding the left to the next column, is what is called carrying the tens and setting down the units, for in adding the first column, we find the amount to be 15, which is 1 ten and 5 units; therefore we write down the 5 and add the 1 (which is 1 ten) to the column of tens.—The reason of this is plain, when we recollect, that in all simple numbers; *ten* at the right is equal to *one* next at the left.

Note.—We do not mention the cipher in adding; because it has no value in itself, and it would be useless to say 6 and 0 (a cipher) are 6.

Thou. Hun. Tens. Units.			
1634	8963	34678	38621
3687	6324	86123	27313
8093	8910	97638	96789
6817	861	10123	46382
1121	67	26346	99168
21352			

7. A lady being asked how old she was, answered, she was 19 years old on the day of her wedding, and had been married 7 years; how old was she? Ans. 26 years.

8. The lesser of two numbers is 1600; their difference is 179; what is the greater number? Ans. 1779.

9. George Washington was born A. D. 1732, and lived 67 years; in what year did he die? Ans. 1799.

10. America was discovered by Columbus 1492 years after the birth of our Saviour, and the deluge happened 2348 before his birth; how many years intervened? Ans. 3840 years.

11. According to the census taken in 1820, the number of inhabitants in the several New England states was as follows: Maine 298,335; New Hampshire 244,161; Vermont 235,764; Massachusetts 523,287; Rhode Island 83,059; Connecticut 275,248; how many inhabitants did New England at that time number? Ans. 1,659,854.

12. A butcher bought four fat oxen; the first ox slaughtered weighed 1045 pounds, the second 20 pounds more than the first, the third 40 pounds more than the second, and the fourth 55 pounds more than the first; what was the weight of the four oxen? Ans. 4315 pounds.

13. A father having only two children, in making his will gave his daughter Maria three thousand dollars, and his son William fifteen hundred dollars more than Maria; what was William's portion, and what was the amount of the whole estate?

Ans. { William's portion $4,500.
Whole estate, 7,500.

14. Four lads counting their oranges, found that Joseph

had 80, Jacob 50 more than Joseph, George had 99, and Henry 2 more than George: how many had they all.

Ans. 410.

15. A gentleman paid four hundred dollars for a span of fine horses, three hundred for a carriage, and eighty for a set of harness; what did they all cost him? Ans. $780.

16. A merchant settling his accounts finds that he owes A, 80 dollars, B, $120, C, $150, and D, $480; how much does he owe in all? Ans. $830.

17. Maps and Globes were invented by Anaximander, 600 years before the birth of our Saviour; how long have they been in use up to the year 1831? Ans. 2431.

ADDITION OF DECIMALS.

Decimal or *Federal* money, (which is the coin of the United States,) increases in a ten-fold proportion, and in simplicity is next to whole numbers. The rules, which you observed in adding whole numbers, are applicable here.

TABLE.

10 mills	make	1 cent,	marked	c.
10 cents	....	1 dime,	....	d.
10 dimes	....	1 dollar,	....	$.
10 dollars	....	1 eagle,	....	E.

RULE.

Place dollars under dollars, cents under cents, and mills under mills, (remembering to place a comma or separatrix directly after dollars,) and add the same as in whole numbers; then point off from the right hand of the amount, as many figures for decimals as are equal to the greatest number of decimal parts in any of the given numbers. Or place the separatrix directly below the decimal point in the numbers added. We usually express eagles and dollars, in dollars; and dimes, cents, and mills, in cents and mills; thus, 4 eagles, 6 dollars, 5 dimes, 6 cents and 2 mills, may be expressed 46 dollars 56 cents and 2 mills; or thus, $46, 56cts. 2m.

When you write dollars and cents only, if the cents be less than ten, place a cipher between the cents and dollars, thus, $4 and 6 cents must be written $4,06 cts.

A dollar is unity; dimes, cents and mills are, as the table of decimals expresses, tenths, hundredths, and thousandths; thus, $1, 1 dime, 6 cents and 3 mills, may be expressed $1 and 163 thousandths of a dollar, or $1, 16 cts. 3 mills.

EXAMPLES.

1.

Hun.,	Tens,	Units,	Tenths,	Hundredths,	Thousandths,
$			d.	c.	m.
2	6	3,	6	3	7
	1	6,	1	8	4
1	6	3,	8	1	2
6	4	0,	8	0	1
$10	8	4,	4	3	4

2.

$	cts.	m.
6 5,	1 7	3
1 8,	0 2	1
1 6,	1 1	6
8 1,	6 8	7

3.

$	cts.	m.
6,	8 4	8
8,	9 5	6
7,	1 6	3
3,	6 3	4

Ten in an inferiour denomination being equal to one in the next superiour, it is plain that we may add the same as in whole numbers.

4. Add 1 dollar, 65 cents and 3 mills, 5 dollars, 65 cents and 8 mills, 20 dollars, 8 cents and 4 mills, 14 dollars and 1 mill.

$	cts,	m.
1,	6 5	3
5,	6 5	8
2 0,	0 8	4
1 4,	0 0	1
$4 1,	3 9,	6

Care should be taken to supply ciphers in the place of vacant denominations, thus, 14 dollars and 1 mill must be written $14,00 cts. 1 mill.

Ans. Expressed in words, forty-one dollars, thirty-nine cents, 6 mills.

NOTE.—Ciphers at the right hand of decimals, or in the place of decimals, do not alter the value, therefore they may be omitted; thus, $5,00, is the same as $5.

5. Find the sum of 39 cents, 4 dimes, 5 dimes, 38 cents, 40 cents, 3 mills, 80 cents, and 2 dimes.

cts.	m.
,3 9	
,4 0	
,5 0	
,3 8	
,4 0	
,0 0	3
,8 0	
,2 0	
$3, 0 7	3

The student will perceive that every dime is ten cents, consequently 2 dimes make 20 cents; 3 dimes, 30 cents; 4 dimes 40 cents; &c. So for 2 dimes we write 20 cents, for 3 dimes, 30 cents, and for 4 dimes, 40 cents, &c.

6. What is the sum of 180 dols. 1 ct., 136 dols. 40 cts., 300 dols. 10 cts., 100 dols. 1 ct. 1 mill. 4 dols. and 2 mills?

Ans. $720,52,3

7. A man has 5 notes, viz.: one of $36,75 cts., one of

$84,25 cts., one of $40,80 cts., one of $101,90 cts., and one of $40,11 cts.; what is the amount of the notes?

Ans. $303,81 cts.

8. I received of A, B, and C, the following sums: A paid me $140,50 cents, B $500,58 cents, C $1000; can you tell me how much I received from the three?

Ans. $1641,08 cts.

9. A paid me $300, B paid me $400, and C paid me as much as A and B both, what did they all pay me, and what did I receive from C?

They all paid me $1400.
I received of C $ 700.

10. A man has four farms, one is worth $2000, one $2560, one $1206, and one $5600; what is the worth of the four?

Ans. $11366.

NOTE.—Where half (½) cents occur, it is evident that every two halves make a whole; you will then add one to the cents for every two halves; thus in the eleventh sum, we have 3 halves, that is, one cent and a half, so you will write down the half and add one to the cents.—And it is also evident, where a half is given to find the whole, that the half twice repeated or added will equal the whole; one third (⅓) three times repeated will equal the whole; and one fourth (¼) four times repeated will equal the whole.

11. Bought 5 gallons of molasses for $2,50 cts., 2 pounds of coffee for 37½ cts., 2 skeins of silk for 12½ cts., 1 pound of tea for 37½ cts.; how much did the whole cost me?

Ans. $3,37½ cts.

12. One half of a vessel is worth fifteen hundred dollars; what is the whole worth? Ans. $3000.

13. One third of a man's estate is in land which is worth $2000; what is his whole estate? Ans. $6000.

14. If a man receive one thousand dollars for one quarter (¼) of his property; what should he receive for the whole?

Ans. $4000.

QUESTIONS ON SIMPLE ADDITION.

What is simple addition? A. Collecting several numbers in one. What is the number called that arises from the operation of the work? A. Sum or amount. How do you place your numbers for adding? A. Units under units, tens under tens, &c. Where do you commence adding? A. At the right hand. How do you proceed in the work? A. Add the right hand column, and write down the right hand figure of the amount under the column added, and add the left to the first figure of the next column; and so proceed through all the columns, remembering to set down the whole amount of the left hand column.

How do you prove addition? A. By adding the columns downwards. Why should that prove it? A. It is putting together the same numbers that composed the first amount, only in a different order. How do you write down Decimal or Federal money for adding? A. Dollars under dollars, cents under cents, and mills under mills. After your numbers are written down, how do you proceed in adding? A. The same as in whole numbers. Why do you add the same? A. Because one in a superiour column, is equal to ten in the next inferiour column, the same as in whole numbers. In federal money what are the figures called at the left hand of the separatrix? A. Dollars.— What are the two next at the right called? A. Cents. What is the next at the right called? A. Mills. If a denomination be wanting what do you write in its place? A. A cipher or ciphers to fill the vacant places. Do ciphers at the right hand of a decimal alter its value? A. They do not. When you have the value of a half given, how do you find the value of the whole? A. Add the given sum to itself, and the amount will be the value of the whole. If you have the value of one third given, how do you find the value of the whole? A. Set down the value of the third three times, and add; the sum will be the value of the whole

SIMPLE SUBTRACTION,

Is taking a *less* number from a *greater*. The *greater* number is called the *Minuend*, and the *less* the *Subtrahend*. The *number* produced from the operation of the work is called the *remainder* or *difference*.

Subtraction being exactly the reverse of addition, your knowledge of that rule will be of great service in this; you have already learned by addition that 4 and 3 make 7; and now you will readily perceive that if either be taken away from 7, the other will remain. You also understand that addition is collecting numbers together; you must now learn that subtraction is taking them apart.

RULE.—Place units under units, tens under tens, &c. as in addition, with the greater number above. Draw a line under them.— Begin at the right hand, and subtract the units of the subtrahend from those of the minuend, and write the difference directly below. Proceed in like manner with the remaining figures at the left. But if a figure in the subtrahend exceed that above it in the minuend, you must add 10 to the upper figure, and from the amount take the figure below; remembering to add 1 to the next figure of the subtrahend; this is adding equals to both the given numbers, for the convenience of work; and adding equals to both numbers, their difference must ever remain the same.

Subtraction is denoted by a horizontal line; thus, 6−3=3 signifies that the difference between 6 and 3 is 3.

PROOF. 1st Method. Add the *difference* between the given numbers to the subtrahend, and if the *amount* equal the *minuend*, the work is right. 2nd. Method. Or you may subtract the *difference* from the *minuend*, and if the *remainder* equal the *subtrahend*, the work is right.

SUBTRACTION TABLE.

2	from	2	leaves	0	4	from	4	leaves	0	6	„	10	„	4
2	„	3	„	1	4	„	5	„	1	6	„	11	„	5
2	„	4	„	2	4	„	6	„	2	6	„	12	„	6
2	„	5	„	3	4	„	7	„	3	7	from	7	leaves	0
2	„	6	„	4	4	„	8	„	4	7	„	8	„	1
2	„	7	„	5	4	„	9	„	5	7	„	9	„	2
2	„	8	„	6	4	„	10	„	6	7	„	10	„	3
2	„	9	„	7	4	„	11	„	7	7	„	11	„	4
2	„	10	„	8	4	„	12	„	8	7	„	12	„	5
2	„	11	„	9	5	from	5	leaves	0	8	from	8	leaves	0
2	„	12	„	10	5	„	6	„	1	8	„	9	„	1
3	from	3	leaves	0	5	„	7	„	2	8	„	10	„	2
3	„	4	„	1	5	„	8	„	3	8	„	11	„	3
3	„	5	„	2	5	„	9	„	4	8	„	12	„	4
3	„	6	„	3	5	„	10	„	5	9	from	9	leaves	0
3	„	7	„	4	5	„	11	„	6	9	„	10	„	1
3	„	8	„	5	5	„	12	„	7	9	„	11	„	2
3	„	9	„	6	6	from	6	leaves	0	9	„	12	„	3
3	„	10	„	7	6	„	7	„	1	10	from	10	leaves	0
3	„	11	„	8	6	„	8	„	2	10	„	11	„	1
3	„	12	„	9	6	„	9	„	3	10	„	12	„	2

The student should be required mentally to answer the following questions. If there be a class, let each student answer his question directly after reading it.

2 from 4 leaves how many?
6 ---- 8 ---- how many?
8 ---- 12 ---- how many?
4 ---- 9 ---- how many?
5 ---- 8 ---- how many?
3 ---- 11 ---- how many?

1. John having 9 cents, paid 6 cents for apples; how many had he left?

2. George having 11 oranges, gave 4 to his mother; how many had he left?

3. A man paid 15 dollars for a cow, and sold her for 20; how much did he gain by the trade?

4. Charles bought a knife for 15 cents, but soon became sick of his bargain, and sold it for 12 cents; how much did he lose?

5. Charles gave Henry 24 cents, George gave him 20; how many did Charles give more than George? How many are left after taking 20 from 24?

6. Joseph has 16 peaches, Henry 4, and George 24; how many has George more than the other two?

7. Henry bought an English Reader for 30 cents, and a writing book for 20; but George offers for his writing book what his English Reader cost him; how much will Henry gain by selling?

8. Charles carried 24 eggs to market, and sold them for 8 oranges; how many more eggs had he than oranges?

1. *Examples to be performed on the Slate.*

From 4 6 Minuend.
Take 3 2 Subtrahend.
——
1 4 Remainder or difference.

DEMONSTRATION:—Commencing at the right hand as the rule directs, we say 2 from 6 leaves 4, placing it directly under; because it is plain if 2 units be taken from 6 units, there will remain 4 units. Then we say 3 from 4 leaves 1, placing it directly under; for it is evident, if 3 tens be taken from 4 tens, there will remain 1 ten; consequently we have 14 for the difference between 46 and 32.

PROOF. According to the 1st method.

4 6
3 2
——
1 4
——
4 6

DEMONSTRATION:—Nothing can be more plain, than the proof of this rule. Because it is clear, that it can take no more than the difference between two numbers, to make the less equal to the greater. Consequently, when we add 14, the difference between 46 and 32, to 32 the less, we find the amount to be 46, a number equal to the greater.

PROOF. According to the 2nd method.

4 6
3 2
——
1 4
——
3 2

DEMONSTRATION.—You already understand, that the difference between two numbers when added to the less, gives a sum equal to the greater, and you will now readily perceive, that if the difference between two numbers be taken from the greater, it must leave a number equal to the less. Because it can take no more than the difference between two numbers, when taken from the greater, to diminish it, so that it shall be equal to the less; for you perceive, when we take 14 from the minuend, it is reduced to 32, a number equal to the subtrahend. Consequently subtraction may be made to prove itself.

NOTE.—We might very properly have offered subtraction, as a proof of addition, had you been acquainted with the rule. It is a saying of the school boy, that it is a poor rule that will not work both ways. We have already proved subtraction by addition; then to make the rule good according to the test of the school boy, we will prove addition by subtraction. Let 46 and 32 be added, the amount is 78; now it is plain, if the numbers which compose this amount, be taken away, nothing will be left; thus, if from 78 you subtract 46, the first number, 32, will remain; and if 32, the other number, be taken away from 32 the remainder, nothing will remain. Now you may treasure up in your mind this fact; that if you subtract from the amount of any sum the several numbers that compose the amount, you destroy it or reduce it to nothing.

2. From 4 2
Take 2 4
——
1 8

In this example, we find that the 4 units of the subtrahend cannot be taken from the 2 in the minuend; therefore we add 10 to the 2, which increases it to 12; then we say, 4 from 12 leaves 8, placing it under. Then we add 1 to the 2, the next figure of the subtrahend, which increases it to 3; we then say, 3 from 4 leaves 1, placing it directly under, which leaves 18 for the difference of the 2 given numbers.

DEMONSTRATION.—The reason of this operation is plain when we recollect that the 1 ten which we add to the subtrahend is equal to the 10 units which we add to the minuend; because one in a superiour column is equal to 10 in an inferiour column; and adding equal sums to two numbers, or subtracting equal sums, the difference between the two numbers must ever remain the same; thus,

1 5+1 0=2 5—6=1 9
1 2+1 0=2 2—6=1 6
—— —— ——
3 3 3

NOTE.—Adding 10 to a figure in the minuend before we can subtract, and then one to the next left hand figure of the subtrahend, is by some called *borrowing*.

3. Again, to show the principles of this rule in a different, though, if possible, in a clearer light, we introduce the following example.

(3)

From 5 4
Take 2 6
——
2 8 difference.

DEMONSTRATION.—When no figure in the subtrahend is greater than that directly above it in the minuend, the student finds no difficulty. And even when a figure in the subtrahend is greater than that directly above it in the minuend, the difficulty vanishes, when he properly understands the local value of numbers. In this example our minuend (54) consists of 5 tens and 4 units; and our subtrahend (26) consists of 2 tens and 6 units. Now to take the 6 units of the subtrahend from the 4 units of the minuend is impossible; yet it is evident that the minuend (54) is greater than the subtrahend (26;) therefore, we will resolve our minuend, 5 tens and 4 units, into 4 tens and 14 units, which can be evidently done. And now we can take the 6 units of the subtrahend from the 14 units of the minuend, and have 8 units remain; we then take the 2 tens of the subtrahend from the 4 tens of the minuend, and have 2 tens left; then our remainder or difference is 2 tens and 8 units, or 28, which is the same.

Although further examples may seem unnecessary to illustrate this rule; yet as the following example includes every variety in subtraction, it may not be thought amiss to introduce it here.

(4)

From 3 2 0 5 0 4 1
Take 7 8 7 0 0 2

2 4 1 8 0 3 9 Rem.

We begin at the right hand as our rule directs; and since we cannot take 2 from 1, we add 10 to the 1, which makes 11, and then say, 2 from 11 leaves 9, placing it directly under. We then add 1 to the next left hand figure of the subtrahend (for reasons already given under the 2d example) which is a cipher, (0,) and say 1 from 4 leaves 3, placing it under. Next we say take a cipher (0) from a (0) and a cipher (0) remains. We next say, take 7 from 5 we cannot, but according to our rule, we add 10 to the 5 and then say, 7 from 15 leaves 8, placing it directly under. Next, according to our rule, (which has been explained,) we add 1 to the 8, which makes 9, and then say, 9 from a cipher (0,) we cannot; but when we add 10 to the cipher (0,) we say 9 from 10 leaves 1, placing it under. We now add 1 to the 7, which makes 8, and say, 8 from 2 we cannot subtract; we then add 10 to the 2, which increases it to 12, we then say 8 from 12 leaves 4, placing it under. Lastly, we say 1 from 3 leaves 2, placing it under. We obtain this 1 in the vacant place of the subtrahend at the left hand, on account of our having added 10 to the next right hand figure of the minuend.

5. What is the difference between 478 and 320? Ans. 158.

6. If 1 be taken from a 1000; what will remain? Ans. 999.

7. If one be subtracted from a million; what will remain? Ans. 999,999.

8. What is the difference between one million and ten thousand? Ans. 990,000.

9. At the census taken in 1810, the number of inhabitants in the six N. England states, was 1,471,973; and at the census taken in 1820, the number of inhabitants was 1,659,854;

what was the increase of population in New England between 1810 and 1820? Ans. 187,881 increase.

10. At the census in 1810, the number of inhabitants in the State of New-York was 958,066; and on taking the census in 1820, the number of inhabitants had increased to 1,372,812; then what was the increase between 1810 and 1820? Ans. 414,746 increase.

11. According to the two preceding sums, how much more was the increase of population in the State of New-York, than in the six New England States between 1810 and 1820? Ans. 226,865.

12. The number of square miles in New England is 65,047, and the number of square miles in New-York is 46,000; then how many more square miles are there in New England than in New-York? Ans. 19,047.

13. Newspapers were first printed at Paris in 1631; how many years since, up to the year 1829? Ans. 198 years.

14. Bought 2000 yards of shirting for 466 dollars, and sold 1476 yards for 369 dollars; how many yards have I left, and how many dollars do I want to make up the first cost? Ans. I have 524 yards, and want $97.

15. What number must be subtracted from 2081, that the remainder may be 1104? Ans. 977.

16. America was discovered by Christopher Columbus in 1492; how many years have since elapsed up to the year 1829? Ans. 337 years.

17. Union College was incorporated in 1794; how many years since?

18. Yale College at New-Haven was incorporated in 1700; how many years since?

19. Gen. George Washington died in 1799, aged 67 years; in what year was he born? Ans. 1732.

20. How long since the Declaration of Independence which was declared in 1776—reckoning to the year 1829?

Ans. 53

SUBTRACTION OF DECIMALS.

Subtraction of decimal or federal money, is exactly the same in the operation of the work, as subtraction of whole numbers. Because ten in an inferiour denomination, is equal to one in the next superiour, through all the denominations. Consequently the same proportion exists as in whole numbers.

RULE.—Place whole numbers the same as if no decimals were annexed: and decimals, so that tenths stand under tenths, hundredths under hundredths, and thousandths under thousandths, &c. Or, which is the same thing, place dollars under dollars, cents under cents, and mills under mills; so that the separatrix in the subtrahend shall stand directly under that of the minuend; and care must be taken to place the separatrix in the result directly under the separatrix in the given numbers.

Here, as in addition of decimals, a dollar is a unit; and dimes, cents, and mills, according to their order, are tenths, hundredths, and thousandths of a dollar; eagles and dollars are expressed in dollars; dimes, cents and mills are expressed in cents and mills; thus, 4 eagles, 8 dollars, 6 dimes, 8 cents, and 3 mills, are expressed 48 dollars, 68 cents and 3 mills, or $48, 68 cts. 3 m.

A dollar being a unit, a separatrix must always be placed directly after dollars when any of the inferiour denominations follow; thus, five dollars and sixty-eight cents must be expressed in figures $5,68 cts.

NOTE.—Care must be taken to place ciphers in the place of vacant denominations, whenever they occur in either of the given sums, so that the significant figures may stand in their proper places; thus, you should write 1 dollar and 5 mills, $1,00, cts. 5 m. and a separatrix may be placed between cents and mills, as well as between dollars and cents, for the ease of expressing numbers.

EXAMPLES.

1. From 4 dollars and 68 cents, take 3 dollars and 20 cents.

$ *cts.*
4, 6 8
3, 2 0

$1, 4 8 Ans.

2. What is the difference between 84 dollars, 91 cents and 6 mills, and 45 dollars, 64 cents and 2 mills?

$ *cts.* *m.*
8 4, 9 1, 6
4 5, 6 4, 2

$3 9, 2 7, 4 Ans.

3. From 60 dollars, 80 cents and 5 mills, take 1 dollar and 1 mill.

$ *cts.* *m.*
6 0, 8 0, 5
1, 0 0, 1

$5 9, 8 0, 4 Ans.

4. From 1 dollar, take 80 cents and 5 mills.

$ *cts.* *m.*
1, 0 0, 0
, 8 0, 5

$0, 1 9, 5

5. A miller bought a quantity of corn, for which he paid

$480; he afterwards sold it for 580 dollars, 68 cents and 5 mills; what did he gain by the sale? Ans. $100, 68 cts. 5m.

6. A merchant borrowed $600, for which he gave his note; he paid at one time $240, at another $150; how much remains due on the note? Ans. $210.

2. Joseph bought 10 oranges for 30 cents, he sold 6 for 24 cents; how many oranges has he left, and how many cents does he want to make up the first cost?
Ans. he has 4 oranges left, and wants 6 cents.

8. From one dollar, take one dime and one mill.
Ans. ,89 cts. 9 m.

9. From two dollars, take two dimes. Ans. $1,80 cts.

10. From two dimes, take two cents. Ans. ,18 cts.

11. From seven dollars, take six dimes. Ans. $6,40 cts.

12. From eight dollars and eight dimes, take one dime and one mill. Ans. $8,69 cts. 9 m.

13. From ten dollars, take one dollar, fifty cents and four mills. Ans. $8,49 cts. 6 m.

14. From forty dollars, take forty cents and five mills.
Ans. $39,59 cts. 5 m.

15. If one mill be taken from a thousand dollars, what will remain? Ans. $999,99 cts. 9 m.

16. A gentleman found four bags of money to the amount of $500; the first bag contained $95, the second $130, and the third $101; I wish to know what the fourth contained?
Ans. $174.

17. A clergyman's salary is $1500 a year, and he spends $800,50 cts.; how much of his salary does he lay up?
Ans. $699,50 cts.

18. A gentleman bought a horse for $100, which proving unsound, he is willing to lose $12,37½ cts.; how must he sell the horse? Ans. $87,62½ cts.

19. A merchant bought a piece of broadcloth for $205, which proved to be damaged, so that he sold it for $175,12½ cents; how much did the merchant lose? Ans. $29,87½ cts.

20. On a note of $105,37½ cts. if you received at one time $42,20 cts. at another $37 84 cts. what remains due?
Ans. $25, 33½ cts.

QUESTIONS ON SIMPLE SUBTRACTION.

NOTE.—The teacher should not wait for the student to go over the rule with all the examples before the questions are asked that are nexed to each rule. When the student commences a rule, the teac. should commence questioning him, and explain the nature and princi ples of the rule, requiring the student to give reasons for all the operations of his work.

What is simple subtraction? A. Taking a less number from a greater. What are the two given numbers called? A. Minuend and subtrahend. What is the number called that arises from the operation of the work? A. Difference or remainder. What does the difference or remainder show? A. It shows how much the minuend is greater than the subtrahend. What does minuend signify? A. A number to be lessened by another. What does subtrahend mean? A. A number to be taken from a greater. How do you place the given numbers in subtraction? A. So that units stand under units, tens under tens, &c. How do you proceed in the work? A. Commence at the right hand, and take the figures in the subtrahend from those directly above them in the minuend, placing the difference in each place directly under. When the lower figure in any place exceeds that directly above it, what must be done? A. Add ten to the upper figure, and take the lower figure from the amount, placing the difference directly under, and then add one to the next lower figure of the subtrahend. Why does not this adding ten to the figure in the minuend and then one to the next lower figure of the subtrahend, alter the true difference between the given numbers? A. Because it is adding equals to both the given numbers, and adding equals to both, the difference between two numbers must ever remain the same; and we add equals to both, because ten in an inferiour column, (which we add to the minuend,) is only equal to the one which we add to the next superiour column of the subtrahend.—How do you prove subtraction? A. By adding the difference to the subtrahend, and if the amount equals the minuend, the work is right; because it is evident, that it can take no more than the difference between two numbers to make the less equal to the greater. Can subtraction be made to prove itself? A. Yes. How is that done? A. By subtracting the difference from the minuend; and if it leave a number equal to the subtrahend the work is right, because it is evident, that if the difference between two numbers be taken from the greater, it reduces it equal to the less.

What is subtraction of decimals? A. Taking something that is *less* from that which is *greater*, where one or both of the given sums contain parts of an integer. How do you place decimals for subtracting? A. Tenths under tenths, hundredths under hundredths; or in federal money, which is the same, dimes under dimes, cents under cents, &c. What are integers in federal money? A. Dollars. What are dimes? A. Tenths. What are cents? A. Hundredths. What are mills? A. Thousandths. What is the decimal expression called when dimes, cents and mills are taken together? A. Thousandths of a dollar. How do you subtract decimals? A. The same as in whole numbers. Why do you subtract the same? A. Because ten in an inferiour column or denomination, is equal to one in the next superiour, the same as in whole numbers. Why do we call dimes and cents, cents only? A. We call them all cents, because the left hand figure in cents expresses tens in cents, which is the same as dimes. When your subtrahend contains denominations which are not named in the minuend, what do you do? A. Supply the vacant denominations of the minuend with ciphers, then place the subtrahend under, so that cents may occupy the place of cents, and mills the place of mills. If you are required to subtract dimes from dollars, what do you first do? A. Join two ciphers to the right hand of the minuend, placing a separatrix between them and dollars; then place your dimes under the left hand cipher joined to the minuend, and to the dimes join a cipher, and they will stand as cents, and then subtract. How do you point off in subtraction of decimals? A. Place the separatrix in the result, directly below the separatrix in the given numbers. Why is federal money introduced under the head of decimals? A. Because dimes, cents and mills form a decimal expression of which a dollar is the integer.

SIMPLE MULTIPLICATION,

Is repeating one of two numbers as often as the other contains a unit; or, it is the shortest method of performing addition, where the same number is to be repeated a given number of times. The two given numbers are called *multiplicand* and *multiplier*. The *multiplicand* is the number to be repeated. The *multiplier* is the repeater, or number by which you multiply. The number produced by the operation of the work, is called the product. This is the most useful rule in practical arithmetick. When the price of one is given, by this rule, we obtain the price of any number, or quantity; when length and breadth are given, by it, we find the area or surface; in reduction, it affords the greatest facility in reducing higher denominations to lower, and its principles are advantageously applied in all practical business of buying and selling.

NOTE.—The multiplicand and multiplier taken together, are called factors or substitutes.

Multiplication is denoted by this character, ×; thus, 6×3=18, which signifies, that the product of 6 multiplied by 3, is 18.

NOTE.—No pains should be spared by the student in making himself master of the following table. The task is easy if persevered in; but when the student suffers himself to pass over it superficially, no time afterwards spent in work is scarcely sufficient to make it familiar to his mind. A good knowledge of it will greatly facilitate his progress, and save him the trouble of repeatedly reviewing the same work to detect mistakes.

MULTIPLICATION TABLE.

2 times 1 are 2
2 × 2 = 4
2 „ 3 „ 6
2 „ 4 „ 8
2 „ 5 „ 10
2 „ 6 „ 12
2 „ 7 „ 14
2 „ 8 „ 16
2 „ 9 „ 18
2 „ 10 „ 20
2 „ 11 „ 22
2 „ 12 „ 24

3 × 1 = 3
3 „ 2 „ 6
3 „ 3 „ 9
3 „ 4 „ 12
3 „ 5 „ 15
3 „ 6 „ 18
3 „ 7 „ 21
3 „ 8 „ 24
3 „ 9 „ 27
3 „ 10 „ 30
3 „ 11 „ 33
3 „ 12 „ 36

4 × 1 = 4
4 „ 2 „ 8
4 „ 3 „ 12
4 „ 4 „ 16
4 „ 5 „ 20
4 „ 6 „ 24
4 „ 7 „ 28
4 „ 8 „ 32
4 „ 9 „ 36
4 „ 10 „ 40
4 „ 11 „ 44
4 „ 12 „ 48

5 × 1 „ 5
5 „ 2 „ 10
5 „ 3 „ 15
5 „ 4 „ 20
5 „ 5 „ 25
5 „ 6 „ 30
5 „ 7 „ 35
5 „ 8 „ 40
5 „ 9 „ 45
5 „ 10 „ 50
5 „ 11 „ 55
5 „ 12 „ 60

6 × 1 = 6
6 „ 2 „ 12
6 „ 3 „ 18
6 „ 4 „ 24
6 „ 5 „ 30
6 „ 6 „ 36
6 „ 7 „ 42
6 „ 8 „ 48
6 „ 9 „ 54
6 „ 10 „ 60
6 „ 11 „ 66
6 „ 12 „ 72

7 × 1 = 7
7 „ 2 „ 14
7 „ 3 „ 21
7 „ 4 „ 28
7 „ 5 „ 35
7 „ 6 „ 42
7 „ 7 „ 49
7 „ 8 „ 56
7 „ 9 „ 63
7 „ 10 „ 70
7 „ 11 „ 77
7 „ 12 „ 84

8 × 1 = 8
8 „ 2 „ 16
8 „ 3 „ 24
8 „ 4 „ 32
8 „ 5 „ 40
8 „ 6 „ 48
8 „ 7 „ 56
8 „ 8 „ 64
8 „ 9 „ 72
8 „ 10 „ 80
8 „ 11 „ 88
8 „ 12 „ 96

9 × 1 = 9
9 „ 2 „ 18
9 „ 3 „ 27
9 „ 4 „ 36
9 „ 5 „ 45
9 „ 6 „ 54
9 „ 7 „ 63
9 „ 8 „ 72
9 „ 9 „ 81
9 „ 10 „ 90
9 „ 11 „ 99
9 „ 12 „ 108

10 × 1 = 10
10 „ 2 „ 20
10 „ 3 „ 30
10 „ 4 „ 40
10 „ 5 „ 50
10 „ 6 „ 60
10 „ 7 „ 70
10 „ 8 „ 80
10 „ 9 „ 90
10 „ 10 „ 100
10 „ 11 „ 110
10 „ 12 „ 120

11 × 1 = 11
11 „ 2 „ 22
11 „ 3 „ 33
11 „ 4 „ 44
11 „ 5 „ 55
11 „ 6 „ 66
11 „ 7 „ 77
11 „ 8 „ 88
11 „ 9 „ 99
11 „ 10 „ 110
11 „ 11 „ 121
11 „ 12 „ 132

12 × 1 = 12
12 „ 2 „ 24
12 „ 3 „ 36
12 „ 4 „ 48
12 „ 5 „ 60
12 „ 6 „ 72
12 „ 7 „ 84
12 „ 8 „ 96
12 „ 9 „ 108
12 „ 10 „ 120
12 „ 11 „ 132
12 „ 12 „ 144

The student should be required mentally to answer the following questions.

What will 3 apples come to, at 3 cents each?

What will 8 bushels of wheat cost, at 8 shillings a bushel?

8 times 8 are how many?

What will 10 bushels of oats come to, at 2 shillings a bushel?

What are 9 yards of calico worth, at 3 shillings a yard? 3 times 9 are how many?

What must you pay for 12 cows, at 12 dollars each?

What will 4 cows come to, at 10 dollars each? What will 5 cows? what will 6 cows? what will 7 cows? what will 8 cows? what will 10 cows?

What will 11 ploughs cost, at 11 dollars each?

What will 9 hogs cost, at 9 dollars each?

A pound contains 20 shillings; how many shillings in 2 pounds? in 3 pounds? in 4 pounds? in 5 pounds? How many are twice 20? how many are 3 times 20? 4 times 20? 5 times 20?

Twelve pence make 1 shilling; how many pence in 2 shillings? in 3 shillings? in 4 shillings? in 5 shillings? in 6 shillings? in 7 shillings? in 8 shillings? in 9 shillings? in 10 shillings? in 11 shillings? in 12 shillings? How many are twice 12? 3 times 12? 4 times 12? 5 times 12? 6 times 12? 7 times 12? 8 times 12? 9 times 12? 10 times 12? 11 times 12? 12 times 12?

CASE I.—*When the multiplier is a single, significant figure.*

RULE.—Place the multiplier under the right hand figure of the multiplicand, and draw a line underneath. First, multiply the right hand figure of the multiplicand by the multiplier; and when the product does not exceed 9, place it directly under; but if the product exceed 9, place down the right hand figure of the product; and add the left to the product of the next figure of the multiplicand, and so proceed, till you have multiplied all the figures of the multiplicand by the multiplier; remembering to set down the whole product of the left hand figure.

METHODS OF PROOF.—1st. Make the multiplicand a multiplier, and if it produce the same result, the work is right.

2nd. Multiplication may be proved by addition.

Write the multiplicand down as many times as the multiplier expresses a unit; then add, and if the same result be produced, the work is right.

3d. Multiplication may be proved by subtraction.

From the product, subtract the multiplicand as many times, as the multiplier expresses a unit; and if it diminish it to nothing, the work is right.

4th. Lastly, multiplication may be proved by division.

Divide the product by either of the factors, and if the operation produce the other, the work is right. Although this is the best method of proof, yet it should be omitted, till the student has become acquainted with division.

Other methods of proof might be given; but these are introduced on account of their being the most simple, and best suited to our purpose in illustrating the principles of this rule; and showing the relation which it bears to the other simple rules.

EXAMPLES.

1. Multiply 8 by 4.

Multiplicand, 8
Multiplier, 4
—
Product, 32

We place the multiplier under the multiplicand, as our rule directs. Then we say 4 times 8 are 32, placing the product under; we then have 32 for a product, which is 8, 4 times repeated.

Demonstration.—That this is a short way of performing addition, is very evident: for we arrive at the result at once, which, in addition, requires the multiplicand 8, to be set down four times and added; thus, 8+8+8+8=32.

1st. **Example.**—Proved according to the first method.

Multiplicand, 4
Multiplier, 8
—
Product, 32

What was before the multiplicand, now becomes the multiplier. We now say 8 times 4 are 32, the same product as was produced by the first operation.

Demonstration.—It is plain, that it can make no difference, whether 8 be repeated 4 times, as, 4 times 8 are 32; or 4 repeated 8 times, as, 8 times 4 are 32, the same result is produced.

1st. **Example.**—Proved according to the 2nd method.

8
8
8
8
—
32 Amount.

Demonstration.—Here, we set down the multiplicand 4 times; because in the example, our multiplier expresses 4 units; we then add, and the result clearly demonstrates, that when the same number is repeated by multiplying, it may be repeated by adding; and consequently proves the work. But proof by addition is a more tedious method than proof by multiplication, and is therefore only introduced to show the relation between the two rules. It is also plain, that it can make no difference, whether we add the multiplicand as many times as our multiplier expresses a unit; thus, 8+8+8+8=32; or add the multiplier as many times as our multiplicand expresses a unit, thus, 4+4+4+4+4+4+4+4=32; the same result, you perceive, must be produced.

1st. **Example.**—Proved according to the 3d method.

Multiplicand, 8
Multiplier, 4
Carried up.

Dem.—You have just learned, that multiplication is a short way of performing addition. You

Brought up.

```
Product,  32
           8
          --
          24
           8
          --
          16
           8
          --
           8
           8
          --
           0
```

also understand, that subtraction is made to prove addition; consequently it may be made to prove multiplication; for subtraction is the reverse of addition, and 32, (the product in our example,) is 8, four times expressed; then if 8 be taken away from 32, (the product,) 4 times, it must evidently diminish it to nothing; because it is taking away 8, the multiplicand, as many times as it has been repeated by 4, the multiplier.

1st Example.—Proved according to our 4th method.

```
   8
   4
  --
8|32|4
  32
  --
   0
```

Demonstration.—Division being exactly the reverse of multiplication, is consequently made to prove it; because, when we multiply 8 by 4, the 8 is 4 times repeated; and when this 8 is made a divisor, we find that 32 contains it 4 times, which gives us our other factor; and repeating the 8 by this factor, it again produces 32, the same as our first product; which must always be the case; because, it is only repeating the same number a second time, and must produce the same product; and when we come to subtract this product, we can have no remainder; because, taking the same number from itself can leave no remainder.

2. What will five yards of broadcloth cost, at three dollars a yard?

```
 $
 3 Multiplicand.
 5 Multiplier.
---
$15 Ans.

Proof by Addition.   Proof by
 $                   Multiplication.
 3                        5
 3                        3
 3                       --
 3                       15
 3
--
15
```

Here, by multiplying the 3 by 5, we repeat the 3 five times; and it is evident, that five yards are worth five times as much as one. If we were not acquainted with multiplication, we would be under the necessity of setting 3 down five times, and adding as our proof shows.

From this sum, we learn, that when the price of one is given; as 1 yard, 1 pound, 1 ounce, &c. we may obtain the price of the quantity, by multiplying the price of a unit, by the quantity; for the quantity when made a multiplier is considered to be a number containing as many units as the quantity contains yards, pounds, ounces, &c.

3. What will 9 calves come to, at 3 dollars each? Ans. $27.

4. What is the worth of 9 bushels of clover seed, at 9 dollars a bushel? Ans. $81.

5. Multiply 48 by 3.

Multiplicand,	4 8
Multiplier,	3
	1 4 4

Here, we say 3 times 8 are 24, placing down 4, the right hand figure of the product, and keeping the left hand figure in mind; then we say 3 times 4 are 12, and 2 are 14; placing down the whole product, (it being the product of the left hand figure;) this is carrying by 10, or adding the left hand figure to the product of the next figure, which is the same.

	6.	7.	8.	9.
Multiplicand,	3 4 3 2	4 6 7 8 3	8 3 7 3 4 6 3	7 8 3 4
Multiplier,	3	4	5	6
	1 0 2 9 6 Ans.			

10.	11.	12.
8 3 2 3 4	8 3 4 6 3 2 1 2	9 3 4 6 7 3 4 6
7	8	9

CASE II.—*When the multiplier consists of more than one figure.*

RULE.—Place your multiplier under the right hand figures of the multiplicand, so that units shall stand under units, tens under tens, &c: Then, multiply the multiplicand by each figure of the multiplier, commencing at the right hand; and remember to commence the product of each, as you pass to the left, directly under the multiplying figure. If a cipher, or ciphers occur between the significant figures of the multiplicand, you will bring them down in the product, in their proper place, in order to give the product of the figures at the left their proper local value; but if you have any thing to carry, instead of bringing down a cipher, you set down the figure which you have to carry; then add these several products for the whole, or total product.

1. Multiply 30684 by 43.

	43
	92052
	122736
Product.	1319412

Here, when we multiply the multiplicand by 3, we repeat it 3 times. When we multiply by 4, we remove the product one figure further to the left, which gives it ten times the value it would have, placed directly under; and it should have ten time, the value; because it is not, in fact, multiplying the multiplicand by 4, but by 40, for the 4 expresses 4 tens. Had the product of the second figure of the multiplier been placed directly under the product of the first figure, it would have been repeating it 4 times; but placing it one figure to the left increases it ten times, which makes 40; because 4 times 10 are 40. If we take the multiplier apart, and multiply the multiplicand separately by the parts of the multiplier, and add the products, we will have the same result; thus,

```
30684      30684
    3         40
-----    -------
92052    1227360
           92052
         -------
         1319412
```

Here you will perceive we multiply by the parts of our first multiplier; first our multiplicand by 3, and then our multiplicand by 40, which is the local value of the left hand figure of our multiplier; and multiplying the multiplicand by 4, increases it 4 times; then bringing a cipher at the right, increases it 10 times; which evidently makes 40 times repeated. Then, when we add the products, we obtain the same result; but in our first example, giving the product of our left hand figure its local place, was the same in effect.

Had we placed the product of the left hand figure of our multiplier, directly under the product of the first, and then have added, it would have only been repeating the multiplicand 7 times; for when we multiply by 3, (the first figure of the multiplier,) we repeat the multiplicand 3 times; then if we multiply by 4, placing the product directly under the product of the first figure of the multiplier, we repeat it 4 times; and 3 times repeated, and 4 times repeated, make 7 times repeated; as the following work shows.

```
 30684    30684
    43        7
 -----   ------
 92052   214788
122736
------
214788
```

Thus, you see the same result is produced.

```
     2.             3.              4.
  467023         5678496        8320604
     127             363            643
  -------        -------        -------
 3269161
 934046
467023
--------
59311921 Ans.
```

```
     5.             6.              7.
  386304         3760045        6740432
     321             684           1364
```

8. A merchant bought 481 bales of linen; in each bale there were 36 pieces; and in each piece, 24 yards; how many pieces, and how many yards, were there?

Ans. 17316 pieces, 415584 yards.

9. Three hundred and forty seven men shared equally a prize, and each received forty-nine dollars; what was the whole prize? Ans. $17003.

10. If four bushels of wheat make a barrel of flour, and the price of wheat, one dollar a bushel; what will 375 barrels come to? Ans. $1500

Case III—*When ciphers occur between the significant figures of the multiplier, omit them in the operation.*

RULE.—Place the product of each significant figure, directly under that by which you multiply; then add the products together, a their sum will be the total product.

EXAMPLES.

1st. Multiply 384 by 203.

```
  3 8 4
  2 0 3
  -----
1 1 5 2
7 6 8
---------
7 7 9 5 2 Ans.
```

Here, when we multiply by 2, the left hand figure of the multiplier, we commence the product directly under itself, which brings it in the place of hundreds; and it should stand in the place of hundreds; for it is repeating the multiplicand by 200: the multiplying figure, (2,) standing in the place of hundreds.

```
2.
  3604
   102
  ----
  7208
3604
------
367608 Ans.
```

```
3.
    340084
      4005
   -------
   1700420
1360336
----------
1362036420 Ans.
```

```
4.
  10304
    205
  -----
  51520
20608
-------
2112320 Ans.
```

```
5.
    3 4 6 3 4 6 3 2
          1 0 3 0 2
  -----------------



  -----------------
3 5 6 8 0 5 9 7 8 8 6 4 Ans.
```

```
6.
    3 0 4 6 4 0 3
        2 0 6 0 4
  ---------------



  ---------------
6 2 7 6 8 0 8 7 4 1 2 Ans.
```

7. What is the product of 365432 multiplied by 7608? Ans. 2780206656.

8. Multiply eight thousand five hundred and sixteen, by seventy-six thousand and two. Ans. 647233032.

9. Multiply 120345 by 9004. Ans. 1083586380.

10. What is the product of 24393 multiplied by 402? Ans. 9805986

CASE IV.—*When one or both of the factors have ciphers at the right hand.*

RULE.—Multiply the significant figures the same as if there were no ciphers at the right, and add their products; then, join to the right hand of the total product of the significant figures, as many ciphers as there are at the right hand of both the factors.

EXAMPLES.

1. Multiply 4 6 3 by 4 0
4 0
1 8 5 2 0

DEM.—Here, we first multiply by 4, which repeats the multiplicand 4 times; but we wish to have our multiplicand repeated forty times, and to effect this, we bring the cipher of our multiplier at the right of the product, which gives the product of the 4 its local value, which is ten times more than its simple value; and ten times the simple value of 4, is forty; consequently we have our multiplicand forty times repeated.

2. Multiply 3460 by 4200.
3460
4200
692
1384
14532000

DEM.—Here, we first multiply our significant figures, as our rule directs. And since we multiply only by 42, the hundredth part of our multiplier, our product is only a hundredth part of what it should be; we then increase it a hundred times, by joining two ciphers at the right hand of our product. We now have repeated the significant figures of our multiplicand 4200 times; but still, our product is only one tenth of what it should be; because we have only multiplied a tenth part of our multiplicand; then to give our product its proper value, we increase it ten times, by joining another cipher at the right hand; which is for the cipher at the right of our multiplicand.

3. Multiply 21200 by 70. Ans. 1484000.

4. Multiply 340300 by 13400. Ans. 4560020000.

5. Twenty shillings make one pound; how many shillings in four hundred and fifty pounds? Ans. 9000 s.

6. In eighty pounds, how many shillings? Ans. 1600.

7. One hundred and sixty square rods make one acre; how many square rods in 320 acres? Ans. 51200 rods.

8. Six hundred and 40 acres make one square mile; how many acres in three hundred square miles?
Ans. 192000 acres.

9. Multiply 480000 by 12000. Ans. 5760000000.

CASE V.—*To multiply by* 10, 100, 1000, &c.

RULE.—Add as many ciphers to your multiplicand, as there are ciphers in the multiplier; and the multiplying is performed.

EXAMPLES.

1. Multiply 146 by 10. Ans. 1460.

Dem.—We join a cipher to the multiplicand, and our work is done, because it increases the figures of our multiplicand ten times, by giving units the place of tens; and tens the place of hundreds; and hundreds the place of thousands, &c.

2. Multiply 846 by 100. Ans. 84600.

Dem.—Here, we join two ciphers to the multiplicand; and the figures of our multiplicand are increased 100 times, because 6, the first figure of our multiplicand, after the ciphers are joined, becomes hundreds; and the next left hand figure 4, becomes thousands, one hundred times the first value.

3. Multiply 8324 by 1000. Ans. 8324000.

4. Multiply 30460 by 10000. Ans. 304600000.

Case VI.—*To multiply by* 9, 99, 999, &c.

RULE.—Join to the right of the multiplicand, as many ciphers, as your multiplier contains nines; and from the sum produced, subtract the multiplicand. Or you may simply multiply the multiplicand by the number of nines in the multiplier. The first method, frequently shortens the work.

EXAMPLES.

1. Multiply 464 by 9.

4640
464
Ans. 4176

Dem.—Joining a cipher to the right of our multiplicand, is repeating our multiplicand ten times, and by subtracting our multiplicand once, must evidently leave it nine times repeated; as appears, from the same example multiplied by 9.

4 6 4
9
Ans. 4 1 7 6

2. Multiply 4642 by 99.

464200
4642
459558

Ans. 459558.

Dem.—Here, it is plain, by joining two ciphers to the multiplicand, we increase it 100 times; and our multiplicand being once taken away, or subtracted, must leave it 99 times repeated.

3. Multiply 3472 by 999. Ans. 3468528.

4. Multiply 34672 by 9999. Ans. 346685328.

Case VII.—*To multiply by a composite number; that is, when the multiplier can be produced by the multiplying of any two figures in the multiplication table.*

RULE. Multiply the multiplicand first by one of the figures, and that product by the other, and this last product will be the total product required.

EXAMPLES.

1. Multiply 243 by 35.

```
      2 4 3
          7
    -------
    1 7 0 1
          5
    -------
Ans. 8 5 0 5
```

In this example, 7 and 5 are called the component parts of 35; because 7 times 5 are 35; and it will make no difference which we multiply by first.

Dem.—We first multiply by 7, which repeats our multiplicand 7 times; and then we multiply this product by 5, which is repeating, 5 times, a number 7 times as great as our first multiplicand, consequently, our multiplicand is 35 times repeated, because 5 times 7 are 35. Our work perhaps may appear more clear to the young student, by multiplying the same multiplicand directly by 35; thus, the same result, you perceive, is produced.

```
  2 4 3
    3 5
-------
1 2 1 5
7 2 9
-------
8 5   5
```

Note.—This method of work, will not be found of much service, in simple multiplication; but it is sometimes found very convenient in multiplying a compound quantity.

2. Multiply 480 by 36. Ans. 17280.
3. Multiply 1324 by 45. Ans. 59580.
4. Multiply 3684 by 72. Ans. 265248.
5. Multiply 3124 by 81. Ans. 253044.
6. Multiply 4638 by 96 Ans. 445248.
7. Multiply 3040 by 49. Ans. 148960.

PROMISCUOUS EXAMPLES.

1. What will a year's board come to, at $2 a week; allowing fifty-two weeks to the year? Ans. $104.

2. Suppose 80 seamen were concerned in taking a prize at sea, and on dividing their prize money, each seaman received $150; what was the amount of the prize? Ans. $12000.

3. A gentleman bought 307 horses for shipping, at the rate of $105 each; what did he pay for the whole? Ans. $32235.

4. A gentleman dividing his fortune among seven sons, found that the portion of each, was $1050; what was the gentleman's fortune? Ans. $7350.

5. There are 320 rods in a mile; now suppose the distance between the United States and Europe, to be 3000 miles;

will you tell how many rods Europe is distant from the United States? Ans. 960000.

6. How much money must be distributed among 85 persons, so that each may have 110 dollars? Ans. $9350.

7. What is the product of 980, repeated 99 times? Ans. 97020.

8. If 1781 be multiplied by 1121 what will be the product? Ans. 1996501.

9. If I pay 9 dollars for one barrel of pork; what must I pay for 50 barrels, at that rate? Ans. $450.

10. If one tun of hay cost 8 dollars; what will be the cost of 50 tuns? Ans. $400.

11. What will 20 cows cost, at $18 each? Ans. $360.

12. A dollar in New-York contains 8 shillings; how many shillings in 96 dollars? Ans. 768 shillings.

MULTIPLICATION OF
DECIMAL OR FEDERAL MONEY.

A knowledge of this money, will be found very beneficial in the transaction of business, when the price of one pound, one yard, &c. is given in dollars, cents and mills, to find the price of a quantity.

RULE.—Multiply the price of one by the number expressing the quantity, the same as in whole numbers; and remember to place a separatrix, as many figures from the right hand in the product, as it is, in the given price.

NOTE.—All figures at the left of the separatrix, will be dollars; the next two at the right will be cents; and the next mills.

1st EXAMPLE. If 1 yard of calico, cost 35 cents; what will 5 yards cost?

```
     ,3 5 cts.
        5
   --------
   $1, 7 5 cts. Ans.
```

```
      cts. ,3 5
           ,3 5
By Addition. ,3 5
           ,3 5
           ,3 5
        --------
        $ 1, 7 5
```

DEM.—It is plain, that 5 yards should cost 5 times as much as 1 yard; and by multiplying the price of 1 yard by 5, our product is five times the price of one yard, as will plainly appear by adding the price of one yard, 5 times. The reason of our pointing off as many figures for the decimal parts of a dollar, as there are deicmals in the given number, is plain; because the decimals increase in a tenfold proportion, the same as whole numbers; and when we multiply the decimal that expresses tenths, it is plain, that the left hand figure of the product is a whole number; because, ten tenths, are equal to one unit; then the left hand figure of tenths in our example, (17 tenths,) must be 1 unit; because ten tenths make a unit; and we then have 7 tenths over, which stands in the place of tenths, from our pointing.

2. What will 5 yards of broadcloth come to, at $4,75 cts. a yard? Ans. $23,75 cts.

3. What will 35 bushels of corn come to, at ,60 cts. a bushel? Ans. $21,00 cts.

4. What is the value of 57 yards of muslin, at ,37 cts. a yard? Ans. $21,09 cts,

5. What is the value of 375 acres of land, at $15,50 cts. per acre? Ans. $5812,50 cts.

6. What cost 100 bushels of barley at ,37 cents per bushel? Ans. $37,00.

7. At 2 dollars and 38 cents per ream of paper, what is the worth of 150 reams? Ans. $357.

8. What are three hundred barrels of beef worth, at $9,85 cts. per barrel? Ans. $2955.

9. What will 8 bushels of potatoes come to, at 37 cts. per bushel? Ans. $2,96 cts.

10. What will 10 cords of wood come to, at $1,25 cts. per cord? Ans. $12,50 cts.

11. What cost a farm containing 150 acres, at $24,50 cents per acre? Ans. 3675 dollars.

12. What will 9 dozen of eggs come to, at 9 cents a dozen? Ans. 81 cts.

13. Bought 20 horses for shipping, at seventy-two dollars per head; what did they all cost? Ans. $1440.

14. What are 35 bushels of apples worth, at ,30 cents per bushel? Ans. $10,50 cts.

15. What will 56 pounds of butter come to, at ,22 cts. per pound? Ans. $12,32 cts,

16. If a man spend ,75 cts. per day; what will he spend in 365 days, at that rate? Ans. $273,75 cts.

17. What will 30 sheep cost at 1 dollar and 25 cents per head? Ans. $37,50 cts.

18. What will 45 bushels of corn come to, at ,45 cents per bushel? Ans. $20,25 cts.

19. If a man receive $1,25 cts. a day, what must he receive for 48 days' labour? Ans. $60.

QUESTIONS ON SIMPLE MULTIPLICATION

What is multiplication? A. The shortest method of performing addition, when the same number is to be repeated a given number of times. What are the two given numbers called? A. Multiplicand and multiplier. What is the number called arising from the operation of the work? A. The answer or product. What is the multiplicand? A. A number to be repeated by another. What is the multiplier? A. The repeater, or number by which the multiplicand is to be repeated. What do you understand by the product? A. I understand, that it is the same as setting down the multiplicand as many times as the multiplier expresses a unit, and adding; the amount in addition, and the product in multiplication, must be the same. If you have four for a multiplier, how many times greater is the product than the multiplicand? A. Four times; because the multiplicand has been repeated four times. Could the same be performed by addition? A. It might by setting down the multiplicand as many times as the multiplier expresses a unit; and then adding, the amount would be the same as the product in multiplication; but it would be a tedious method to work by addition, especially when the multiplier is large. If you multiply a number by five, what part of the product is the multiplicand? A. A fifth part; because the multiplicand has been repeated five times to produce the product. How do you place the given numbers for work? A. The multiplier under the multiplicand, so that units stand under units, tens under tens, &c. If your multiplier consists of but one figure, how do you proceed in the work? A. Multiply the right hand figure of the multiplicand by the multiplier, and if the product should not exceed 9, place it directly under; but if the product exceed 9, set down the right hand figure of the product, and add the left to the product of the next figure, because it is of the same kind of the next at the left; and thus proceed with all the figures of the multiplicand; remembering to set down the whole product of the left hand figure. How may multiplication be proved? A. The most simple methods of proof are, by multiplication, addition, subtraction, and division. How do you prove multiplication by itself? A. By making the multiplicand, a multiplier. Why should that prove it? A. Because it is evident, that it can make no difference which of the given numbers we take for the multiplier; thus, we may say, 6 times 8 are 48; or we may make 6 the multiplicand, and say, 8 times 6 are 48; the same result is produced in either case. How do you prove multiplication by addition? A. By setting down the multiplicand as many times as the multiplier expresses a unit, and then adding; and if the amount in addition, be equal to the product in multiplication, the work has been correctly performed; because it is plain, if we take 8 for a multiplicand, and 4 for a multiplier, that the product will be 32; and if we set 8 down 4 times, and add, the amount will be 32; but by multiplication we arrive at the result with less labour. How is multiplication proved by subtraction? A. By subtracting the multiplicand from the product, as many times as the multiplier expresses a unit; and should the operation diminish the product to nothing, the work is right; for it is clear, if the multiplicand be taken away as many times as it has been repeated; it must diminish it to nothing. How is multiplication proved by division? A. By dividing the product by either of the factors; and if the quotient be equal to the other, the work is right; because division is only the reverse of multiplication, or a short way of performing subtraction; and if we divide the product by the multiplicand the quotient must equal the multiplier, which shows how often the multiplicand has been subtracted from the product; and as it can only be subtracted as often as the multiplicand has been repeated, the quotient must always equal the multiplier, leaving no remainder. When the multiplier consists of more than one figure, how do you proceed? A. First, multiply the multiplicand by the right hand figure of the multiplier; then by the next left hand figure of the multiplier; and so on, till the multiplicand has been repeated by each figure of the multiplier, placing the product of each directly under the multiplying figure; then add the several products, in the same order in which they stand, for the total product. What must be done, when ciphers occur between the significant figures of the multiplier? A. Care must be taken to commence the product of the first figure of the multiplier, in each place, directly under the figure by which we multiply. If our multiplier is 36, and instead of removing the product of the 3 one figure further to the left than the product of the 6, we place the product of the 3 directly under the product of the 6, and then add the products, what would it be the same as multiplying by? A. By 9; because we first repeat our multiplicand 6 times, and then 3 times; and not giving the product of the 3 its proper local place, the multiplicand stands 6 times and 3 times repeated; that is, when added, 9 times repeated; but had we placed the product of the 3 one figure further to the left, we would have given it its local value; the products then would have expressed the multiplicand 6 times repeated, and 30 times repeated, and when added, the multiplicand 36 times repeated. When ciphers occur at the right hand of one or both the factors, what must be done? A. The significant figures must be multiplied the same as if no ciphers were at the right; then to the right hand of the total product of the significant figures, we annex as many ciphers as are equal to the ciphers at the right of both the factors; in order to give the significant figures their local place in the product. How do you multiply by 10, 100, 1000, &c.? A. To multiply by 10, we join a cipher to the right of the multiplicand, which increases it ten times; because figures increase in a tenfold proportion, and the cipher removes the place of each one figure further to the left; the figure which first expressed units now expresses tens, and the rest at the left increase in the same proportion; and when we join two ciphers, the unit figure becomes hundreds; and the figure in the place of hundreds expresses 100 times its simple value; and when we join three ciphers, the unit figure becomes thousands, that is, one thousand times its simple value; and the figures at the left, being in a like manner removed to the left, increase in the same proportion. Can you multiply by 9, 99, 999, &c. in any other way than by directly multiplying the multiplicand by the number of nines? A. Yes, we may join as many ciphers to the right of our multiplicand as our multiplier has nines, and from the sum thus produced, subtract the multiplicand; because by joining one

ipher to the multiplicand, we repeat it 10 times; and then subtract-
ıg the multiplicand once, leaves it 9 times repeated; and if our mul-
plier is 99 we join two ciphers which increases our multiplicand 100
mes; then subtracting the multiplicand once, leaves it 99 times re-
eated; and we observe the same rule, in multiplying by 3 nines, or
ny number of nines. How do you proceed to multiply by a compo-
te number? A. First multiply the multiplicand by one of the com-
onent parts, and then that product by the other; and the last pro-
uct will be the total product. When is a number said to be a compo-
te number? A. When it is produced by multiplying together any two
gures in the multiplication table. Is thirty-five a composite number?
. Yes. What are its component parts? A. 7 and 5. Why? A.
ause 7 times 5 are 35. In multiplying by the component parts of
, why should it produce the same product, as multiplying directly by
? A. Because when we multiply by 7, we repeat our multiplicand
times; and when we multiply this product by 5, we repeat a num-
r 5 times, which is 7 times greater than our first multiplicand; and
times 7 times must make 35 times repeated.—What is multiplica-
on of decimal, or federal money? A. It is repeating the parts of an
teger, or integers and parts of integers, a given number of times.—
hat are integers in federal money? A. Dollars. What are the de-
mal parts of an integer? A. Dimes, cents and mills, or cents and
ills, as they are commonly expressed. What do you understand by
decimal? A. I understand, that it is something less than a unit; in
lling tenths, I understand that it takes 10 for a unit; and hun-
edths, that it takes 100 for a unit; and thousandths, that it takes
00 to equal a unit; so that the name of each figure, shows the ex-
t number required to equal a unit. How do you multiply decimal
oney? A. The same as whole numbers. How do you distinguish
cimals from whole numbers? A. By a separatrix placed between
em. Why are decimals multiplied the same as whole numbers? A.
ecause they increase in a tenfold proportion, the same as whole num-
rs. How do you point off decimals in your product? A. As far
om the right, as the separatrix is in the multiplicand. When the
ice of one yard, one pound, or the price of a unit of any kind, is
ven; how do you obtain the price of a quantity? A. By multiplying
e price of one, by the number expressing the quantity. Why should
at give the price of the quantity? A. Because it is repeating the
ice of one as many times, as the quantity expresses, or contains
its. If one yard cost $4,50 cts.; how would you find the price of
yards? A. I would multiply $4,50 cts., the price of one yard, by the
its expressed in the number of yards, that is, by 4. Why should
at give the price of 4 yards? A. Because, it is plain, that 4 yards
ust cost 4 times as much, as one yard; and multiplying the price of
e yard by 4, is repeating the price of one yard, 4 times.

SIMPLE DIVISION,

Is the shortest method of performing subtraction, where the same
umber is to be taken away a given number of times. It shows in a

concise manner how often one number is contained in another, or how often one number may be subtracted from another. The two given numbers in division are called *divisor* and *dividend.* The number arising from the operation of the work, is called the *quotient.* And there is sometimes an uncertain number called the *remainder.* The dividend is the number to be divided. The divisor is the number by which the dividend is to be divided. The quotient shows how many times the dividend contains the divisor, or how many times the divisor can be subtracted from the dividend. The remainder is something left of the dividend after the divisor can no longer be subtracted, and is always less than the divisor.

In multiplication we have the price of a unit given to obtain the price of a quantity. In division we have the price of a quantity given to obtain the price of a unit; so it will be perceived, that division is exactly the reverse of multiplication, as well as a short way of performing subtraction; by it, we change lower denominations into higher; and by it we divide any number or quantity into an equal number of parts or shares, containing equal quantities.

RULE.—Place the divisor at the left hand of the dividend, separated from it by a vertical line, (|). Then, assume as many of the left hand figures of the dividend, as shall be sufficient to contain the divisor something less than ten times, and inquire how often the divisor is contained in the assumed number, and place the result for a quotient, at the right of the dividend, separated therefrom by a small vertical line. Multiply the divisor by this quotient figure, and place the product under the assumed part of the dividend. Then subtract it therefrom, and to the remainder, bring down the next figure of the dividend, which must be divided as before, placing the result in the quotient; and so proceed, till all the figures of the dividend, have been brought down. After multiplying by your quotient figure, if the product be greater than the assumed part of your dividend, you must lessen your quotient figure; but if the remainder (after subtracting) be greater than the divisor, you must increase your quotient figure.—And when you bring down (to the right of your remainder) a figure from the dividend, and it does not then contain the divisor, you must write a cipher in the quotient, to signify, that the divisor is not contained, and then bring down another figure from the dividend; and so continue to do, till your divisor is contained, or all the figures brought down from the dividend.

METHODS OF PROOF.—The best method of proving division, is by multiplication.

1. Multiply the quotient by the divisor, and if the product be equal to the dividend, the work is right. But should you have a remainder, it must be added to the product to make it equal to the dividend.

2. Division may be proved by addition. Add the divisor as many times, as the quotient expresses a unit: and to the amount add the remainder, (if you have any,) and if the sum equal the dividend, the work is right. But the better way to prove division by addition, is to add the remainder to the several products produced from the multiplication of the divisor in the order in which they stand; and if the amount equal the dividend, the work is right.

3. Division may be proved by subtraction. Subtract the divisor from the dividend, as often as the quotient contains a unit; and if it diminish it to nothing, the work is right; but when you have a remainder in dividing, your remainder after subtracting, must equal your remainder, when the work was performed by division.

4. Division may be made to prove itself. Divide the dividend by the quotient, and if your quotient equal your divisor in the first operation, the work is right; and if you had a remainder in the first work, the remainder in the last operation must equal the remainder in the first.

Division is denoted by the following character ÷; thus, 48÷6 signifies, that 48 is to be divided by 6. It is sometimes denoted by a horizontal line, (—) drawn under the dividend, and the divisor placed below the line; thus, $\frac{48}{6}$ signifies, that 48 is to be divided by 6. It is likewise denoted by two vertical lines, | | or inverted parenthesis,)(; thus, 6)48(signifies, that 48 is to be divided by 6.

NOTE.—Under this, as well as the other simple rules, the methods of proof introduced, are in themselves sufficient to illustrate the principles of the rules, and the relation which they bear to one another; and for the sake of becoming acquainted with these principles, the student should as carefully examine every method of proof offered, as the rules themselves. Every difficulty will vanish before the student, every principle will appear easy and simple, and his progress will be rapid and interesting, if he will only attentively examine the illustrations and demonstrations given with the examples.

DIVISION TABLE.

2 ÷ 2 = 1	3 ÷ 3 = 1	4 ÷ 4 = 1	5 ÷ 5 = 1
4 .. 2 .. 2	6 .. 3 .. 2	8 .. 4 .. 2	10 .. 5 .. 2
6 .. 2 .. 3	9 .. 3 .. 3	12 .. 4 .. 3	15 .. 5 .. 3
8 .. 2 .. 4	12 .. 3 .. 4	16 .. 4 .. 4	20 .. 5 .. 4
10 .. 2 .. 5	15 .. 3 .. 5	20 .. 4 .. 5	25 .. 5 .. 5
12 .. 2 .. 6	18 .. 3 .. 6	24 .. 4 .. 6	30 .. 5 .. 6
14 .. 2 .. 7	21 .. 3 .. 7	28 .. 4 .. 7	35 .. 5 .. 7
16 .. 2 .. 8	24 .. 3 .. 8	32 .. 4 .. 8	40 .. 5 .. 8
18 .. 2 .. 9	27 .. 3 .. 9	36 .. 4 .. 9	45 .. 5 .. 9
20 .. 2 .. 10	30 .. 3 .. 10	40 .. 4 .. 10	50 .. 5 .. 10
22 .. 2 .. 11	33 .. 3 .. 11	44 .. 4 .. 11	55 .. 5 .. 11
24 .. 2 .. 12	36 .. 3 .. 12	48 .. 4 .. 12	60 .. 5 .. 12

DIVISION TABLE—CONTINUED.

6 ÷ 6 = 1	7 ÷ 7 = 1	8 ÷ 8 = 1	9 ÷ 9 = 1
12 .. 6 .. 2	14 .. 7 .. 2	16 .. 8 .. 2	18 .. 9 .. 2
18 .. 6 .. 3	21 .. 7 .. 3	24 .. 8 .. 3	27 .. 9 .. 3
24 .. 6 .. 4	28 .. 7 .. 4	32 .. 8 .. 4	36 .. 9 .. 4
30 .. 6 .. 5	35 .. 7 .. 5	40 .. 8 .. 5	45 .. 9 .. 5
36 .. 6 .. 6	42 .. 7 .. 6	48 .. 8 .. 6	54 .. 9 .. 6
42 .. 6 .. 7	49 .. 7 .. 7	56 .. 8 .. 7	63 .. 9 .. 7
48 .. 6 .. 8	56 .. 7 .. 8	64 .. 8 .. 8	72 .. 9 .. 8
54 .. 6 .. 9	63 .. 7 .. 8	72 .. 8 .. 9	81 .. 9 .. 9
60 .. 6 .. 10	70 .. 7 .. 10	80 .. 8 .. 10	90 .. 9 .. 10
66 .. 6 .. 11	77 .. 7 .. 11	88 .. 8 .. 11	99 .. 9 .. 11
72 .. 6 .. 12	84 .. 7 .. 12	96 .. 8 .. 12	108 . 9 .. 12

10 ÷ 10 = 1	100 . 10 .. 10	77 .. 11 .. 7	48 .. 12 .. 4
20 .. 10 .. 2	110 . 10 .. 11	88 .. 11 .. 8	60 .. 12 .. 5
30 .. 10 .. 3	120 . 10 .. 12	99 .. 11 .. 9	72 .. 12 .. 6
40 .. 10 .. 4	11 ÷ 11 = 1	110 . 11 .. 10	84 .. 12 .. 7
50 .. 10 .. 5	22 .. 11 .. 2	121 . 11 .. 11	96 .. 12 .. 8
60 .. 10 .. 6	33 .. 11 .. 3	132 . 11 .. 12	108 . 12 .. 9
70 .. 10 .. 7	44 .. 11 .. 4	12 ÷ 12 = 1	120 . 12 .. 10
80 .. 10 .. 8	55 .. 11 .. 5	24 .. 12 .. 2	132 . 12 .. 11
90 .. 10 .. 9	66 .. 11 .. 6	36 .. 12 .. 3	144 . 12 .. 12

NOTE.—The student will notice in the table, that the dividend is placed first, the divisor next, and the quotient or answer to each, at the right of the parallel lines. The student can make use of language similar to this, in repeating the table, 2 in 2 once, 2 in 4 twice, 2 in 6 three times; that is, 2 is contained, or can be subtracted from 2, once; and 2 is contained, or can be subtracted from 4, twice; &c.

8÷2=how many?
16÷4=how many?
36÷6=how many?
45÷5=how many?
72÷9=how many?
96÷8=how many?

32÷ 4=how many?
28÷ 4=how many?
81÷ 9=how many?
64÷ 8=how many?
72÷12=how many?
48÷ 6=how many?

Questions to be mentally answered by the Student.

1. If 10 oranges cost 20 cents; what will one cost?
2. If 3 apples cost 3 cents; what will 1 cost?
3. If 3 bushels of oats cost 6 shillings; what was that a bushel?

4. George divided 16 apples equally among 4 boys; how many did he give each?

5. Charles wishes to divide 24 oranges among 4 companions; how many can he give each?

6. A lady paid 24 shillings for 8 yards of satin; what did it cost her per yard?

7. A gentleman bought 4 yards of broadcloth for $20; what did he pay per yard?

8. If 3 cents will buy a peach; how many can you buy for 9 cents? how many for 12 cents? how many for 15 cents? how many for 18 cents? how many for 21 cents? how many for 24 cents? how many for 27 cents? how many for 30 cents? how many for 33 cents? how many for 36 cents?

9. If beef is 4 cents a pound; how many pounds can you buy for 8 cents? how many for 12 cents? how many for 16 cents? how many for 20 cents? how many for 24 cents?

10. When stage fare is 5 cents a mile; how far can you be carried for 10 cents? how far for 15 cents? how far for 20 cents? how far for 25 cents? how far for 30 cents? how far for 35 cents? how far for 40 cents?

11. A teacher gave 48 cents to 8 boys; how many did he give each?

12. Twelve yards of broadcloth cost 60 dollars; what was the cloth a yard?

Examples.—To be performed on the slate.

1. Divide 12 by 4

Dividend,
Divisor 4) 12 (3 Quotient.
12
—
0

Dem.—We first set down 4 our divisor, and at the right hand write 12, our dividend, separated from our divisor by a small vertical line. We then inquire how often 4 is contained in 12; and considering division to be exactly the reverse of multiplication, we readily determine. In multiplication, we have two factors given to find their product; and 4, our divisor, is one of those factors, and 12, our dividend, is the product arising from the multiplication of 4, by the other factor; consequently we must think of some figure by which we may multiply 4 to produce 12. The mind readily fixes itself on 3, as the other factor, for a quotient figure; because the divisor and the quotient are the factors in multiplication, and the dividend the product. We then put 3 at the right of our dividend for a quotient, separated from the dividend by a small vertical line; we next multiply our divisor by 3, by saying 3 times 4 are 12; placing the product under our dividend; we then subtract and find that we have no remainder.

PROOF.—According to the first method.

Multiplicand, 4
Multiplier, 3
———
Product, 12

DEM.—Here, we take the divisor and quotient, and multiply them together; which clearly demonstrates the relation which this rule bears to multiplication. In division, we only separate the factors and give them different names; the 4 which we here have for a multiplicand, we had before, for a divisor; and the 3, our multiplier, we had then for a quotient; they are multiplied together in division the same as in multiplication, only under different names, producing the same product. When we divide by 4, our quotient is a fourth part of our dividend; and when we multiply 4 by 3, our multiplier is a fourth part of our product; consequently, our dividend and product must be alike; because 3 is the fourth part of either. Multiplying 4 by 3 produces 12; it is then plain, that 12 contains the 4, our multiplicand, 3 times; therefore when we divide by 4, we must suppose the 12 to contain 4 three times.

PROOF.—According to the second method.

4
4
4
—
12

When we divide twelve by 4, we inquired how often 4 was contained in 12, and we found it to be contained 3 times. Consequently, if we put down the divisor three times and add, our amount must equal the dividend; therefore, it is plain, that division may be proved by adding the divisor as often as the quotient expresses a unit; as our example shows by its producing the dividend.

PROOF.—According to the third method.

12
4
—
8
4
—
4
4
—
0

DEM.—As division is a short way of performing subtraction, it is plain, that it may be proved by subtraction, or the same work be performed by subtraction. In division we take out the three 4's from the dividend at one operation; consequently the work is greatly abridged, especially, when our quotient contains many figures, as our proof shows, which is the manner of performing the work by subtraction, and illustrates the relation which this rule bears to subtraction. By subtracting 12 in division, at once from our dividend, we had no remainder; and by subtracting 4 three times, we have no remainder, so that the same result is produced in both cases. But in performing this work by subtraction we would necessarily have to count the number of subtractions, which is at once shown by our quotient in division.

PROOF.—According to the fourth method.

Dividend,
Divisor, 3) 12 (4 Quotient,
12
—
0

That division may be made to prove itself is evident, because when we divided 12 by 4, we found that 12 contained 4 three times; then it follows, that 12 contains 3 four times, as our proof shows; then it is plain, if 4 can be subtracted from 12 three times, that 3 may be subtracted from 12 four times.

Here we offer an example where the dividend will have to be separated into different parts.

2. Divide 2460 by 5.
Dividend, Quo.,
Divisor, 5) 2460 (492
20
—
46
45
—
10
10
—
0

Here we first assume *two* of the left hand figures of our dividend, because the left hand figure does not contain the divisor; we then inquire how often 5 is contained in 24, and we suppose it contained 4 times, placing the 4 at the right for a quotient; we then say, 4 times 5 are 20, placing it under the assumed part of the dividend; we next subtract 20 from 24; and to the right hand of the remainder, we join 6, the next figure of our dividend, and then consider how often 5 can be subtracted from 46, and finding it to be 9 times, we then set 9 in the quotient, and multiply 5 by 9, which gives us 45, which we place under 46; we then subtract and find the remainder to be 1, to which we join a cipher, the next figure of our dividend; and we again inquire how often 5 is contained in 10: we suppose it to be twice, setting 2 in the quotient; we then say 2 times 5 are ten, placing it under the remaining part of our dividend; then subtracting, we find it leaves nothing of our dividend; that is, we have no remainder. The work is now done, and the quotient shows how many times 5 may be subtracted from the dividend; for the operation is already performed in the shortest possible way; that is, by division.

DEM.—Our divisor, 5, in our second example, expresses greater simple value than 2, the left hand figure of our dividend; consequently we cannot have in our quotient, units of the first order of our dividend; but when we assume the next figure of our dividend we have 24, which according to the laws of notation expresses 24 hundreds; we then can have units in our quotient of the second order of our dividend, which is hundreds; so our first quotient figure must express hundreds; because that part of the dividend producing the first figure of the quotient, is hundreds. We then place 4 in the quotient, without regard to the local part of the dividend producing it; because every figure at the right hand of that part of the dividend used, successively brings one in the quotient possessing the same local place of the part of the dividend producing it. By bringing down our next quotient figure to the remainder, we have 46 tens, according to the rules of notation, (for there is another figure to be brought down from our dividend,) in which the divisor is contained 9 times, which is nine tens; because the part of our dividend producing it, is tens. When we join the next figure of our dividend to the remainder, we have ten units, consequently our next quotient figure, 2, must be 2 units; because the part of the dividend producing the 2, is units.

NOTE.—By proving this example by multiplication the student will discover, that our remainders arise from the carriage in multiplication; considering our quotient and divisor as the two factors that have produced the dividend.

That no obscurity may shadow our work in this rule, we repeat the same example, with our dividend divided into its proper parts; and likewise separating our quotient, giving to each part its proper value.

Divisor, 5) 2460 Dividend.
The first part of our dividend is=2400
5×400=2000 400, the 1st quotient.
1st remainder= 400
add 60
The 2nd part of our dividend= 460 90 the 2nd quotient.
5×90= 450
2nd remainder= 10
add 0
The 3rd part of our dividend= 10 2, the 3rd quotient.
5×2= 10 492 Answer, or total
Last remainder= 0 sum of the quotients.

Here it is plain that our dividend can be resolved into 2400+60+0. Although in our usual method of work, we would consider the first part of our dividend, 24 and our quotient, 4; yet our first quotient figure expresses 400, and the first part of our dividend 2400; and our remainder 460, as may be seen in the operation, and so with regard to the rest.

3. Divide 46346 by 6.
Dividend,
Divisor 6) 46346 (7724 Quotient.
42*
. 43
. 42*
.. 14
.. 12*
... 26
... 24
.... 2*
Proof 46346

This is the shortest method of proving this rule by addition. The asterisks shew the numbers which are to be added; and the dotted lines, the order in which they are to be added.

DEM.—This method of proof is plain, when we consider, that division is only a short way of performing several subtractions. Here, we add the remainder to the several products or subtrahends, which gives a sum equal to the dividend; which in subtraction, would be called the minuend; the remainder added to the last subtrahend equals that part of the dividend directly above, and so of the rest, till the amount equals the dividend. When we consider that these subtractions have exhausted the dividend; (except the remainder 2,) it is then plain, if these subtrahends be collected together with the remainder, the sum must equal the dividend.

CASE I.—*The work is usually performed by what is called short division, when the divisor does not exceed* 12, *and the quotient is then placed directly under the dividend. When the divisor exceeds* 12, *it is called long division; and the quotient must then be placed at the right hand of the dividend as in the examples given above.*

EXAMPLES.

Dividend
1. Divisor 2) 4 6 7 8 3 2
2 3 3 9 1 6 Ans.
2
Proof 4 6 7 8 3 2

We first say 2 in 4 twice, placing the 2 under; then 2 in 6, three times; then 2 in 7, three times; and 1 over, to which we suppose the 8 to be joined, and say 2 in 18 nine times; then 2 in 3 once, and we have 1 over; to which we suppose 2, the next figure, to be joined, and say, 2 in 12, six times.

DEM.—To suppose the next figure of the dividend to be joined to the remainder, wherever we have a remainder, is considering the remainder to be so many tens, which must always be correct; for the remainder arises from the carriage in multiplication, and as it takes 10 of an inferiour column or place, to make 1 in the next superiour; it follows, that 1 in a superiour column or place, is equal to 10 in the next inferiour; therefore we may always consider the remainder to be tens as long as there are figures at the right in the dividend.

2. 3) 4 6 7 8 9
Ans. 1 5 5 9 6–1 Rem.
3
Proof. 4 6 7 8 9

NOTE.—Where there is a remainder, in the proof of the work, it must be added to the product.

3. 4) 6 3 7 8 4

4. 4) 6 8 3 4 6

5. 5) 7 6 3 7 8 4 6 3

6. 6) 1 2 3 6 7 3 4

7. 7) 8 6 6 3 2 4 6

8. 8) 8 3 4 6 7

9. 9) 6 7 6 8 3 6

10. 10) 8 6 7 0 3 2 4

11. 11) 3 4 6 6 7 8

12. 12) 6 7 6 1 2 6

13. Suppose 47244 dollars are to be divided among 4 persons; what can each receive? Ans. $11811.

14. Five men found a purse of money containing 65465 guineas; how many guineas must each man receive for his portion? Ans. 13093 guineas.

15. A gentleman purchased 10 acres of fine land, for which he paid $2250; what did he pay per acre? Ans. 225.

16. If a gentleman pays 144 shillings for eight weeks board; what does he pay per week? Ans. 18 shillings.

17. Suppose a man's wages to be $1092 a year; what is that per calendar month, there being 12 months in a year? Ans. $91.

18. How many barrels of pork can be bought for 1980 dollars, at 10 dollars per barrel? Ans. 198.

19. When flour is 5 dollars a barrel; how many barrels can be bought for 3240 dollars? Ans. 648.

20. If 12 yards of broadcloth cost 72 dollars; what is the cost of 1 yard? (See the following note.) Ans. $6.

NOTE.—It must be plain to the student, that 1 yard costs only a twelfth part as much as twelve yards; and by dividing 72 by 12, our quotient, ($6,) is a twelfth part of 72 dollars.

CASE II.—*When the divisor exceeds* 12.

RULE.—You will place the given numbers here, as in the preceding case; remembering to place the quotient at the right of the dividend. If the left hand figure of the divisor is greater than that in the dividend, you must assume one figure more in the dividend than the figures in the divisor; and then you can judge very near how many times the divisor is contained in the assumed part of the dividend, by considering how many times the left hand figure of the divisor, is contained in the two left hand figures of the dividend; making an allowance for the carriage. When an equal number of figures of the dividend contains the divisor, you will consider how many times the left hand figure of the divisor, is contained in the left hand figure of the dividend, making an allowance for carriage. By observing these rules, you will generally be able to place a proper figure in your quotient.

EXAMPLES.

In this example, we find that the left hand figure of the divisor, is greater than that at the left in the dividend; we then according to our rule, assume one more figure in the dividend than we have in the divisor; and then inquire how often 4, the left hand figure of the divisor, is contained in the two left hand figures of the dividend, and we find that it is contained nine times; but considering what we shall have to carry to the product of the 4, we know that the product would exceed the assumed figures of our dividend. We then say 8

1. Divide 369463 by 46.

```
                 Dividend.
Divisor, 46 ) 369463 ( 8031 Quotient.
              368
              ----
              .146
              .138
              ----
              ....83
              ....46
                ----
              ....37 Rem.
              ------
     Proof.   369443
```

times, and find that we have a remainder of only 1; to which we bring down the next figure of our dividend; but we find that 46, our divisor, is not contained in 14; we then signify it, by placing a cipher in the quotient, and likewise to give 8, the left hand figure of our quotient, its proper local place; for thousands in our dividend, produced the 8; consequently it must possess the place of thousands in the quotient. We then bring down from our dividend 6, the next right hand figure; and inquire how often 46 is contained in 146, we suppose 3 times, and proceeding according to the rule, we find that we have 8 for a remainder; to which we bring down 3, the next figure in our dividend; we then inquire how often 4 is contained in 8, and find it contained twice; but considering the carriage we know that the product would exceed this part of our dividend; therefore we set 1 in the quotient and bring down our divisor and subtract, and we find our remainder to be 37.

2. Suppose a man's income to be 1825 dollars a year; what is that per day, there being 365 days in a year? Ans. $5.

3. Divide 20304 by 24. Ans. 846.
4. Divide 20304 by 846. Ans. 24.
5. Divide 3264 by 136. Ans. 24.
6. Divide 937387 by 54. Ans. 17359 Rem. 1.
7. Divide 2674236 by 634. Ans. 4218 Rem. 24.
8. Divide 742364 by 462. Ans. 1606 Rem. 392.

9. If a man spend 1008 dollars in a year; what is that per calendar month? Ans. $84.

10. The President of the United States has a salary of 25000 dollars per year: what is that per day, allowing 365 days to the year? Ans. $68, Rem. 180.

11. Sixteen men by contract, are to receive for a job of work, two thousand three hundred and thirty-six dollars; what is each man's part of the dividend? Ans. $146.

12. A man sold a farm for $5000, containing 250 acres; what did he receive per acre? Ans. $20.

13. The product of a certain number is 63342 and the multiplier 34; what is the multiplicand? Ans. 1863.

14. The product of a number is 2336, and the multiplicand 146; what is the multiplier? Ans. 16.

Case III.—*To divide by* 10, 100, 1000, &c.

RULE.—Cut off as many figures from the right hand of your dividend, as there are ciphers in the divisor. The figures so cut off at the right, will be the remainder, and the other figures of your dividend at the left, will be the quotient.

EXAMPLES.

1. Divide 36784 by 10.

Quotient.	Rem.
3 6 7 8	4

Proof.

```
10)36784(3678
   30
    67
    60
     78
     70
      84
      80
       4
```

Dem.—In multiplication we have proved that the joining of a cipher to the multiplicand, increases it ten times; and division being the reverse of multiplication, we, instead of joining a figure, must cut one off, which diminishes the dividend, so that the figures in the quotient, have only a tenth part of the value which they have in the dividend; because the figure which expresses tens in the dividend, only expresses units in the quotient; and it is plain, if we should divide by 10 according to the usual mode of dividing numbers, the cipher of our divisor would come under the 4, and in subtracting, leave 4 a remainder.

2. Divide 36048 by 100. Ans. 360. Rem. 48.
3. Divide 461340 by 1000. Ans. 461. Rem. 340.
4. Divide 634210 by 10000. Ans. 63. Rem. 4210.

Note.—If we cut off one figure from the right of our dividend, the left hand figures, which we call the quotient, express one tenth of the dividend, excluding the remainder; and the figure cut off at the right, shows what is left of the dividend after dividing it into ten equal parts. When we cut off two figures, the left hand figures express one hundredth part of our dividend; and when we cut off three, the left hand figures express only one thousandth part of the dividend; because the figure which before expressed thousands, after the operation, only expresses units; so that the effect may be easily discovered by the laws of notation.

Case IV.—*When the divisor is a composite number; that is, when it can be produced by multiplying any two figures in the table together.*

RULE.—Divide the dividend first by *one* of the figures, and then that quotient by the other, and the last quotient will be the answer.

Note.—The total remainder is found by multiplying the last reainder by the first divisor, and to the product add the first remainder. Or multiply the whole divisor by the last quotient; and subact the product from the dividend; and the difference will be the ue remainder.

EXAMPLES.

1. Divide 4 6 8 7 7 by 1 2.

Dividend.
)ivisor, 4) 4 6 8 7 7
)ivisor, 3) 1 1 7 1 9—1 Rem.
uotient, 3 9 0 6; 1×4=4+1=5 rem.
1 2
Proof
4 6 8 7 2
5 Add 5, the Rem.
4 6.8 7 7

Three and four we take for the component parts of 12; because 3 times 4 are 12; or we might take 2 and 6; because 2 times 6 are 12. Or according to our rule, we may obtain our remainder by subtracting the product of our quotient and divisor from the dividend thus,

4 6 8 7 7 Dividend.
4 6 8 7 2 Prod. of Divisor and Quo.
5 True Rem.

Dem.—The reason of this rule is plain, because it is evident, as we nd in our example, that the 3d part of the 4th of any number, is the 2th of the whole.

2. Divide 146738 by 8. Ans. 18342, Rem. 2.
3. Divide 167834 by 16. Ans: 10489, Rem. 10.
4. Divide 56732 by 24. Ans. 2363, Rem. 20.
5. Divide 937387 by 54. Ans. 17359, Rem. 1.
6. Divide 634679 by 64. Ans. 9916, Rem. 55.

7. Suppose a privateer takes a prize worth $68526, hich is to be divided among 81 seamen; what is the share f each? Ans. $846.

8. A man bought ninety-six horses for shipping, for which e gave 6720 dollars; what did they cost him on an aveage? Ans. $70.

Case V.—*When there are ciphers at the right of the divisor.*

RULE.—Cut off the ciphers from the right hand of the divisor, and like number of figures from the right of the dividend. Divide the emaining figures without regard to the figures cut off from either; nd to the right hand of the remainder, annex the figures cut off from he right of the dividend, which will give the true remainder.

EXAMPLES.

1. Divide 86348634 by 67000.

```
Divisor.  Dividend.  Quotient.
67|000 ) 86348|634 ( 1288
         67
         ---
         193
         134
         ---
          594
          536
          ---
           588
           536
           -------
           52634 Rem.
Proof.  86348634
```

Note.—The student, at first view, will perhaps look on our remainder, and wonder at its being so great; but his surprise vanishes, when he takes into consideration that our true divisor is not 67, but 67,000.

Dem.—Cutting off the ciphers from the divisor, and an equal number of figures from the right of our dividend, is dividing them both by 10, 100, 1000, &c.; and it is evident, as often as the whole divisor is contained in the whole dividend, so often must a like portion of the divisor be contained in a like portion of the dividend; so it saves the needless repetition of the ciphers as they are repeated in the common way, as the following example shows.

```
Divisor.  Dividend.  Quotient.
67000 ) 86348634 ( 1288
        67000
        ------
        193486
        134000
        ------
         594863
         536000
         ------
          588634
          536000
          ------
           52634 Rem.
Proof.  86348634
```

Note.—This last work shows, that the ciphers of the divisor come under the figures cut off from the dividend, and coming at the right hand, the figures cut off from the dividend, come in the remainder of course.

2. Divide 6467321 by 460. Ans. 14059, Rem. 181.

3. Divide 76173 by 320. Ans. 238, Rem. 13.

4. Divide 4673625 by 21400. Ans. 218, Rem. 8425.

5. Divide 149596478 by 120000. Ans. 1246, Rem. 76478.

6. Divide 46646300 by 670000. Ans. 69, Rem. 416300.

7. Divide 6346211 by 20000. Ans. 317, Rem. 6211.

8. Divide 64613214 by 4000. Ans. 16153, Rem. 1214.

9. A gentleman sold 300 acres of land for 2100 dollars; at what rate did he sell it per acre? Ans. $7.

10. A merchant sold 300 pipes of wine for $56100; at what rate did he sell it per pipe? Ans. $187.

DIVISION
OF DECIMAL OR FEDERAL MONEY.

RULE.—Divide the same as in whole numbers. And remember to point off from the right hand of the quotient, as many figures for decimals as you have decimal places in your dividend; and should you add ciphers to your remainder for the sake of continuing your division, you will count them as decimals belonging to your dividend.

EXAMPLES.

1. If 6 bushels of wheat cost $7,50 cents; what is the cost of 1 bushel? Ans. $1,25 cents.

$ *cts.*
6) 7, 5 0
Ans. $1, 2 5 cts.
6
Proof. $7, 5 0

Here we say, 6 in 7 once, and 1 over; then 6 in 15 twice, and 3 over; then 6 in 30, five times.

Dem.—When we divide the price of 6 bushels by 6, our quotient is one sixth part of our dividend; and it is evident, that one bushel should cost only a sixth part as much as six bushels.

From this example we may lay down this general rule—That where the price of a quantity is given we obtain the price of one, by dividing the price of the quantity by the number expressing the quantity; and the quotient will be the price of 1. The reason of our pointing off for decimals is evident from the principles of subtraction; since this is only a concise method of performing subtraction.

Note.—When the quotient contains two or three decimal places, the remainder, if any, will not be noticed in the answer or quotient.

2. Divide $52,24 cents equally among 24 persons; what will each receive? Ans. $2,17 cts. 6 mills.

3. A merchant bought 235 yards of calico for $58,75 cents; what did he pay per yard? Ans. 25 cts.

4. A merchant paid $1615 for 380 yards of broadcloth; what did it cost him per yard? Ans. $4,25 cts.

Note.—If the price is given in dollars, and you have a remainder after dividing, annex a cipher, and seek how often the divisor is contained; and if you again have a remainder, annex another cipher, and so continue your division, till you have annexed three ciphers to your dividend, which will reduce your dividend to mills; consequently you will have as many figures to point off from the right of your quotient as the decimals which you have joined to your dividend.

5. If 40 yards of calico cost twelve dollars; what is that per yard? Ans. 30 cts.

6. A man paid fifty-six dollars for 50 sheep; what did he pay per head? Ans. $1,12 cts.

7. A man sold 200 bushels of oats for 64 dollars; what did he receive per bushel?. Ans. 32 cts.

8. Bought 250 acres of land, for 1375 dollars; what is that per acre? Ans. $5,50 cts.

9. A merchant bought 30 barrels of flour for $112,50 cts.; what does it stand him in per barrel? Ans. $3,75 cts.

10. If 30 yards cost 40 dollars; what is the cost of one yard? Ans. $1,33 cts., 3 mills.

11. Bought fifty bushels of barley for twenty-four dollars; what was paid per bushel? Ans. ,48 cts.

12. Paid for a fine cheese weighing 32 pounds, two dollars and fifty-six cents? what was paid a pound? Ans. 8 cts.

13. A man laboured fifty days, and received for his labour forty dollars; what was that per day? Ans. 80 cts.

14. A man paid six dollars for 20 bushels of apples; what did they cost him a bushel? Ans. 30 cts.

QUESTIONS ON SIMPLE DIVISION.

NOTE.—The teacher should continue to question the student until he becomes perfectly familiar with all the answers given to the questions in this, and the other rules. Perhaps no method of instruction can be introduced, that has so good a tendency to exercise the faculties of the student, as that of asking him questions and explaining the principles on which the rules are founded; it brightens the faculties and facilitates thought.

What is division? A. A short way of performing subtraction.—What are the two given numbers called? A. Divisor and dividend. What is the number called that arises from the operation of the work? A. Quotient or answer. Do you sometimes have an uncertain number? A. Yes. What is it called? A. Remainder. What is the dividend? A. The number to be divided. What is the divisor? A. The number by which the dividend is to be divided. What does the quotient show? A. It shows how many times the divisor is contained in the dividend, or how often it may be subtracted; or when the dividend is divided into as many parts as the divisor expresses units, the quotient shows the quantity or number contained in each part. What do you understand by the remainder? A. That there are so many left after the divisor is contained as many times as the quotient expresses a unit, or after subtracting the divisor as often as the quotient expresses a unit there are so many left; but not enough to contain the divisor once more. When is your work called short division? A. When the divisor does not exceed twelve. If the divisor exceeds twelve, what is it called? A. Long division. Where do you generally place your quotient in short division? A. Directly under the dividend. In long division where is the quotient usually placed? A. At the right hand of the dividend. Where do you place your divisor? A. At the left of the dividend. How do you proceed in the work? A. Assume a sufficient number of figures at the left hand of the dividend to contain the divisor not exceeding nine times, and set the result in the quotient; multiply the divisor by this quotient figure, and place the product under the assumed part of the dividend, and subtract it therefrom; and to the right of the remainder annex the next right hand figure of the dividend, and again seek how often the divisor is contained in this portion of the dividend, setting the result in the quotient, and so proceed till all the figures of the dividend have been brought down. If your remainder at any time be greater than your divisor, what must be done? A. Increase the quotient figure. If the product of the divisor and quotient figure exceeds that part of your dividend directly above it; what must be done? A. Lessen the quotient figure. How is division proved? A. It may be proved by addition, by subtraction, by multiplication, and by division. How is division proved by addition? A. Add the divisor as many times as the quotient expresses a unit, and add the remainder, if there be any, to the amount; and if the whole amount equal the dividend, the work is right; because it is evident, if the divisor be added as often as it has been taken away from the dividend, it must again produce the dividend. Is there any other way of proving division by addition? A. Yes, by adding the remainder to the several lines of products in the order in which they stand, and if the amount equal the dividend, the work is right; this is on the principle of proving subtraction, adding the remainder and subtrahend, for obtaining the minuend; and it is plain, if these several subtrahends and the remainder be added, the amount must equal the minuend; because it is collecting in one sum the subtractions which destroyed the dividend. How is division proved by subtraction? A. By subtracting the divisor from the dividend as often as the quotient contains a unit, and if it reduce it to nothing the work is right. When there is a remainder in division the same remainder must be left in subtraction; this proof is evident, because division is only a short way of performing these several subtractions. How can division be proved by multiplication? A. By multiplying the divisor by the quotient, or the quotient by the divisor; and if the product equal the dividend the work is right; because it is plain, if 12 be divided by four, the quotient will be three; then if we repeat the 4 three times our product must be twelve; if there be a remainder you must add it to the product, because in subtracting the divisor there was something left of the dividend. How is division proved by itself? A. By dividing the dividend by the quotient, and if your last quotient equal the first divisor, the work is right; because it is as plain, that the quotient can be subtracted from the dividend as often as the divisor contains a unit, as it is, that your divisor can be subtracted from the dividend as often as your quotient expresses a unit; thus, when we divide twelve by four, it is plain, our quotient must be three, and when we divide twelve by three, it is plain our quotient must be four, which is our first divisor. When you have the price of a quantity given, how do you obtain the price of one? A. By dividing the price of the quantity, by the number that expresses the quantity, and the quotient will be the price of one. Suppose the price of twelve yards to be 48 dollars; how would you find the price of one yard? A. By dividing $48, the price of the quantity, by 12, the number expressing the quantity; and the quotient will be the price of one yard. Why should that give the price of one yard? A. Because one yard must be worth a twelfth part as much as twelve yards, and by dividing the price of twelve yards by 12, our quotient is a twelfth part of our dividend. How may you divide by 10, 100, 1000, &c.; A. I divide by 10, by cutting off one figure from the right of the dividend and the figures at the left will be the quotient; the one cut off, will be the remainder. Why should that give the true quotient? A. Because the figures at the left of the one cut off, have only a tenth part of the value which they possessed before; for the figure that before expressed tens, now only expresses units, and those at the left have diminished in the same proportion; and in dividing by 100, we cut off two of the right hand figures of the dividend, which gives those at the left only one hundredth part of the value which they possessed before, and the figures at the right are the remainder; because in dividing in the usual way, the ciphers would come under those figures, and consequently they would come down for a remainder; and so when we divide by 1000, the figure expressing thousands, after the operation of cutting off, only expresses units. When there are ciphers at the right hand of the divisor, what may be done? A. They may be cut off, and also an equal number of figures from the right of the dividend; and then the operation is performed with the remaining figures of both, the same as if those cut off belonged to neither; and the figures cut off from the right hand of the dividend, must be brought down to the right hand of the remainder, for the true remainder. Why does that appear to be correct? A. Because as often as the whole divisor is contained in the whole dividend, so, like portions of the divisor must be contained in like portions of the dividend; therefore it saves, by cutting off the ciphers and a like number of figures from the dividend, the needless repetition of ciphers. How do you proceed to divide by a composite number? A. First divide the dividend by one of the component parts of the divisor, and then that quotient by the other. How do you find the true remainder? A. Multiply the last remainder by the first divisor, and to the product add the first remainder, and it will give the true remainder. If your divisor be eight, what would you call the component parts? A. Two and four. Why? A. Because twice four are eight. Why should you obtain the same quotient, by first dividing by 4 and then dividing that quotient by 2, that you would by dividing your dividend directly by 8? A. Because it is plain, that the half of a fourth of any number, is the eighth of the whole. If you divide a number by sixteen and have no remainder, what part of your dividend is your quotient? A. The sixteenth part. How can you prove it? A. By repeating the quotient sixteen times, the product will then equal the dividend.

What is division of decimal or federal money? A. It is dividing a number that is composed of integers, and decimal parts of an integer, or simply the parts of an integer. How do you proceed in dividing.

A. The same as in whole numbers, only point off from the right of the quotient as many figures for decimals, as are equal to the decimal places in the dividend. In dividing dollars, if you have a remainder; what do you do, after all the figures are brought down from the dividend? A. Join a cipher to the remainder, and then seek how often the divisor is contained as before, and if something again remain, annex another cipher to the remainder, and divide as before. What will the two figures in the quotient next to dollars express? A. Cents. What does the next to cents express? A. Mills. Why do we divide decimals the same as whole numbers? A. Because they increase in a ten fold proportion from the right to the left.

SUPPLEMENT TO MULTIPLICATION.

Multiplying by a mixed number, as $6\frac{1}{2}$, $5\frac{3}{4}$, &c. is taking the multiplicand as many times as there are units in the multiplier; and likewise taking a part of the multiplicand, as many times as there are like portions of a unit in the fraction of the multiplier.

RULE.—Multiply the multiplicand by the whole number of the multiplier; and take such parts of the multiplicand for the product of the fraction, as there are like parts of a unit contained in the fraction.

NOTE.—Multiplying by 1, is taking the multiplicand once; multiplying by 2, is taking it twice; multiplying by 3, is taking the multiplicand 3 times, and so on. Multiplying by a fraction, is taking a part of the multiplicand, as many times, as there are like parts of a unit in the multiplier. Multiplying by $\frac{1}{2}$ is taking one half of the multiplicand; multiplying by $\frac{1}{4}$, is taking one fourth of the multiplicand; multiplying by $\frac{1}{6}$, is taking one sixth of the multiplicand; multiplying by $\frac{2}{6}$, is taking one sixth of the multiplicand, twice.

EXAMPLES.

1. Multiply 36 by $4\frac{1}{2}$.

2) 36
$4\frac{1}{2}$
18=$\frac{1}{2}$ of 36, the multiplicand.
144
162 Ans.

2. Multiply 24 by $5\frac{1}{3}$.

3) 24
$5\frac{1}{3}$
8=$\frac{1}{3}$ of 24, the multiplicand.
120
128 Ans.

DEMONSTRATION.—The reason of this work appears plai from the preceding note; because in the first example we wis to repeat our multiplicand by a half, and the half of 36, i evidently obtained by dividing thirty-six by 2; and th quotient being properly the product of a part of a unit, the product of the unit figure is placed directly under.

3. Multiply 36 by $4\frac{2}{6}$

6) 36

$4\frac{2}{6}$

$$\left.\begin{array}{l} 6=\frac{1}{6}\text{ of }36 \\ 6=\frac{1}{6}\text{ of }36 \end{array}\right\}=\frac{2}{6}\text{ of }36.$$

144

Ans. 156

4. Multiply 34 by $5\frac{3}{4}$

34

$5\frac{3}{4}$

$$\left.\begin{array}{l} 17=\frac{1}{2}\text{ of }34 \\ 8\frac{1}{2}=\frac{1}{4}\text{ of }34 \end{array}\right\}\frac{1}{2}\text{ and }\frac{1}{4}=\frac{3}{4}$$

170

$195\frac{1}{2}$ Ans.

5. Multiply 55 by $5\frac{1}{5}$. Ans. 286.

6. Multiply 365 by $6\frac{1}{7}$. Ans. $2242\frac{1}{7}$.

7. Multiply 264 by $10\frac{1}{8}$. Ans. 2673.

8. Multiply 6742 by $6\frac{5}{8}$.

6742

$6\frac{5}{8}$

* $4213\frac{6}{8}=\frac{5}{8}$ of 6742.

† 40452

Ans. $44665\frac{6}{8}$

6742 × by $\frac{5}{8}$,

5 Numerator.

Denominator 8) 33710

$4213\frac{6}{8}=\frac{5}{8}$ of 6742.

Note.—To multiply by a fraction, we sometimes find it more convenient to multiply the multiplicand by the numerator of the fraction and divide the product by the denominator; *see* ☞ the quotient is then the product of the fraction; which product* in all cases must be added, as in this 8th example, with the product† of the whole number.

9. Multiply 2456 by $9\frac{11}{12}$. Ans. $24355\frac{4}{12}$.

10. Multiply 346 by $2\frac{5}{7}$. Ans. $939\frac{1}{7}$.

11. Multiply 4673 by $\frac{7}{9}$. Ans. $3634\frac{5}{9}$.

12. Multiply 6621 by $\frac{3}{4}$. Ans. $4965\frac{3}{4}$.

FAMILIAR EXAMPLES.

To exercise the learner in Addition, Subtraction, Multiplication, and Division.

1. If you add 444, 375, 250, and subtract 300 from the amount; what will remain? Ans. 769.

2. John borrowed of George at one time 500 dollars, at another $250; John paid at one time 150 dollars, at another 120 dollars; how much is John indebted to George?
Ans. $480.

3. What number is that which being multiplied by 30 the product will be 1350? Ans. 45.

4. What number is that which being divided by 72 the quotient will be 36? Ans. 2592.

5. What will 48 yards of broadcloth come to at $4½ per yard? Ans. $216.

6. What will 96 yards of Irish linen come to at ¾ of a dollar per yard? Ans. $72.

7. At one dollar and a half per yard; what will fifty yards cost? $75.

8. Four men in partnership have, in stock, 6346 dollars; of which A put in $1500, B $900, and C $2500; what did D put in? Ans. 1446.

9. If 4 yards of broadcloth cost 16 dollars; what will 8 yards cost? Ans. $32.

$$\begin{array}{rl} & \$ \\ 4) & 16 \\ \hline & \$\,4 \text{ the price of 1 yard.} \\ & 8 \\ \hline \text{Ans.} & \$\,32 \text{ the price of 8 yards.} \end{array}$$

NOTE.—From this example the student may draw very important instruction, which may be employed in almost all the transactions of business in buying and selling. Where the price of a quantity is given to obtain the price of a unit, we divide the *price* of the quantity *by the quantity*; the quotient is then the price of the unit, that is, one pound, one yard, &c. And when we have the price of a unit given, to obtain the price of a quantity we multiply the price of a unit by the number expressing the quantity. Consequently when the price of a quantity is given to find the price of some other quantity, we only have to observe the following rule.

RULE.—Divide the price of the given quantity by the number expressing the quantity, and your quotient will be the price of one; then multiply the price of one by the number expressing the other quantity, and the product will be the price of the required quantity.

10. If 8 yards of calico cost 24 shillings; what will 4 yards cost? Ans. 12*s*.

DEM.—It is plain, that 1 yard must cost the eighth part of the price of eight yards; and by dividing 24 shillings, the price of eight yards, by 8, our quotient, 3 shillings, is an eighth part of 24 shillings; consequently 3 shillings is the price of 1 yard; it is also evident, that 4 yards must cost 4 times as much as one yard; then by multiplying 3 shillings, the price of one yard, by 4, our product must be the price of 4 yards; because it is repeating 3 shillings, the price of 1 yard, 4 times.

```
     Shillings.
  8) 2 4
     ———
       3 the price of 1 yard.
       4
     ———
Ans. 1 2 s. the price of 4 yards.
```

11. If 4 barrels of flour cost 20 dollars; what will 30 barrels cost? Ans. $150.

12. If 28 cords of wood cost 42 dollars; what will 6 cords cost? Ans. $9.

13. If a man receive 7 cents for the loan of 1 dollar per year; what should he receive for the use of 100 dollars the same time? Ans. $7.

NOTE.—To reduce cents to dollars, cut off two of the right hand figures; and all to the left will be dollars; the two figures cut off will be cents; thus 1450 cents=$14,50 cts.; to bring mills into dollars, cut off three of the right hand figures; thus, 14505 mills=14 dollars, 50 cts. and 5 mills.

14. If a man receive 49 cents for the use of 7 dollars; what should he receive for the use of 50 dollars, the same time? Ans. $3,50 cts.

15. If a man receive 28 dollars for the use of 400 dollars a year; what should he receive for the use of $1000 the same time? Ans. $70.

16. If a man's wages for 365 days amount to $456,25 cents; what will his wages amount to for 60 days? Ans. $75.

17. Suppose you receive $36, for 24 yards of fine linen; what should you receive for 48 yards? Ans. $72.

18. What will 12½ yards of cloth come to, at $4 per yard? Ans. $50.

19. Suppose 50 yards cost $37,50 cents; what is the cost of one yard? Ans. ,75 cts.

20. If one yard cost seventy-five cents; what will fifty cost? Ans. $37,50 cts.

QUESTIONS.

What is a mixed number? A. A whole number and a fraction, as $4\frac{2}{7}$, express 4 and two sevenths. How do you multiply by $\frac{1}{2}$? A. By taking half of the multiplicand, that is, dividing the multiplicand by 2, and taking the quotient for the product. How do you multiply by one third? A. Take one third of the multiplicand. How do you multiply by $\frac{1}{12}$? A. Take one twelfth of the multiplicand. What is multiplying by one? A. It is taking the multiplicand once? What is multiplying by 3? A. It is taking the multiplicand three times. When you multiply by a whole number and a fraction, where do you place the product of the fraction? A. Directly under the product of the unit figure of the multiplier, because multiplying by a simple fraction is only multiplying by a part of a unit. Where the price of a quantity is given, how do you find the price of any other quantity? A. Divide the *price* of the given quantity *by the given quantity*, which will give the price of a unit, then multiply the price of a unit, by the other quantity, and the product will be the price of the required quantity. If six yards of cloth cost 12 shillings, how would you find the cost of 3 yards? A. I would divide 12 shillings by six, the number of yards, the quotient will then be the price of 1 yard; then multiply the price of one yard by 3, the number of yards, and the product will be the price of 3 yards. How much more should a merchant receive for 3 yards than for 1? A. Three times as much.

REDUCTION,

Is the changing of numbers from one denomination to another without altering their value. It is of two kinds, *Descending* and *Ascending*. Reduction *Descending* is changing *higher* denominations into *lower*, as pounds into shillings, and shillings into pence, &c.; it is performed by multiplication. Reduction *Ascending* is changing *lower* denominations into *higher*, as pence into shillings, shillings into pounds; performed by division.

REDUCTION DESCENDING.

RULE.—Multiply the number in the highest denomination given by as many of the next lower as make one in that higher; and remember to add to the product, the figures of the next lower denomination, and in like manner proceed till your given sum is reduced to the denomination required.

PROOF.—Reduction descending is proved by reduction ascending, and reduction ascending, by reduction descending; and, if by reversing the work, the given sum be produced, the work is right.

FEDERAL MONEY.

10 Mills	make	1 Cent,	ct.
10 Cents	make	1 Dime,	di.
10 Dimes, or 100 cts.	make	1 Dollar,	$.
10 Dollars	make	1 Eagle,	E.

The following is a brief account of the coin in use in the United States, abstracted from an "act establishing a mint, and regulating the coins of the United States," passed April 2d, 1792.

The money of account of the United States shall be expressed in dollars or units, dimes or tenths of a dollar, cents or hundredths of a dollar, and mills or thousandths of a dollar.

The following are the denominations of coin in the United States.

Gold—	The Eagle	shall be of the value of ten dollars.
	The Half Eagle	„ five dollars.
	The Quarter Eagle	„ two and a half dollars.
Silver—	The Dollar	„ Spanish milled dollar.
	The Half Dollar	„ half the d., or 50 cts.
	The Quarter Dollar	„ 1-4th the d., or 25 cts.
	The Dime	„ 1 tenth the d., or 10 cts.
	The Half Dime	„ 1-20th the dollar, or 5 cts.
Copper—	The Cent	„ 1 hundredth of the dollar.
	The Half Cent	„ 1 half the cent.

The standard for gold coin, shall be eleven parts of pure gold and one part of alloy, so that every twelfth part is alloy. The alloy must be silver and copper, but the silver must not exceed one half in the alloy.

The weight of the Eagle, shall be two hundred and forty-seven and a half grains of pure gold, or two hundred and seventy grains of standard gold; and the other gold coins shall be proportional.

The standard proportion for silver coin of the United States, shall be one thousand four hundred and eighty-five parts of pure silver, and one hundred and seventy-nine parts of alloy; and the alloy must be pure copper.

The weight of the Dollar, shall be three hundred and seventy-one and one fourth grains of pure silver, or four hundred and sixteen grains of standard silver; and the other silver coins must weigh proportional according to their value.

The copper coins shall be of pure copper. The weight of the Cent shall be eleven penny weights of copper; and the Half Cent shall weigh in the same proportion. The proportional value of gold to silver is as fifteen to one, or by a law of Congress, fifteen pounds in weight of pure silver, shall be equal in value to one pound in weight of pure gold.

All coins issued from the mint of the United States, shall be a lawful tender for the payment of any debt at the values here given, but should they fail in weight, the values must be proportional to their weight.

EXAMPLES.

1. In 2 dollars; how many cents? Ans. 200 cts.

Demonstration.—It is plain, that we must have one hundred times as many cents as dollars to equal our dollars in value, because it takes a hundred cents to equal a dollar; and by multiplying by 100,

$
2
100
Ans. 200 cts.

our product expresses one hundred times as many as our multiplicand, though of no greater value; because cents express only the hundredths of a dollar. But from our rules given in multiplication, this work may be abridged; for to multiply by 100 we have only to annex two ciphers at the right of our multiplicand, and we have the product of our multiplicand, multiplied by one hundred; and to reduce 2 dollars to cents, we have only to annex at the right hand of the 2, two ciphers; for $2, by annexing two ciphers, stand thus, 200 cts.=2 dollars.—Now it must appear plain to the student, that as 1 dollar is 100 cts., and two dollars, 200 cts.; $3 must be 300 cts., and 4 dollars, 400 cts., and so on.

2. Reduce 2 dollars to mills. Ans. 2000.

$2
100
200
10
Ans. 2000 mills.

Dem.—It has already been proved that multiplying by 100, reduces dollars to cents. And it is also evident that by multiplying cents by ten, our product is mills; because we want ten times the number of mills, that we have of cents, to equal our cents in value; for one mill is only the tenth of a cent. But we may abridge our work by annexing three ciphers to the dollars; our product will then express mills, because a mill is the thousandth of a dollar, and annexing three ciphers to dollars, is multiplying the dollars by one thousand; thus, $2, by annexing three ciphers, becomes 2000 mills.

3. In $3 and 96 cents; how many cents? Ans. 396 cts.

$ *cts.*
3,96
1 00
3 00
Add the 96
Ans. 396 cts.

This may be shortened by annexing the cents to the dollars; it will then stand reduced to cents; thus, $3,96 cts. becomes by taking away the separatrix, 396 cents, because, joining the cents is multiplying the dollars by one hundred, and the cents have their proper places, counting the same that they do standing alone; thus, $4,56 cents, by taking away the separatrix, expresses 456 cents; the reason of this is plain, because by annexing the cents the dollars have the place of hundreds.

4. In $4,91 cts. 3 mills; how many mills? Ans. 4913 mills.

Note.—This is reduced to mills by taking away the separatrix and joining all the figures together, because dollars take the place of thousands, their proper local value when expressed in mills, and so on, of the rest. And to prove your work, you have only to place your separatrix; thus, 4913 mills, proved $4,91 cts. 3 mills.

5. Reduce 36 dollars 95 cents and 4 mills, to mills.
Ans. 36954 mills.

6. In $45,95 cents; how many cents? Ans. 4595 cts.

7. In $45; how many mills? Ans. 45000 m.
8. In $36 and 1 cent; how many cts.? Ans. 3601 cts.
9. In $400 and 1 mill; how many mills? Ans. 400001.
10. In 1 dollar and 1 dime; how many dimes? Ans. 11 dimes.
11. In 1 dollar and 5 dimes; how many cents? Ans. 150 cts.
12. In one dollar, one dime, one cent and one mill; how many mills? Ans. 1111 mills.
13. In 15 dimes; how many cents? Ans. 150 cts.
14. In 9 dimes; how many mills? Ans. 900 mills.

ENGLISH MONEY.*

4 Farthing (qrs.)	make	1 Penny,	marked	*d.*
13 Pence	"	1 Shilling	"	*s.*
20 Shillings	"	1 Pound	"	£.

A Groat is four pence.

NOTE.—Farthings are often written thus, 1 farthing, $\frac{1}{4}$ of a penny, 2 farthings, $\frac{1}{2}$ of a penny, and 3 farthings, $\frac{3}{4}$ of a penny.

EXAMPLES.

1. In 4 pounds; how many shillings? Ans. 80 s.

£
4
20
Proof 2|0)8|0 s. Ans.
4 £

DEM.—It is plain, since it takes 20 shillings to make one pound, that we must have twenty times as many shillings as pounds, to equal our pounds in value; and multiply £4, our multiplicand, by 20, our product expresses twenty times as many as our multiplicand; and it should express 20 times as many, to equal the pounds in value; because a shilling is the twentieth of a pound.

2. Reduce £5 and 2*s.*, to shillings. Ans. 102s.

£ *s.*
5 2
20
Proof 2|0) 10|2s. Ans.
£5 2s.

Here, according to our rule, we multiply our highest denomination, and add to the product the 2 shillings, the next inferiour denomination, and our product then expresses the number of shillings contained in 5 pounds and 2 shillings, because 20 times 5 are 100, and 2 make 102.

---

* This is the money of account now in England. And it was the only mode of reckoning in the United States till after an act of Congress in 1792, establishing a mint and regulating the standard of our coins; but since, it has gradually grown out of use.

3. Reduce £48 17s. 6d. and 3 farthings to farthings.

Ans. 46923 qrs.

£	s.	d.	qrs.	
48	17	6	3	
20				
977				Shillings in £48 17s.
12				
11730				Pence in £48 17s. 6d.
4				
Ans. 46923				Farthings in £48 17s. 6d. 3qrs.

DEM.—We first multiply our pounds by 20, because it takes 20s. to make a pound; and add the shillings, because our product expresses shillings. We then multiply our shillings by 12, because a unit in shillings is equal to 12 pence; and we add the pence in the given sum to the product, because our product is pence; we then multiply the pence by 4, because a unit in pence is equal to four times as many farthings; and we add the farthings in our given sum to the product, because our product expresses farthings.

4) 4 6 9 2 3 qrs.

12) 1 1 7 3 0–3 qrs.

2|0) 9 7|7–6 d.

Proof. £4 8 17s. 6d. 3 qrs.

4. In £6; how many shillings? Ans. 120s.

5. In £1; how many shillings and pence? Ans. 20s. 240d.

6. In £2; how many pence? Ans. 480.

7. In £3; how many farthings? Ans. 2880qrs.

8. In £16 14s. 6d.; how many pence? Ans. 4014d.

9. In £34; how many shillings, pence, and farthings? Ans. 680s., 8160d., 32640qrs.

10. In £96; how many half pence? Ans. 46080.

11. In £46 12s. and 6d.; how many farthings? Ans. 44760qrs.

12. In £86 14s. 6d. 2qrs; how many farthings? Ans. 83258qrs.

13. In £39; how many shillings, pence, and farthings? Ans. 780s., 9360d., 37440qrs.

14. In 48 Guineas at 28 shillings each; how many shillings and pence? Ans. 1344s., 16128d.

15. In 24 Moidores at 36 shillings each; how many shillings? Ans. 864s.

16. In 320 Pistoles at 22 shillings each; how many shillings and pence? Ans. 7040s., 84480d.

17. In 4 dollars at 8 shillings each; how many shillings, and pence? Ans. 32s., 384d.

18. In 32 dollars at 8 shillings each; how many pence, and half pence? Ans. 3072d., 6144 half pence.

REDUCTION ASCENDING.

RULE.—Divide the given sum by that number which it requires of the given denomination, to make a unit in the next higher denomination; and so proceed, dividing in each operation the last quotient by the number which it requires to make a unit in the next higher, till you have reduced the given sum to the denomination required; the last quotient, with the several remainders, if any, will be the true answer.

EXAMPLES.

1. In 16qrs.; how many pence? Ans. 4d.

4) 16qrs.
4d. Ans.
Proof. 4
1 6qrs.

Dem.—The reason of our dividing by 4 is plain; because it takes four farthings to make a penny, and one fourth the given number in pence must equal the farthings in value; for every penny is equal to four farthings; and when we divide by 4, our quotient expresses one fourth part of our dividend, though the same in value; because a number in pence is equal to four times the same number in farthings.

2. In 145 farthings how many shillings?

Ans. 3s. 0d. 1qr.

qrs.
4) 145
12) 36d. 1qr.
Ans. 3s. 0d. 1qr.

Note.—The remainder is of the same name of the dividend that produced it.

Dem.—When we divide by 4, our quotient is one fourth part of the dividend in number, but the same in value; because any number of pence is equal to four times the same number in farthings; and when we divide the pence by twelve our quotient is one twelfth of our dividend in number, but the same in value; for any number of shillings is equal to twelve times the same number in pence.

3. In 35339 farthings; how many pounds?

Ans. £36 16s. 3qrs.

qrs.
4) 35339
12) 8834–3 qrs.
2|0) 73|6–2d.
Ans. £36–16s. 2d. 3qrs.

Dem.—Here, we first divide by 4, because four farthings make a penny, then by 12, because 12 pence make a shilling, and lastly, by 20, be-

Carried over.

```
      Ans. £36 16s. 2d. 3qrs.
             20
Proof.    736s.=£36 16s.
             12
          8834d.=£36 16s. 2d.
             4
        35339qrs.=£36 16s. 2d. 3qrs.
```

Brought over. cause 20 shillings make a pound.

4. In 46382 farthings; how many pence? Ans. 11595d. 2qrs.

5. In 16486 pence; how many shillings? Ans. 1373s. 10d.

6. In 85 shillings; how many pounds? Ans. £4 5s.

7. In 2880 farthings; how many pounds? Ans. £3.

8. In 46080 half pence; how many pounds? Ans. £96.

9. In 1344 shillings; how many guineas? Ans. 48g.

10. In 864 shillings; how many moidores? Ans. 24m.

11. In 660 shillings; how many pistoles? Ans. 30.

12. In 3468 farthings; how many guineas? Ans. 2guin. 16s. 3d.

13. In 36463 pence; how many pounds? Ans. £151 18s. 7d.

14. In 74981 half-pence; how many pounds? Ans. £156 4s. 2½d.

15. In 3452 sixpences; how many pounds? Ans. £86 6s. 0d.

REDUCTION ASCENDING AND DESCENDING

1. In 476 pounds; how many guineas? Ans. 340guin.

```
       £
      476
       20
 28 ) 9520 ( 340 Ans.
      84
      112
      112
        0
```

Demonstration.—It is plain, that we reduce pounds to shillings by multiplying by 20; then our product, which is shillings, must give guineas when divided by 28, because 28 shillings make a guinea.

2. In 340 guineas; how many pounds? Ans. £476.

guin.
340
28
2720
680
2|0)952|0
£476 Ans.

DEM.—It is evident, that twenty-eight times the number of guineas, must give shillings; and one twentieth of the shillings, must be pounds; because 20 shillings are equal to one pound; so it will be perceived, that this is exactly the reverse of the example above.

3. In 324 guineas at 28 shillings each; how many moidores at 36 shillings each? Ans. 252 moi.

4. In 680 pistoles at 22 shillings each; how many pounds? Ans. £748.

5. In 748 pounds; how many pistoles, and shillings? Ans. 680 pistoles, 14960s.

6. In 48 moidores; how many dollars, at 8 shillings each? Ans. 216 dols.

7. In £82 10 shillings; how many pistoles? Ans. 75.

8. In 34642 sixpences; how many shillings, and pounds? Ans. 17321 shillings, £866 1s.

9. In 411 English crowns at 6s. 8d. each; how many pounds? Ans. £137.

10. In 3584 four and a half penny pieces; how many dollars, at 8 shillings each? Ans. $168.

11. In 6480 groats; how many pence and shillings? Ans. 25920d., 2160s.

12. In £25 and 10 guineas; how many shillings, sixpences, and three-pences? Ans. 780s., 1560 six-pences, 3120 three-pences.

13. In 6483 cents; how many dollars? Ans. $64,83 cts.

14. In $580; how many cents? Ans. 58000 cts.

15. In $85,68 cts.; how many cents? Ans. 8568 cts.

16. In 84684 mills; how many dollars? Ans. $84,68 cts. 4 mills.

17. In $68,25 cts. 2 mills; how many mills? Ans. 68252 mills.

18. In 360480 groats; how many shillings, and pounds? Ans. 120160s., £6008.

NOTE.—As the student advances, the teacher should question him, letting the student mentally tell how he performs each operation; why he multiplies, and what it produces for a product; why he divides and the name of the quotient. The teacher, by pursuing this method will find his students making double the progress that they would make by letting them pass on in the ordinary way, from page to page, without ever reflecting on the principles of the rules, or being able to give a single reason for the operations of the work.

TROY WEIGHT.*

24 Grains (grs.)	make	1 Pennyweight,	marked	pwt.
20 Pennyweights	"	1 Ounce	"	oz.
12 Ounces	"	1 Pound	"	lb.

* Gold, silver, jewels, and all liquors, are weighed by this weight.—An ounce of silver is divided into 20 parts, called pennyweights; and an ounce of gold is divided into 24 parts, called carats. Heat is applied to distinguish the fineness of metals. A pound of silver that will abide the fire without loss, is 12 ounces fine, but if it lose 3 pennyweights by trial, its fineness is 11oz. 17pwt., &c. Gold that loses nothing in trial, is 24 carats fine, but, if it lose 2 carats, it is 22 carats fine.

Alloy is inferiour metal mixed with gold or silver, to diminish its fineness. The true standard of gold coin in England, is 22 carats of gold and 2 carats of copper; the alloy being one eleventh part of the fine gold. The true standard of silver is 11 oz. 2 pwts. of fine silver, mixed with 18 pwts. of copper.

Note.—175 ounces *Troy* are equal to 192 ounces *Avoirdupois*, and 175 pounds *Troy* are equal to 144 pounds *Avoirdupois*. 1 pound Troy contains 5760 grains, and 1 pound Avoirdupois contains 7000 grains.

EXAMPLES.

1. In 2 pounds; how many ounces? Ans. 24oz.

2lb.
12 ounces in a pound.
24oz. in 2lb. Ans.

Dem.—It is evident we must have twelve times as many ounces as pounds to equal our pounds; and when we multiply by 12, our product expresses twelve times as many as our multiplicand, though expressing the same quantity; because 1 pound equals 12oz., 2lb. must equal 24oz., and 3lb. must equal 36oz., &c.

2. In 5lb.; how many pennyweights? Ans. 1200pwts.

5lb.
12
60 } ounces in
20 } 5lb.
1200pwts. in 5lb.

Proof
2|0)120|0
12)60oz.
5lb.

Demonstration.—We first multiply by 12, because 12oz. make a pound; we next multiply by 20, because 20pwts. make an ounce.

3. In 4lb. 4oz. 6pwt. 4grs.; how many grains?

Ans. 25108grs.

4lb. 4oz. 6pwt. 4grs.
12
52 } ounces in 4lb. 4oz.
20
1046 } pwts. in 4lb. 4oz. 6pwts.
24
4188
2092
25108 } grs. in 4lb. 4oz. 6pwts. 4 grs.

DEM.—When we multiply by 12, we add the 4oz. to the product; because the product is ounces, and ounces must be added to ounces; we next multiply by 20, and add in the 6pwts., because our product is pennyweights; and lastly, we multiply by 24, because 24 grs. make a pennyweight, and add the 4 grains to the product, because our product is grains; and every inferiour denomination must be added to a product of the same name.

4. In 47lb. 10oz.; how many ounces, pennyweights and grains? Ans. 574oz. 11480pwt. 275520grs.

5. In 483840 grains; how many pennyweights, ounces and pounds? Ans. 20160pwts. 1008oz. 84lb.

6. In 50486oz.; how many pounds? Ans. 4207lb. 2oz.

7. Bring 14678 pennyweights into pounds. Ans. 61lb. 1oz. 18pwts.

8. In 8lb. 4oz. 6pwts. 15grs.; how many grains? Ans. 48159grs.

9. In 4 pounds of silver; how many pennyweights? Ans. 960pwts.

10. How many tea spoons can be made from 3 pounds of silver, each spoon weighing six pennyweights? Ans. 120 spoons.

11. How many table spoons, each weighing 22pwts., can be made from 5 pounds of silver? Ans. 54 and 12 pwts. over.

AVOIRDUPOIS WEIGHT.

16 Drachms (dr.)	make	1 Ounce,	marked	oz.
16 Ounces	"	1 Pound,	"	lb.
28 Pounds	"	1 Quarter of a hund. wt.		qr.
4 Quarters	"	1 Hundred wt. or 112lb.		cwt.
20 Hundred	"	1 Tun	"	T.

All coarse and drossy goods, such as hay, grain, grocery, and chandlery wares, and all metals, except gold and silver, are weighed by this weight.

A barrel of pork or beef weighs 200lb. A quintal of fish 112 pounds.

12 particular things make one dozen; 12 dozen one gross, and 144 dozen 1 great gross; 20 particular things make a score.

NOTE.—The student should commit to memory the tables, as they occur, before he proceeds to work the sums.

EXAMPLES.

1. In 3 pounds; how many ounces? Ans. 48oz.

```
      3    { 1lb. Avoir-
   16oz.=  { dupois.
   ----
     18
     3
   ----
   48oz. in 3lb. Ans.

        oz.
   16 ) 48 ( 3lb.
        48          Proof.
      ----
         0
```

DEM.—We multiply by 16, because in this weight 16oz. make a pound, and we must have 16 times as many ounces as pounds to equal our pounds in quantity; and 48, our product, is 16 times as much as 3, our multiplicand.

2. In 2T. 3cwt. 2qr. 16lb. 5oz. 11 drachms; how many drachms? Ans. 1251419dr.

```
   2T. 3cwt. 2qr. 16lb. 5oz. 11dr.
      20
   -------
      43 { cwt. in
       4 { 2T 3cwt.
   -------
     174 { qrs. in 2T.
      28 { 3cwt. 2qrs.
   -------
    1408
    348
   -------
    4888 { lbs. in 2T. 3cwt.
      16 { 2qrs. 16lb.
   -------
   29333
   4888
   -------
   78213 { oz. in 2T. 3cwt.
      16 { 2qrs. 16lb. 5oz.
   -------
  469289
  78213
   -------
 1251419 { dr. in 2T. 3cwt. 2qrs.
         { 16lb. 5oz. 11dr.
```

DEM.—We first multiply our tuns by 20, because every tun is equal to twenty hundred; then by 4, because every hundred is equal to 4 quarters; we next multiply by 28, because every quarter contains 28 pounds; we then multiply by 16, because every pound contains sixteen ounces; and again by 16, because every ounce contains 16 drachms. When we reduce our tuns to cwt. we add our 3cwt. because our product is of that name; and when we multiply by 4, we add 2qrs. to the product, because our product is quarters; and when we multiply by 28, we add to the product our 16 pounds because our product expresses pounds; and when we multiply pounds by 16, we add in our 5oz. because the product is ounces; and lastly, when we multiply ounces by 16, we add our 11dr. to the product, because our product is drachms.

3. In 4 tuns; how many hundred, quarters and pounds?
Ans. 80cwt. 320qrs. 8960lbs.

4. In 12 tuns 15cwt. 1 qr. 19lb. 6oz. 12dr.; how many drachms?
Ans. 7323500dr.

5. In 6720 pounds; how many tuns? Ans. 3 tuns.

6. In 573440 drachms; how many parcels each 28 pounds?
Ans. 80 parcels.

7. In 3024lb.; how many cwt.? Ans. 27cwt.

8. In 3076 pounds; how many cwt.?
Ans. 27cwt. 1qr. 24lb.

9. In 124cwt. 1qr. 13lb.; how many pounds?
Ans. 13929lb.

10. In 30911 pounds; how many cwt.?
Ans. 275cwt. 3qrs. 27lb.

11. In 8100 pounds; how many tuns?
Ans. 3T. 12cwt. 1qr. 8lb.

12. In 36 pounds Avoirdupois; how many pounds Troy?
Ans. 43lb. 9oz.

Note.—To reduce pounds avoirdupois to pounds Troy, multiply your pounds avoirdupois by 7000, and divide the product by 5760, the number of grains in a pound Troy.—And to reduce Troy pounds to pounds avoirdupois, multiply by 5760 and divide by 7000, the number of grains in a pound avoirdupois.

APOTHECARIES WEIGHT.*

20 Grains (gr.)	make	1 Scruple, sc.	marked	℈.
3 Scruples	„	1 Drachm, dr.	„	ʒ.
8 Drachms	„	1 Ounce, oz.	„	℥.
12 Ounces	„	1 Pound, lb.	„	℔.

* By this weight, apothecaries compound their medicines; but they buy and sell by avoirdupois weight, all above grains. A pound Troy is the same as a pound in apothecaries' weight according to the tables.

EXAMPLES.

1. In 3℔. 6oz. 4dr. 2sc. 18gr.; how many grains?
Ans. 20458gr.

3lb. 6oz. 4dr. 2sc. 18gr.
12
42 { ounces in
8 { 3lb. 6oz.
340 { drachms in 3lb.
3 { 6oz. 4dr.
1022 { scruples in 3lb.
20 { 6oz. 4dr. 2sc.
20458 { grains in 3lb. 6oz.
{ 4dr. 2sc. 18gr.

Dem.—We first multiply our pounds by 12, because every pound makes 12 ounces, and we add our ounces in the given sum to the product; we next multiply by 8, because every oz. contains 8 drachms, and to the product we add the drachms in our given number; and we next multiply by 3, because every drachm is equal to 3 scruples, and to the product, we add the scruples in our given sum; and lastly, we multiply by 20, because every scruple makes twenty grains, and to the product, we add the grains in our given sum.

2. In 49232 grains; how many pounds?
Ans. 8lb. 6oz. 4dr. 1sc. 12gr.

3. In 1219 drachms; how many pounds?
Ans. 12lb. 8oz. 3dr.

CLOTH MEASURE.*

2¼ Inches (in.)	make	1 Nail,	marked	na.
4 Nails, or 9 inches	„	1 Quarter of a yd.		qr.
4 Quarters, or 36 inches	„	1 Yard.	„	yd.
3 Quarters	„	1 Ell Flemish,		E. Fl.
5 Quarters	„	1 Ell English,		E. E.
6 Quarters,	„	1 Ell French,		E. Fr.

* Linens are bought of the Dutch by the Ell Flemish and sold in America by the yard, but the Flemish Ells are sold in England by the Ell English. Thirty-seven inches and one fifth make an Ell Scotch, marked E. Sc. Three quarters and 2 thirds of a qr. a Spanish Var.

Note.—This measure is used for measuring cloth, tape, &c.

EXAMPLES.

1. In 24 yards; how many quarters and nails?
Ans. 96qrs. 384nails.

24yds
4
96 qrs. in 24yds.
4
384 nails in 24yds.

Proof
4)384 nails in 24yds.
4)96qrs.
24yds.

Dem.—We first multiply by 4, because we must have four times as many quarters, as yards, to equal our yards in quantity; we next multiply by 4, because we must have four times as many nails, as quarters, to equal our quarters in quantity.

Demonstration of the Proof.—We first divide the nails by 4, because it takes 4 nails to make a quarter; and we next divide the quarters by 4, because it takes 4 quarters to make a yard.

NOTE.—In reduction it must be kept in mind, that we only change the name of our denominations, not altering their value or quantity.

2. In 1520 nails; how many yards?	Ans. 95yds.
3. In 800 quarters; how many yards?	Ans. 200yds.
4. In 340yds. 2qrs. 3na.; how many nails?	Ans. 5451na.
5. In 4234 nails; how many ells English?	Ans. 211 ells 3qrs. 2na.
6. In 3464 yards; how many ells Flemish?	Ans. 4618 ells 2 qrs.
7. In 6846 ells French; how many ells English?	Ans. 8215 E. E. 1qr.
8. In 60 ells French; how many yards?	Ans. 90yds.
9. In 60 ells English; how many yards?	Ans. 75yds.
10. In 60 ells Flemish; how many yards?	Ans. 45yds.
11. In 4 pieces of cloth, each 24 yards; how many nails?	Ans. 1536na.
12. In 8 pieces of cloth, each containing 25 ells English; how many yards?	Ans. 93yds. 3qrs.
13. In 1280 nails; how many pieces of cloth, each 20 yards?	Ans. 4 pieces.
14. In 644 ells Flemish, how many ells French?	Ans. 322 ells Fr.
15. In 84642 nails; how many ells Flemish?	Ans. 7053 ells Fl. 1qr. 2na.

DRY MEASURE.*

2 Pints (pt.)	make	1 Quart,	marked	qt.
8 Quarts	----	1 Peck,	----	pk.
4 Pecks	----	1 Bushel,	----	bu.
36 Bushels	----	1 Chaldron,	----	Ch.

* This measure is used in measuring fruit, potatoes, apples, pears, &c. which are given in rounding measure. Grain, salt, &c. are measured by striked measure, unless sold by weight.

NOTE.—In dry measure, a gallon contains 268^{4} cubick inches.

A Winchester bushel is 8 inches deep, and 18½ inches in diameter, and contains 2150⅖ cubick inches. This is the bushel in common use.

1. In 34 bushels; how many pecks, quarts, and pints?
Ans. 136pks. 1088qts. 2176pts.

```
      34bu.
       4
     136pks.
       8
    1088qts.
       2
  2)2176pts. Ans.
P 8)1088qts.
r 4)136pks.
o    34bu.
o
f
```

Dem.—We first multiply by 4, because 4 pecks make a bushel; we next multiply by 8, because 8 quarts make a peck; and lastly, we multiply by 2, because 2 pints make a quart.

Note.—It is evident, that reduction descending may be proved by reduction ascending, because it is only reversing our work; that is, dividing by the same numbers, which we multiplied by. And in reduction ascending, it is plain that we must, in proving our work, multiply by the numbers which were our divisors in the first operation, making our last divisor our first multiplier, and so on.

2. In 1024 pints; how many bushels? Ans. 16bu.

3. In 12 chaldrons of coal; how many pecks?
Ans. 1728pks.

4. In 40 bushels; how many pints? Ans. 2560pts.

5. How many pecks are there in 4 bins, each bin containing 25 bushels? Ans. 400pks.

6. In 46321 pecks; how many chaldrons?
Ans. 321ch. 24bu. 1pk.

7. A man wishes to barrel 300 bushels of corn for shipping, each barrel will hold 3 bushels and 3 pecks; how many barrels will it require? Ans. 80 barrels.

LIQUID MEASURE.* (*See top next page.*)

4	Gills (gi.)	make	1	Pint,	marked	pt.
2	Pints	----	1	Quart,	----	qt.
4	Quarts	----	1	Gallon,	----	gal.
31½	Gallons	----	1	Barrel,	----	[illegible].
42	Gallons	----	1	Tierce,	----	tier.
63	Gallons	----	1	Hogshead	----	hhd.
2	Hogsheads	----	1	Pipe,	----	P.
2	Pipes	----	1	Tun,	----	Tun.

* All spiritous liquors are measured by this measure; Rum, Brandy, Gin, Whiskey, &c.

Note.—The wine gallon contains 231 cubick or solid inches, and the ale gallon 282 cubick or solid inches.

EXAMPLES.

1. In 44 barrels; how many gallons, quarts, and pints?
Ans. 1386gal. 5544qts. 11088pts.

2) 44 bl.
31½
22
44
132
1386 gal. in 44 bl.
4
5544 qts. in 44 bl.
2
11088 pts. in 44 bl.

Dem.—Here, we first multiply by 31½, because 31½ gallons make a barrel; we next multiply by 4, because 4 quarts make a gallon; and lastly, by 2, because 2 pints make a quart.

2. In 26 hogsheads; how many gallons, and quarts?
Ans. 1638gals. 6552qts.

26 hhd.
63
78
156
1638 gal.
4
6552 qts.

Dem.—Here, we multiply by 63, because we must have 63 times as many gallons as hogsheads to equal our hogsheads in quantity. We next multiply by 4, because we must have 4 times as many quarts as gallons to equal our gallons in quantity.

3. In 2144 pints; how many hogsheads?
Ans. 4hhd. 16gal.
4. In 9 tuns of wine; how many gallons? Ans. 2268gal.
5. In 6048 gills; how many barrels? Ans. 6bl.
6. In 756 quarts; how many hogsheads? Ans. 3hhd.
7. In 168 barrels; how many gallons, and hogsheads?
Ans. 5292gal. 84hhd.
8. In 58 barrels of cider; how many gallons, quarts, and pints? Ans. 1827gal. 7308qts. 14616pts.
9. In 2016 pints; how many barrels? Ans. 8bl.
10. In 1008 quarts; how many hogsheads? Ans. 4hhd.

LONG MEASURE.[*]

3	Barley corns (b. c.) make	1	Inch,	marked	in.
12	Inches	1	Foot,	----	ft.
3	Feet	1	Yard,	----	yd.
5½	Yards, or 16½ feet	1	Rod, perch, or pole,		rd.
40	Rods	1	Furlong,	----	fur
8	Furlongs	1	Mile,	----	m.
3	Miles	1	League	----	lea.
69½	Statute miles	1	Degree on the earth,		Deg.

and 360 Degrees, the whole circumference of the earth.

EXAMPLES.

1. In 3 feet; how many barley corns? Ans. 108 b. c.

```
    3 feet.
   12
   ──
   36 inches.
    3
   ───
Ans. 108 b. c.
```

We first multiply by twelve, because every foot contains 12 inches; we next multiply the inches by three, because every inch contains three barley corns.

2. In 48 deg. 14 m. 7 fur. 30 rds. 5 yds. 2 ft. 3 in. and 1 b. c.; how many barley corns? Ans. 636952762 b. c.

```
Deg. m. fur. rd. yds. ft. in. b. c.
 48  14  7   30   5   2   3   1
 69½
 ───
  24
 446
288
────
3350 } miles in 48deg. 14m.
   8
─────
26807 } furlongs in
   40 } 48deg. 14m. 7fur.
───────
1072310 } rods in 48deg. 14m.
        } 7fur. 30rds.
```

Dem.—We first multiply by 69½, because 69½ miles make a degree; we next multiply by 8, because 8 furlongs make a mile; we then multiply the furlongs by 40, because 40 rods make a furlong; we then multiply by 5½ be

Carried up.

---

[*] By this measure we find the distance of one place from another, where length is considered without regard to breadth or thickness.—4 inches make a hand in measuring horses.

Note.—66 Feet, or four rods, make a gunter's chain. 60 Geometrical miles make a degree. 5 Feet a geometrical pace. 6 Feet a fathom, or one French toise. 6 points make 1 line; 12 lines an inch. A league is a denomination used to measure distance at sea.

1072310	*Brought up.*
5½	
536155	
5361555	
5897710	yards in 48deg. 14m. 7fur. 30rds. 5yds
3	
17693132	feet in 48deg, 14m. 7fur. 30rds. 5yds. 2ft.
12	
212317587	in. in 48deg. 14m. 7fur. 30rds. 5yds. 2ft. 3in.
3	
636952762	b. c. in 48deg. 14m. 7fur. 30rd. 5yd. 2ft. 3in. 1b. c.

cause 5½yds. make a rod; we then multiply by 3, because 3 feet make a yard; we then multiply by 12, because 12 inches make a foot; and lastly, we multiply by 3, because every inch contains 3 barley corns.

3. How many rods from Albany to New-York, it being 160 miles? Ans. 51200rds.

4. In 3 miles; how many barley corns? Ans. 570240 b.c.

5. How many barley corns will reach round the globe, it being 360 degrees in circumference? Ans. 4755801600 b.c.

6. In 2640 yards; how many rods? Ans. 480rds.

		yards.	
	5½	2640	
	2	2	
Half yards.	11)	5280	half yds.
Ans.		480	rods.

DEM.—We multiply by 2, to reduce our yards to half yards, we then multiply 5½, the number of yards in a rod, by 2, to reduce the divisor to half yards; we then divide the number of half yards contained in our given number, by the number of half yards in a rod, and as often as our divisor is contained, it is evident, so many rods are in the given number, because our quotient shows how often our divisor may be subtracted from our given number.

7. How often will a carriage wheel, 16 feet and 6 inches in circumference, turn round in going from Watertown to Utica; the distance being 84 miles? Ans. 26880 times.

8. In 15840 yards; how many miles and leagues?
Ans. 9 miles, 3 leagues.

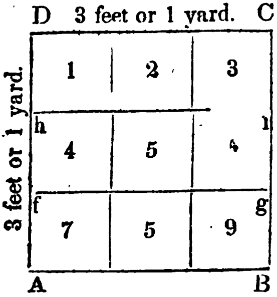
9. In 9 leagues; how many yards and feet?
Ans. 47520yds. 142560ft.

10. In 126720 feet; how many leagues?
Ans. 8 leagues.

LAND OR SQUARE MEASURE.*

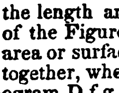
144	Square inches	make	1	Square foot.
9	Square feet		1	Square yard.
30¼	Square yards or 272¼ Square feet		1	Square rod, perch or pole.
40	Square rods		1	Square rood.
4	Square roods or 160 rods		1	Square acre.
640	Square acres		1	Square mile.

* This measure is used in reckoning the area of land in acres, rods, &c., or to find the surface of any thing, where length and breadth are considered without regard to thickness; as the annexed figure plainly shows:



A SQUARE.—The student already understands, that 3 feet in length make a yard in long measure. He must now learn, that it requires 3 feet in length and 3 in breadth to make a yard in square measure.

And this is evident, from our Figure; each side of which measures a yard, consequently it is a square yard. It is also plain, that it contains 9 square feet; because the area or surface of a plain figure, is estimated by the number of little squares contained in it; each side of the little squares 1, 2, 3, and so on, being 1 foot, it is obvious, that there are in the whole Figure A B C D, as many square feet as there are squares in the figure. Or perhaps it may appear more plain to the young student, if the figure A B C D be measured by the little square, (E,) whose side is 1 foot; then as often as the little square (E) is contained in the whole square A B C D, so many square feet the whole Figure must contain; and it is evident, that the little square (E) is contained 9 times in the whole square A B C D. From the Figure, it is likewise plain, that the area or surface of any square may be found by multiplying the length and breadth together; thus, 3×3=9 square feet, the area of the Figure annexed; and it is also plain, from the Figure, that the area or surface may be obtained by multiplying the length and breadth together, where the length and breadth are unequal, as in the Parallelogram D f g C, for in that it will be seen that the side D C is 3 feet, and the side D f, 2 feet, and when multiplied, they give the area thus, 3×2=6 square feet, the area of the Parallelogram D f g C; and if the figure be 3 feet long and 1 foot wide, as the Figure D h i C, it is plain that the area or surface is 3 square feet; because 3×1=3 square feet, the area of the Figure D h i C.



EXAMPLES.

1. In 50 acres; how many roods, and perches?

Ans. 200 roods, 8000 perches.

50 acres.
4
200 roods.
40
Ans. 8000 perches.

Dem.—We first multiply our fifty acres by 4, because we must have 4 times as many roods as acres, to equal our acres in quantity; we next multiply the product in roods by 40, because we must have 40 times as many rods or perches as we have roods, to equal our roods in quantity.

2. How many acres in 24000 rods? Ans. 150 acres.

3. How many feet of boards will it take to lay a floor 20 feet long, and 18 feet wide? Ans. 360ft.

4. If a field is 60 rods long, and 48 rods wide; how many acres does it contain? Ans. 18 acres.

48 rods
60
160)2880(18 Ans.
160
1280
1280
0

Dem.—We first multiply the length and breadth together, which gives the number of square rods in the whole field, because the breadth is 48 rods, and counting back one rod on the length, it is plain, we have 48 square rods; and counting back two rods, we have 96 square rods; it is then evident, by repeating the 48 rods, which may be considered as the front, by 60 rods, the distance back, the product must be the number of square rods in the whole lot; and when we divide these rods by 160, the number of rods in an acre, our quotient must express acres, because our quotient shows how often 160 square rods are contained in the whole number of rods.

5. How many bricks will it take to lay a floor 20 feet long, and 18 feet wide; each brick being 8 inches square?
18×20=360×144=51840÷64=810. Ans. 810 bricks.

6. In a square mile; how many square rods, square yards, and square feet?

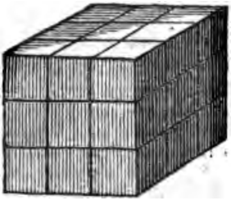
Ans. 102400 rods, 3097600yds. 27878400 feet.

SOLID OR CUBIC MEASURE.*

1728 Solid inches make	1 Solid foot.
40 Feet of round timber or 50 Feet of hewn timber	1 Tun or load.
128 Solid feet, or a pile 8 feet long, 4 feet wide, and 4 feet high	1 Cord of wood.

---

* By this measure, things that have length, breadth, and thickness, are measured.



We have seen in square measure, that the product of the length and breadth gave the area or surface of a square yd., a long square, or Parallelogram.

It will now appear obvious on inspecting the annexed figure, that a cubic yard is 3 feet in length, 3 feet in width, and 3 feet in thickness, that is, a square solid or cube, having six equal sides. Suppose the figure were 3 ft. in length, 3 ft. in breadth, and but 1 foot in thickness, it would then evidently contain ($3\times3\times1=9$) 9 cubick feet, that is, 9 solid squares, each square containing 1 solid foot; because length, breadth and thickness, in feet, multiplied together, give cubick feet; if 2 feet in thickness, it would contain ($3\times3\times2=18$) 18 cubick feet; and, as it is 3 feet in thickness, it must contain ($3\times3\times3=27$,) 27 cubick feet.

It is also evident, that a cubick foot, that is, a solid square, 12 inches in length, 12 inches in breadth, and 12 inches in thickness, will contain ($12\times12\times12=1728$) 1728 solid or cubick inches.

EXAMPLES.

1. In 9 tuns of round timber; how many feet and inches?
Ans. 360 solid feet, 622080 solid inches.

9 Tuns.
40
360 solid feet.
1728
2880
720
2520
360
Ans. 622080 solid inches.

Dem.—We first multiply the tuns by 40, because 40 feet of round timber make a tun; we next multiply the 360ft. by 1728, because 1728 inches make a solid foot.

2. In 9 tuns of hewn timber; how many solid feet, and inches?
Ans. 450ft. 777600in.

3. In 2688 solid feet; how many cords? Ans. 21cds.

4. In 10 cords of wood; how many solid feet?
Ans. 1280 feet.

5. In a pile of wood, 32 feet long, 5 feet high, and 4 feet wide; how many cords? Ans. 5 cords.

Note.—Multiply length, breadth, and thickness together, and the product will be the solidity, in the same denomination of the given numbers: thus, 32ft.×5ft.×4ft.=640ft.÷128ft.=5 cords.

6. In 221184 solid inches; how many cords? Ans. 1cd.

TIME.

60 Seconds, (s.)	make	1 Minute, marked	m.
60 Minutes,		1 Hour,	h.
24 Hours,		1 Day,	d.
7 Days,		1 Week,	w.
4 Weeks,		1 Month,	mo.
13 Months, 1 day and 6 h.		1 Julian year,	yr.

By the Calendar the year is divided into 12 months.—The number of days in each month, may be easily retained by the following verse:

"Thirty days hath September, April, June, and November;
February twenty-eight alone, and all the rest have thirty-one."

February has 29 days every bissextile, or leap year.

Note.—The civil or solar year contains 365 days, being short of the true year, according to the best computations, about 5 hours, 48 minutes, and 48 seconds; so that the time runs forward through the seasons nearly one day in four years. And it was on this account that Julius Cesar caused one day to be added to February every fourth year; reckoning the 24th day of this month twice, because the 24th day was the sixth (sextilis) before the kalends of March, there being in this year two of these sextiles, which gave the year the name of Bissextile year; but this has since been corrected and every 4th year February has 29 days, and in honour of Julius Cesar it is called the Julian year. The periodical year is the time which the earth takes in performing one revolution round the sun, in leaving any fixed point in the heavens and returning again to the same point. On account of certain points in the Ecliptick having a backward or retrograde motion, the Equinox will apparently meet the sun, so that the sun will arrive at the Equinox or first point of Aries, before his entire revolution is completed, and this time is called the tropical year.

EXAMPLES.

1. In 15 weeks; how many days, hours, minutes, and seconds? Ans. 105d. 2520h. 151200m. 9072000sec.

Demonstration.—Here, we first multiply by 7, because we must have seven times the number of days that we have weeks to equal our weeks in time; we next multiply by 24, because we must have twenty-four times as many hours as days to equal the days; we then multiply by 60, because we must have 60 times as many minutes as hours to equal the hours in time; we lastly multiply by 60, because we must have 60 times as many seconds as minutes.

15 weeks.
7
105 days.
24
420
210
2520 hours.
60
151200 minutes.
60
6|0)907200|0 seconds.
Proof. 6|0)15120|0
24)2520(7) 105
24 15
120
120

2. In 413 days; how many hours and minutes?

Ans. 9912h. 594720m.

3. In 413280 minutes; how many hours, days and weeks?

Ans. 6888h. 287d. 41w.

4. How many hours in 4 years, allowing 365 days and 6 hours to the year?

Ans. 35064h.

5. How many seconds in 60 years, allowing 365 days 6 hours, to the year?

Ans. 1893456000s.

6. From the 2d of March to the 19th of November; how many days? Ans. 262d.

7. From April the 19th to the 20th of January in the next year; how many days? Ans. 276d.

NOTE.—In reckoning time, exclude the first day and reckon the last.

CIRCULAR MEASURE OR MOTION.*

60	Seconds, sec. (″)	make	1	Minute, min. marked	′
60	Minutes,		1	Degree, deg.	°
30	Degrees,		1	Sign,	S.
12	Signs, or 360 degrees,		1	Circle of the zodiack.	

NOTE.—Every circle, great or small, contains 360 degrees.
The sun passes 1 degree on the earth in 4 minutes.

EXAMPLES.

1. How many minutes in the 12 signs of the zodiack?

Ans. 21600.

12S.
30
360 degrees.
60
Ans. 21600 min.

DEMONSTRATION.—We first multiply by 30, because 30 degrees make a sign; we next multiply by 60, because 60 minutes make a degree.

---

* This measure is used in measuring latitude and longitude, and in computing the revolutions of the earth and other planets round the sun. The great circle of a sphere, containing the 12 signs, through which the sun passes, is called the zodiack.

2. In six months the sun passes through 6 signs of the zodiack; how many degrees, minutes, and seconds?

Ans. 180deg. 10800min. 648000sec.

3. In 1124304 seconds? how many signs?

Ans. 10S. 12deg. 18min. 24sec.

The following denominations are not included in the tables:

112 Pounds ---- make ---- 1 Quintal of fish.
24 Sheets of paper -------- 1 Quire.
20 Quires --------------- 1 Ream.

SUPPLEMENT TO REDUCTION.

EXERCISES.

1. In 36 pounds; how many shillings, sixpences and threepences?

Ans. 720 shillings, 1440 sixpences, 2880 threepences.

2. In 19200 half pence; how many pence, shillings, and pounds? Ans. 9600 pence, 800 shil., 40 pounds.

3. In 48 guineas; how many dollars at 8 shillings each?

Ans. $168.

4. In 1 mile; how many rods? Ans. 320rds.

5. In 12244 pints; how many hogsheads?

Ans. 24hhd. 18gal. 2qts.

6. In 25600 rods; how many acres? Ans. 160 acres.

7. In 684 quarters; how many Ells English?

Ans. 136 Ells, 4qrs.

NOTE.—When it is required to know how many sorts of coin, and of each an equal number, are contained in a given sum; reduce the several sorts of coin to the lowest denomination mentioned in either, and add them together for a divisor; then reduce your given sum to the same denomination for a dividend, and divide, and the quotient will show how many your dividend contains of each kind. Observe the same directions with regard to weights and measures.

8. In 56 guineas; how many pistoles, pounds, dollars and shillings, and of each an equal number?

Ans. 30 of each kind, 38 shillings over.

1 pistole 22s.	56 guineas.
1 pound 20	28 the shillings in a guinea.
1 dollar 8	448
1 shilling 1	112
For a divisor 51s.	1568 shillings for a dividend.

Div. Dividend.
51)1568(30 of each kind.
   153
   ——
    38 over.

Dem.—The reason of this is obvious, because our divisor contains the amount of one coin of each kind, and our quotient shows how often this divisor can be subtracted from the dividend, consequently the quotient shows how often we can have a coin of each kind, because as often as the divisor, which contains a coin of each kind, can be subtracted from the dividend, so often, we must have a coin of each kind.

9. In 312 moidores; how many guineas, pistoles, pounds and dollars, and of each an equal number? Ans. 144 of each.

10. A silversmith received 20lb. of silver, with instructions to make it into spoons of 2oz.; cups of 3oz.; and tea-pots of 10 ounces, and of each a like number: what was the number? Ans. 16 of each.

11. A man desirous of racking off a barrel of brandy, in pint bottles, quart bottles, and two quart bottles, and of each an equal number, wishes to know how many bottles he must have of each kind? Ans. 36.

12. How many times will a ship 132 feet long sail her length in crossing the Atlantick, allowing the distance to be 3000 miles? Ans. 120000 times.

13. How often will a chariot wheel, 18 feet 4 inches in circumference, turn round in running 22 miles?

Ans. 6336 times.

14. A merchant imported from London 46 bales of broadcloth, each containing 24 pieces, and each piece 40 Ells English; how many yards were there? Ans. 55200 yards.

15. The cargo of a ship from the West Indies, consisted of 500 hogsheads of molasses, and 200 pipes of wine; how many gallons in all, and how much was the ship's burden, allowing every pint to weigh 1 pound?

Ans. 56700gal. 202 tuns, 10cwt.

16. It is required to divide 10 pounds 13 shillings, between a man, woman, and boy; the man must have 9 times as much as the woman, and the woman 7 times as much as the boy; what will be the share of each?

Ans. { 189 shillings, the man's share, 21 shillings, the woman's, and the boy's 3 shil.

The b. 1 share. The w. 7 shares. The m. 63 shares.
1 × 7 = 7 × 9 = 63 The man's shares.
£ s. 7 The woman's shares.
10 13 1 The boy's share.
20 Divisor 71 Shares in all.
71) 213 (3 boy's share or quotient, which shows that
213 7 he may draw his one, three times.
0 21 shillings, woman's share.
9
189 shillings, man's share.

189 man's share.
Proof. 21 woman's share.
3 boy's share.
2|0)21|3
£10 13s. equals the given sum.

Dem.—It is plain that the given sum must be divided into 71 parts, because considering the boy to have one part or share, the woman must have 7, because she must have 7 times as much as the boy; and for the same reason the man must have 63, because he must have 9 times as much as the woman, whose share is 7, and 9 times 7 are 63. Then when these shares are added, we find they amount to 71, then dividing the given sum by 71, must give the boy's share, because he has but one share in the 71; and we find by dividing, that our quotient is 3 shillings for the boy's share; then the 3s. multiplied by 7 must give the woman's share, because she has seven times as much as the boy; that is, 21s for the woman's share; the man must have 9 times as much as the woman; then multiplying the 21, the woman's share, by 9, must give the man's share, which we find to be 189 shillings; then by adding these shares we find they amount to the given sum, which proves our work.

17. Let 192 dollars be divided among three men, A, B, and C, in such a manner, that as often as A receives $5, B may have $8; and as often as B receives $8, C may have $11; what number of dollars will each receive?

Ans. A $40, B $64, C $88.

QUESTIONS ON REDUCTION.

What is reduction? A. Changing one denomination to another. By changing denominations do you alter the value? A. No; because if we change from a superiour to an inferiour denomination, the inferiour must express as many more as shall equal the superiour in value. How many kinds of reduction are there? A. Two, reduction descending and reduction ascending. What is reduction descending? A. Bringing higher denominations to lower. How is reduction descending performed? A. By multiplication. What rule do you observe in multiplying? A. Multiply the highest denomination given by that number which it takes of the next lower to make a unit in that highest, and if the given sum contains any in the next inferiour denomination, add them to the product, and so continue the work till the given sum is reduced to the denomination required. What is reduction ascending? A. Changing lower denominations into higher. How is it performed? A. By division; divide the given number by that number which will make a unit in the next higher, and so continue to do, till you have brought it to the denomination required. Suppose you have remainders, what must you call them? A. Of the same name of the dividend that produced them; and they must be brought down in the regular order of the denominations in the quotient or answer. How do you reduce dollars to cents? A. By multiplying the dollars by 100, and the product will express cents. Why should that reduce dollars to cents? A. Because we must have 100 times as many cents as dollars to express the same in value. If your given sum contains dollars and cents, how may you reduce them to cents? A. By taking away the separatrix, and joining the cents to the dollars. Why should that reduce the whole to cents? A. Because the dollars when united to the cents express hundreds in cents, and the cents count the same as before. How do you reduce cents to mills? A. By multiplying the cents by 10. Why should that reduce cents to mills? A. It is repeating the cents 10 times, and we must have ten times the number of mills, to equal the cents in value. What are the denominations in sterling money? A. Pounds, shillings, pence, and farthings. How do you reduce pounds to shillings? A. By multiplying the pounds by 20. Why multiply the pounds by 20? A. Because it takes 20 shillings to make a pound, and we must have 20 times the number of shillings to equal our pounds in value. How do you reduce shillings to pence? A. By multiplying the shillings by 12, because it takes 12 pence to make a shilling, and we must have 12 times as many pence as shillings to equal the shillings in value. How do you reduce pence to farthings? A. By multiplying the pence by 4, because it takes 4 farthings to make a penny, and we must have 4 times as many farthings as pence to equal our pence in value? How would you reduce pounds to farthings? A. I multiply the pounds by 20, and have shillings for the product; and then multiply the shillings by 12, and have pence for my product; and lastly, I multiply the pence by 4, and the product is farthings. How do you reduce farthings to pounds? A. Divide the farthings by 4, and the quotient will be pence, because one fourth the number of pence will equal the farthings in value; and then divide the pence by 12, because it takes 12 pence to make a shilling; and lastly, divide the shillings by 20, because it takes 20 shillings to make one pound, and one twentieth the number of pounds will equal the shillings in value. How do you reduce pounds to sixpences? A. Multiply by 20, and then that product by 2. Why multiply by 20 and 2? A. Because 20 shillings make a pound and 2 six-pences make a shilling. How do you reduce grains to pounds Troy? A. Divide by 24, because 24 grains make one pennyweight, and the penny weights divide by 20, because 20 pennyweights make an ounce, and then the ounces by 12, because 12 ounces make 1 pound. How do you reduce tuns to pounds? A. First multiply by 20, because each tun will make 20 hundred weight, and then multiply by 4, because each hundred contains 4 quarters, and lastly, by 28, because each quarter contains 28 pounds. How do you reduce guineas to pounds? A. Multiply the guineas by 28, and the product will be shillings, and then divide the shillings by 20, and the quotient will be pounds. How do you reduce pounds to moidores? A. Multiply the pounds by 20, and the product will be shillings, and then divide the shillings by 36, and the quotient will be moidores. Suppose a barrel of wine is to be racked off, in pint bottles, quart bottles, and 2 quart bottles, and an equal number of each is required; how would you proceed to tell the number of bottles required to hold the wine? A. Add, in one sum, the number of pints that would fill the bottles once, for the divisor, and then reduce the barrel to pints for a dividend, and the quotient would show how often the bottles could be filled.

COMPOUND ADDITION,

Is collecting together two or more numbers of different denominations in one sum.

RULE.—Place the numbers, so that those of the same denomination may stand directly under each other. Then add the right hand denomination the same as a sum in simple addition, and divide the amount by as many as it takes of that denomination to make a unit in the next higher denomination; set down the remainder under the denomination added, and carry the quotient to the next higher denomination; and so proceed with all the denominations until you come to the left hand denomination, and there set down the whole amount the same as in simple addition.

PENCE TABLES.

		s.	d.	1	shilling is	12d.
20	pence is	1	8	2	....	24
30	......	2	6	3	....	36
40	......	3	4	4	....	48
50	......	4	2	5	....	60
60	......	5	0	6	....	72
70	......	5	10	7	....	84
80	......	6	8	8	....	96
90	......	7	6	9	....	108
100	......	8	4	10	....	120
110	......	9	2	11	....	132
120	......	10	0	12	....	144

NOTE.—The principles in compound quantities do not essentially differ from those of abstract numbers, that is, simple, refined, pure, not broken numbers. In the one case, we carry by 10; in the other, by the number of units which it takes to equal a unit in the next higher

nomination; so that the student will find the work easy, if he has ac-
ired a knowledge of the tables, and will only keep in mind the rule
en for the operation of the work. Yet it is to be regretted,
it the wisdom of our legislature, has never reduced compound
antities to the decimal system, which would greatly simplify the
erations of work in arithmetick. The French have reduced all com-
und quantities to the decimal scale, except the division of time,
iich does not appear to be capable of change. A wise policy may
tate never to adopt hastily a new system; but after the utility of a
tem has become obvious, neither national pride nor narrow preju-
es should prevent its adoption; no matter by what individual or
lividuals the system may have been simplified, or by what nation
opted, the benefits are the same.

EXAMPLES.

STERLING MONEY.

1. In £41 13s. 6d. 2qrs., £48 11s. 4d. 3qrs., £96 16s.
d. 3qrs.; how many pounds, shillings, pence, and farthings,
ıen added together?

£	s.	d.	qrs.
41	13	6	2
48	11	4	3
96	16	10	3
£187	1	10	0 Ans.

DEM.—We first add the denomination of farthings, and find it amounts to 8, which we divide by 4, because 4 farthings are equal to one penny, and we find that the quotient is 2, and the remainder a cipher; so we set down the cipher and carry 2 to
pence, the next higher denomination, because the 8 farthings
equal to 2 pence; by adding the pence, we find the amount 22,
ich we divide by 12, because it takes 12 pence to make a shilling,
l the quotient is 1, and the remainder 10; we set down 10 under
column of pence, and carry the quotient, which is 1 shilling, to the
llings; and in adding the shillings, we find they amount to 41, which
divide by 20, because it takes 20 shillings to make 1 pound, we
l the quotient 2, and the remainder 1, which we set down under
lings, and carry the quotient, which is pounds, to the pounds; and
ıdding the left hand denomination, we carry the same as in simple
k, and set down the whole amount in adding the left hand column.
ıll other sums in compound addition we observe the same rules;
ıys carrying by that number which is equal to a unit in the next
ıer.

roof, the same as in simple work, only carry as in the first adding.

2.

£	s.	d.
44	13	6
38	17	8
19	10	11
16	18	10
120	0	11 Ans.

3.

£	s.	d.
63	11	6
84	9	11
110	16	8
130	11	10

4.

£	s.	d.
63	16	11
89	11	10
67	14	9
111	16	10

5. What is the amount of £47 16s. 11d. 3qrs., £105 14s. 11d. 2qrs., £89 18s. 4d. 1qr., and £844 13s. 10d. 0qrs., when added? Ans. £1088 4s. 1d. 2qrs.

6. What is the amount of one hundred and five pounds, fourteen shillings, sixpence, one farthing; eighty-four pounds, ten shillings, four pence, three farthings; and five hundred pounds, fifteen shillings, ten pence, three farthings, when added together? Ans. £691 0s. 9d. 3qrs.

7. What is the amount of 2s. 6d., 4s. 8d., 9s. 3d., 4s. 9d.? Ans. £1 1s. 2d.

8. A man bought a wagon for £18 16s. 8d., a plough for £2 10s., a span of horses for £55 10s. 6d.; what must he pay for the whole? Ans. £76 17s. 2d.

9. Three men found a prize, and having divided it equally among them, each man received £18 4s. 1d.; what was the amount of the prize? Ans. £54 12s. 3d.

TROY WEIGHT.

1.	*lb.*	*oz.*	*pwts.*	*gr.*
	36	7	11	10
	34	5	8	15
	81	8	18	14
Ans.	152	9	18	15

NOTE.—Observe to carry by 24 from grains to pennyweights, and by 20 from pennyweights to ounces, and by 12 from ounces to pounds.—Proof may be performed by adding downwards.

2.				3.				4.			
lb.	*oz.*	*pwt.*	*gr.*	*lb.*	*oz.*	*pwt.*	*gr.*	*lb.*	*oz.*	*pwt.*	*gr.*
39	8	10	21	81	10	18	22	63	10	11	19
84	9	11	18	18	11	10	19	18	10	9	3
13	6	15	17	66	10	15	16	6	7	4	4

5. Bought a set of silver spoons weighing 1lb. 10oz. 9pwts. 17grs., a silver cup weighing 5oz. 10pwts. 14grs., and a silver tankard weighing 1lb. 11oz. 7grs.; what was the weight of the whole? Ans. 3lb. 6oz. 0pwts. 14grs.

AVOIRDUPOIS WEIGHT.

1.	*T.*	*cwt.*	*qr.*	*lb.*	*oz.*	*dr.*
	13	12	1	15	6	11
	14	14	3	24	9	10
	18	15	2	11	15	11
Ans.	47	2	3	24	0	0

NOTE.—We carry from drachms to ounces by 16, because 16 drachms make an ounce, and from ounces to pounds by 16, because 16 ounces make a pound; and from pounds to quarters by 28, because 28 pounds make a quarter; and from quarters to hundreds by 4, because 4 quarters make a hundred; and from hundreds to tuns by 20, because 20 hundred make a tun.

2.

cwt.	qrs.	lb.	oz.	dr.
18	3	16	5	14
105	2	26	14	13
108	3	16	11	11
64	1	13	10	15

3

cwt.	qrs.	lb.	oz.	dr.
180	1	16	11	12
14	2	15	10	13
14	1	15	15	14
	2	27	11	15

4. Bought 4 fat oxen, weighing as follows, viz., 12cwt. 3qrs. 16lb., 13cwt. 2qrs. 12lb. 13oz.; 15cwt. 1qr. 21lb. 11oz., 16cwt. 1qr. 24lb. 8oz.; what was the weight of the whole in tuns? Ans. 2T. 18cwt. 1qr. 19lb.

APOTHECARIES' WEIGHT.

1.

	dr.	sc.	gr.
	5	1	15
	6	2	18
	4	0	17
oz.	6	2	11
Ans. 2	7	2	1

2.

oz.	dr.	sc.	gr.
10	7	2	18
9	6	1	19
11	3	2	18
10	5	1	17

3.

lb.	oz.	dr.	sc.	gr.
10	11	6	1	14
13	10	5	2	18
11	11	4	1	11
7	9	3	0	10

CLOTH MEASURE.

1.

	yds.	qrs.	n.
	44	3	2
	53	1	1
	11	3	3
	14	1	3
Ans.	124	2	1

2.

yds.	qrs.	n.
546	2	3
46	1	2
36	2	1
1	3	1

Dem.—It is evident, that we must carry by 4 from nails to quarters, because 4 nails make a quarter; and by 4 from quarters to yards, because 4 quarters make a yard.

3.

Yds.	qr.	n.
36	1	3
44	2	1
55	2	3
68	1	2

4.

E. Fl.	qr.	n.
41	2	3
36	1	2
44	2	1
36	1	3

5.

E.E.	qr.	n.
84	3	3
68	3	2
1	4	1
6	1	3

6.

E.Fr.	qr.	n.
38	3	3
98	5	2
36	3	2
54	4	3

7. Bought 4 pieces of cloth, the first 96yds. 3qrs. 2n.; the second 84yds. 3qrs. 3n.; the third 75yds. 1qr. 2n.; the fourth 96yds. 1qr. 1n.; how much did the four pieces contain?

Ans. 353½ yards.

LONG MEASURE.

1.

	Deg.	*m.*	*fur.*	*rd.*	*ft.*	*in.*	*b.c.*
	63	44	6	30	14	10	2
	56	54	7	35	11	6	2
	10	57	6	16	7	8	1
Ans.	131	18	5	3	1	1	2

2.

Deg.	*m.*	*fur.*	*rd.*	*ft.*	*in.*	*b.c.*
79	65	7	13	11	10	1
34	65	6	35	14	11	2
84	55	6	14	15	11	2

3.

Deg.	*m.*	*fur.*	*rd.*	*yds.*	*ft.*	*in.*	*b. c.*
84	65	6	35	4	2	6	2
95	55	7	14	5	1	11	1
37	1	2	0	0	2	3	2

*

*Note.—When halves or fractions occur with the remainder after the division, they must be carried back in the inferiour denominations according to the value; and if by carrying back it increases the inferiour denominations so as to equal a unit, or exceed a unit, at the left, a unit must then be added to the superiour, and the excess set down in its proper place; for fractions must never be placed in a sum, when there are denominations at the right.

LAND, OR SQUARE MEASURE.

1.

	Rood.	*Pol.*	*ft.*	*in.*
		19	180	135
		25	250	120
		13	90	85
Ans.	1	18	250	16

Dem.—In adding the inches, we find they amount to 340 inches, which contain 144 twice, and 52 remain, which we retain, and carry the 2 to the feet; we find the feet amount to 522, which we divide by 272¼, which we find contained once, and 249¾ remain, and as we cannot set down a fraction in the superior denominations, we reduce the ¾ of a foot to inches, which produce 108 inches, and the 52 inches added which was our remainder in inches, we have 160 inches, which are equal to 1 foot and 16 inches; we then set down the 16in. under inches, and add the foot to 249 feet, which gives 250 feet, which we place under feet, and carry 1 to the poles, and by adding we nnd the poles amount to 58, which divided by 40, the number of poles in a rood, gives 1 rood and 18 poles.

A.	*rds.*	*pol.*	*ft.*	*in.*
75	3	37	245	30
87	1	37	75	114
415	3	18	69	135

I

A.	*rds.*	*pol.*	*ft.*
87	2	39	150
185	3	36	27
19	1	7	9

4. A gentleman has three farms, which measure as follows, viz.: 150 A. 3 roods, 14 poles; 175 A. 2 roods, 18 poles; 100 A. 1 rood, 39 poles; what do they measure in all?

Ans. 426 A. 3 roods, 31 poles.

SOLID, OR CUBICK MEASURE.

1.

	Ton.	*ft.*	*in.*
	47	36	1295
	11	41	375
	14	35	171
Ans.	74	13	113

2.

Yds.	*ft.*	*in.*
47	18	1475
87	26	145
13	9	47

3.

Cds.	*ft.*
38	120
9	98
65	127

4. A man bought 3 piles of wood, which contain as follows, viz.: 37 cords, 51 feet; 14 cords, 120 feet; 19 cords, 95 feet; how many cords did he purchase? Ans. 72 cords, 10 feet.

LIQUID MEASURE.

1.

	Hhd.	*gal.*	*qt.*	*pt.*
	81	38	1	1
	75	18	3	0
	18	55	0	1
Ans.	175	49	1	0

2.

T.	*hhd.*	*gal.*	*qt.*
11	2	54	3
17	3	18	1
95	1	15	2

3.

T.	*hhd.*	*gal.*
9	2	55
19	3	62
81	1	60

4. A merchant bought three casks of whiskey, each containing 65gal. 1qt. 1pt.; what number of gallons did they contain? Ans. 196gal. 1 pint.

DRY MEASURE.

1.

	Bu.	*pk.*	*qt.*	*pt.*
	45	3	5	1
	84	2	7	0
	95	1	6	1
Ans.	226	0	3	0

2.

Bu.	*pk.*	*qt.*	*pt.*
41	3	3	1
84	1	2	0
67	2	1	0

3.

Ch.	*bu.*	*pk.*	*qt.*	*pt.*
75	24	3	3	1
65	19	1	1	0
18	31	2	2	1

4. A, sent to market at one time, 38 bushels 2 pecks, at another 29 bushels 3 pecks, at another 57 bushels 1 peck; what number of bushels did he send to market?

Ans. 125 bushels, 2 pecks.

TIME.

1.

	Y.	mo.	w.	d.	h.	m.	s.
	44	12	3	6	23	55	45
	38	10	2	5	13	50	49
	84	11	3	3	21	59	48
Ans.	168	9	2	2	11	46	22

2.

Y.	mo.	w.	d.	h.
34	6	3	4	23
50	8	2	5	06
85	9	2	3	07

3.

Y.	d.	h.	m.
34	364	23	45
75	325	08	44
320	225	14	59

4.

Y.	mo.	w.	d.	h.
47	8	3	5	14
88	11	3	6	22
79	9	2	5	8

CIRCULAR MEASURE, OR MOTION.

1.

	S.	deg.	m.	sec.
	3	25	17	24
	1	15	49	50
	8	12	55	11
Ans.	13	24	2	25

2.

S.	deg.	m.
4	23	48
6	29	48
7	26	55

3.

S.	deg.	m.	sec.
4	23	56	59
6	8	44	48
3	9	6	36

4. Add 11 signs, 24 degrees, 55 minutes, 25 seconds; 8 signs, 19 degrees, 59 minutes, 17 seconds; 8 signs, 29 degrees, 11 minutes, 18 seconds. Ans. 29S. 14deg. 6m.

COMPOUND SUBTRACTION,

Is taking a less sum or quantity from a greater of different denominations.

RULE.—Place the less sum or quantity under the greater, so that those of the same denomination may stand directly under each other; begin with the least denomination, and if it exceed that directly above it, subtract it from as many of that denomination as shall equal a unit in the next higher, and to the difference add the upper denomination; remembering always, when you have thus subtracted, to add one to the next lower denomination; but when the lower denomination does not exceed that directly above, you merely set down the difference, and proceed to the next without carrying.

Proved by adding the difference to the subtrahend; and if the amount equal the minuend, the work is right.

STERLING MONEY.

1. If a boy has 3 shillings and 9 pence given him, and he pays out 2 shillings and six pence; how much has he left? Ans. 1s. 3d.

	s.	d.
	3	9
	2	6
Ans.	1s.	3d.
Proof.	3s.	9d.

Dem.—It is plain, by taking 6 pence from 9 pence there will 3 pence remain; and by taking 2 shillings from 3 shillings, 1 shilling must remain; and because the difference between the pence is three pence; and the difference between the shillings, 1 shilling; the whole difference must be 1 shilling and three pence.

2. A man lent his neighbour 4 pounds, 6 shillings, and 8 pence; he afterwards received 2 pounds, 16 shillings, and 9 pence; what is yet his due? Ans. £1 9s. 11d.

	£	s.	d.
	4	6	8
	2	16	9
Ans.	1	9	11
Proof.	4	6	8

Dem.—The pence in the subtrahend exceeding the pence in the minuend, we subtract the 9 pence from twelve, (the number which it takes to make a unit in shillings,) by saying, 9 from 12, 3; and to 3, the difference, we add 8, the pence in the minuend, which gives 11 pence to set down in the place of pence; we then add 1 to the shillings in the subtrahend, and say 17 from 20, (the number of shillings which it takes to make a pound;) 3 is the difference, to which we add 6, the shillings in the minuend; we then have 9 shillings to set down, and we add 1 to the pounds in the subtrahend; we then say, 3 from 4 leaves 1, which we set directly under. The 12 pence which we add to the minuend are just equal to the 1 shilling which we add to the subtrahend, so that it is adding equals to both the given sums; and adding equals to both; the difference must ever remain the same; this, however, by some, is called borrowing.

3.	£.	s.	d.	qrs.
	46	16	11	2
	10	18	10	3
Ans.	35	18	0	3

Note.—The student should bear in mind that the principles here are the same as in simple numbers, only that it takes different numbers here, to make a unit in the next higher; as in farthings, it takes 4 to make a unit in pence; and in pence it takes 12 to make a unit in shillings; and in shillings it takes 20 to make a unit in pounds.

4.

£	s.	d.	qrs.
14	13	10	1
3	10	1	0

5.

£	s.	d.	qrs.
7756	16	10	2
6050	10	10	3

6.

£.	s.	d.
884	4	6
60	9	7

7.

£.	s.	d.
89	8	8
78	18	9

8.

£.	s.
405	8
68	15

9.

£.	s.	d.
907	6	8
59	8	6

10.

£.	s.	d.
895	16	7
705	8	8

11.

£.	s.
48	8
7	9

12. Borrowed £29 16s., and paid £16 11s. 9d.; how much remains due? Ans. £13 4s. 3d.

13. From £397 16s. 6d.; take £144 11s. 4d. 3qrs. Ans. £253 5s. 1d. 1qr.

14. From seven pounds; take seven pence, three farthings. Ans. £6 19s. 4d. 1qr.

15. How much does eighty-nine pounds, sixteen shillings, eleven pence, three farthings, exceed forty-eight pounds, eleven shillings, ten pence, two farthings? Ans. £41 5s. 1d. 1qr.

16. From one thousand pounds; take one farthing Ans. £999 19s. 11d. 3qrs.

17. How much does two thousand pounds, exceed 19s. 6½d.? Ans. £1999 0s. 5½d.

TROY WEIGHT.

1. From 8lb. 11oz. 16pwt. 19gr.; take 1lb. 5oz. 12pwt. 23gr. Ans. 7lb. 6oz. 3pwt. 20 gr.

1.

	lb.	oz.	pwt.	gr.
	8	11	16	19
	1	5	12	23
Ans.	7	6	3	20
Proof.	8	11	16	19

2.

lb.	oz.	pwt.	gr.
86	10	17	9
78	3	8	20

AVOIRDUPOIS WEIGHT.

	lb.	*oz.*	*dr.*
	18	11	13
	16	15	12
Ans.	1	12	1

lb.	*oz.*	*dr.*
96	14	14
80	15	13

T.	*cwt.*	*qr.*	*lb.*
72	15	3	27
44	3	0	27

APOTHECARIES' WEIGHT.

1.

	℔	℥	ʒ
	84	4	6
	30	9	7
Ans.	53	6	7
Proof.	84	4	6

2.

ʒ	℈	*gr.*
4	2	8
3	2	9

3.

℔	℥	ʒ	℈	*gr.*
40	7	3	2	7
24	4	5	2	8

DRY MEASURE.

1.

	Ch.	*bu.*	*pk.*	*qt.*
	47	34	3	6
	20	35	0	7
Ans.	26	35	2	7
Proof.	47	34	3	6

2.

bu.	*pk.*	*qt.*	*pt.*
8	3	2	0
5	2	5	1

3.

bu.	*pk.*	*qt.*	*pt.*
45	2	7	0
34	3	5	1

LIQUID MEASURE.

1.

	T.	*hhd.*	*gal.*	*qt.*	*pt.*	*gi.*
	88	3	55	3	0	0
	48	0	62	2	0	2
Ans.	40	2	56	0	1	2
Proof.	88	3	55	3	0	0

2.

Hhd.	*gal.*	*qt.*
44	50	2
20	60	3

3.

Hhd.	*gal.*	*qt.*
54	59	3
45	60	2

CLOTH MEASURE.

1.

	Yd.	*qr.*	*n.*
	47	3	2
	27	2	3
Ans.	20	0	3
Proof.	47	3	2

2.

E.Fl.	*qr.*	*n.*
87	2	2
59	2	3

3.

E.E.	*qr.*	*n.*
756	2	2
374	3	3

4.

E.F.	*qr.*	*n.*
760	3	0
479	5	2

LONG MEASURE.

1.

	Deg.	*m.*	*fur.*	*pol.*	*yd.*	*ft.*	*in.*	*b.c.*
	58	48	7	28	4	2	2	2
	36	25	6	39	0	2	8	0
Ans.	22	23	0	29	3	2	6	2
Proof.	58	48	7	28	4	2	2	2

2.

Lea.	*m.*	*fur.*	*pol.*
8	2	6	26
5	0	7	29

LAND, OR SQUARE MEASURE.

1.

	A.	*rds.*	*pol.*
	36	3	28
	28	2	39
Ans.	8	0	29

2.

A.	*rds.*	*pol.*
450	3	29
220	3	24

3.

Pol.	*sq.ft.*	*sq.in.*
38	24	62
2	216	143

SOLID, OR CUBICK MEASURE.

1

	T.	*ft.*	*in.*
	44	44	884
	39	39	982
Ans.	5	4	1630
Proof.	44	44	884

2.

Cds.	*ft.*
48	25
35	98

3.

T.	*ft.*
284	39
86	27

TIME.

1.

	Y.	*mo.*	*w.*	*d.*	*h.*	*m.*	*s.*
	250	4	3	6	22	55	49
	220	8	0	5	23	59	55
Ans.	29	9	3	0	22	55	54
Proof.	250	4	3	6	22	55	49

2.

Y.	*d.*	*h.*	*m.*	*s.*
205	325	20	56	24
202	335	23	59	56

CIRCULAR MEASURE, OR MOTION.

1.

	S.	*deg.*	*m.*	*s.*
	6	25	45	38
	4	29	38	49
Ans.	1	26	6	49

2.

S.	*deg.*	*m.*	*s.*
9	29	59	44
8	28	48	24

EXAMPLES,

Showing the use of Compound Addition and Subtraction.

1. Lent a friend at one time £13 16s. 8d., at another, £35 10s. 6d.; he paid at one time, £10 10s. 11d., at another, £9 3s. 6d.; how much remains due? Ans. £29 12s. 9d.

2. A man had 375 acres 2 roods of land, but one of his sons marrying, he gave him 145 acres; how much had he left? Ans. 230 acres 2 roods.

3. Borrowed $375,50 cts., paid at one time $95,20 cts., at another $81,10 cts.; how much remains due?

Ans. $199,20 cts.

4. A gentleman paid his 3 labourers as follows, viz.: A he gave £11 13s. 6d.; B he gave 13s. and 6d. more than A; and C he gave as much as A and B both; how much did the gentleman pay out? Ans. £48 1s.

5. James was bound as an apprentice for 6 years and 7 months; he served 3 years and 9 months, and then bought his time; how much time had he to pay for?

Ans. 2 years 10 months.

6. A merchant put in a bin 47 bushels and 3 pecks of corn; he afterwards sold out 25 bushels and 1 peck; how much had he left in the bin? Ans. 22bu. 2 pk.

7. From a piece of cloth containing 47 yards 2 quarters, a merchant sold four suits, each 6 yards 2 quarters; how much of the piece remains unsold? Ans. 21yds. 2qrs.

8. A gentleman left his son 1005 pounds more than his daughter, whose portion was 4475 pounds 10 shillings; what was the son's portion, and what was the whole estate?

Ans. { £5480 10s. the son's portion.
£9956 the whole estate.

9. From 11 years 6 months 11 days, take 10 years 6 months 29 days? Ans. 11 months 12 days.

10. The revolutionary war broke out between Great Britain and the United States, April 19th, 1775, and the treaty of peace was ratified January 20th, 1783; how long did the war last? Ans. 7 years 9 months 1 day.

COMPOUND MULTIPLICATION,

Is repeating a number of different denominations a certain proposed number of times, or it is the shortest method of performing compound addition where the same number is to be repeated.

RULE.—When the quantity does not exceed 12 yards, pounds, &c. set down the price of one, and place the quantity under the right hand denomination for a multiplier. Multiply the right hand denomination, and divide the product by as many as it takes of that denomination to make one in the next at the left; set down the remainder, (if any,) under the denomination multiplied, and carry the quotient to the product of the next left hand denomination, and so proceed till you come to the left hand denomination, and there set down the product as in simple multiplication.

NOTE. This rule will be found very useful in business; where the price of 1 yard, or 1 pound, is given, by it, we find the price of a quantity; by multiplying the price of one yard, or one pound, or a unit of any kind, by the quantity, the product will be the price of the quantity. Or if the weight of one hogshead or of one bale is given, we find the weight of the whole by multiplying the weight of one, by the whole number; and in like manner we find the weight of any number of boxes or bags where the weight of one is given.

INTRODUCTORY EXAMPLES.

1. If one yard of broadcloth cost 1 pound 3 shillings and 4 pence; what will two yards cost? Ans. £2 6s. 8d.

	£	s.	d.
	1	3	4
			2
Ans.	2	6	8

DEM.—It is plain, that the price of one yard multiplied by 2, must give the price of two yards, because it is repeating every denomination expressing the price of one yard by 2; and it will plainly appear by adding, that this is only a short method of performing addition.

£	s.	d.
1	3	4
1	3	4
2	6	8

DEM.—Here, we find by addition the same result is produced; and the same relation exists when we have three, four, five, or any other number for a multiplier, which may clearly be shown by adding the multiplicand as many times as it has been repeated in multiplication.

2. If one Tun of hay cost £2 12s. 8d.; what will three Tuns cost at that rate? Ans. £7 18s. 0d.

	£	s.	d.
	2	12	8
The number of Tuns,			3
Ans.	£7	18	0

DEM.—Here, we first multiply 8d, by 3, and we have 24 for the product; we then divide this product by 12, because it takes 12 pence to make 1 shilling; and we have 2 for a quotient and nothing for a remainder, so we set down a cipher under the pence, and then say three times twelve are thirty-six, and add the quotient, two, which we carry from the pence; we then have 38, which we divide by 20, because 20s. make a pound; we find 20 contained in 38 once and 18 over, which we set down under shillings, and carry 1 pound, which is the quotient, to the product of pounds, by saying three times 2 are 6 and 1 are 7, which we place under pounds, its proper place. This example we will illustrate by addition, so that the student may not mistake the principles of the rule, or the relation which addition and multiplication bear to each other.

	£	s.	d.
	2	12	8
	2	12	8
	2	12	8
Ans.	7	18	0

Here we again see that the same result is produced by adding the price of one yard instead of multiplying. The student will perceive that we carry in both cases the same; when we multiply the 8 pence by 3, the product is 24 pence, that is 2 shillings, consequently 2 shillings may be added to the product of shillings; and in addition we have 24 pence, that is, 2 shillings which may be added to the shillings, and in multiplying the 12 shillings the product is 36 shillings, and the 2 shillings to carry make 38 shillings, that is, 1 pound and 18 shillings, consequently we set down the 18 shillings and carry the pound to the product of pounds; and by addition we obtain 38 shillings the same as in multiplication, that is, 1 pound and 18 shillings; consequently we set down the 18 shillings and carry the 1 pound, and by adding we obtain 7 pounds, and the same number by multiplying.

3. What will 4 yards of cloth come to, at 3s. 4d. per yard? Ans. 13s. 4d.

4. What cost 5 yards of calico, at 3s. 9d. per yard? Ans. 18s. 9d.

5. What will 6 tuns of hay cost, at £2 3s. 6d. per tun? Ans. £13 1s. 0d.

6. A man bought 7 sheep at 9s. 6d. per head; what did they cost him? Ans. £3 6s. 6d.

7. A man bought 8 bushels of apples, at 2s. 6d. per bushel; what did they cost him? Ans. £1.

8. Bought 9 pieces of shirting, each containing 28yds. 2qrs. 2na.; how many yards in the 9 pieces? Ans. 257yds. 2qrs. 2na.

9. Bought 10 cwt. of beef at £1 18s. 6d. per cwt.; what did the whole cost? Ans. £19 5s.

10. What must a merchant pay you for 11cwt. of pork, at £2 1s. 6d. per cwt.? Ans. £22 16s. 6d.

11. What will twelve horses come to, at £29 16s. 8d. each? Ans. £358.

CASE II.—*When the multiplier exceeds* 12, *and is a composite number, produced by multiplying together any two figures in the multiplication table, it will often be found more convenient, first, to multiply by one of the component figures, and then that product by the other; the last product will be the answer.*

EXAMPLES.

1. What will 16 yards of broadcloth come to, at £2 3s. 3d. per yard? Ans. £34 12s.

	£	s.	d.
	2	3	3
			4
	8	13	0
			4
Ans.	34	12	0

DEM.—When we multiply the price of one yard by 4, we obtain the price of four yards; because it is the same as adding the price of one yard 4 times; and when we multiply the price of four yards by 4, it must give the price of sixteen yards, because 4 times 4 are 16; and when we multiply the price of four yards by 4, it is the same as adding as many times, because if we should add the price of four yards to itself once, we would have the price of eight yards, because 2 times 4 are 8; and if we add the price of four twice to itself, we must have the price of twelve, because 3 times 4 are 12, or 4 and 4 are 8 and 4 are 12; then if we multiply the price of four by 4 we have the price of sixteen, because it is the same as adding the price of four 3 times to itself; thus, 4 once added to 4 make 8, and 4 added to 8 make 12, and 4 again added make 16, that is, adding the price of four, 3 times to itself, or putting down the price of four, 4 times and adding.

NOTE.—It will be recollected that 4 and 4 are the component parts of 16, because 4 times 4 are 16.

2. What will 20 calves come to, at £1 6s. 3d. each? Ans. £26 5s.

3. If one yard of calico cost 2s. 6d.; what will 24 yards cost? Ans. £3.

4. What will 28 pounds of beef come to, at 4 pence per pound? Ans. 9s. 4d.

5. What must a man pay for 36 sheep, at 9 shillings 4 pence each? Ans. £16 16s.

6. What is the weight of 48 pipes of wine, each weighing 18cwt. 2qrs. 14lb. Ans. 894cwt., or 44T. 14cwt.

7. What is the weight of 72 boxes, each weighing 1 quarter 14 pounds? Ans. 27cwt.

8. In 21 pieces of cloth, each containing 24yds. 2qrs. 3n.; how many yards? Ans. 518yds. 1qr. 3na.

9. What will 63 yards of calico come to, at 3 shillings 1 penny per yard? Ans. £9 14s. 3d.

10. What will 56 bushels of corn come to, at 3 shillings 9 pence per bushel? Ans. £10 10s.

CASE III.—*When the multiplier is not the exact product of any two figures in the multiplication table, and is above* 12.

RULE.—Take two figures or factors, whose product comes nearest, though short of the multiplier, and multiply by them as in case second. Then multiply the given sum by the number which will make up the deficiency; then add this last product to the sum produced by the two factors, and their sum will be the answer.

EXAMPLES.

1. What will 17 bushels of wheat come to, at 10s. 6d. per bushel? Ans. £8 18s. 6d.

		s.	d.	
		10	6×2	
	£		5	
	2	12	6	the price of 2 bu.
			3	
	7	17	6	the price of 15 bu.
	1	1	0	the price of 2 bu.
Ans.	8	18	6	the price of 17 bu.

Dem.—It is plain, from the preceding case, by multiplying the price of one by 5, and then that product by 3, that we have the price of fifteen; and it is then evident, if we multiply the price of one by 2, and add the product to the price of fifteen, we must have the price of 17 bushels, which was required.

2. What will 23 yards of cloth come to, at 4 shillings 7 pence per yard? Ans. £5 5s. 5d.

3. What is the worth of 47 pounds of butter, at 9 pence 2 farthings per pound? Ans. £1 17s. 2d. 2qrs.

4. How many cwt. are contained in 19 kegs of tobacco, each weighing 56lb.? Ans. 9cwt. 2qrs.

5. What is the weight of 17 hogsheads of sugar, each weighing 8cwt. 3qrs. 14lb.? Ans. 150cwt. 3qrs. 14lb.

6. What is the weight of 23 chests of tea, each weighing 3cwt. 1qr. 21lb.? Ans. 79cwt. 0qrs. 7lb.

7. What will 19 yards of cambrick come to, at 11s. 6d. per yard? Ans. £10 18s. 6d.

8. What will 52 pounds of tea come to, at 5 shillings 9 pence per pound? Ans. £14 19s.

9. If one pound of coffee cost 2 shillings 3 pence; what must be paid for 26 pounds? Ans. £2 18s. 6d.

10. If one yard of lasting cost 7 shillings 10 pence; what will 65 yards cost? Ans. £25 9s. 2d.

Case IV.—*To find the value of a hundred weight, by having the price of one pound given.*

RULE.—If the price be farthings, multiply 2 shillings 4 pence, by the farthings in the price of one pound, and the product will be the price of 1cwt., or 112 pounds. If the price be in pence, multiply 9 shillings 4 pence by the price of one pound, and the product will be the price of 1cwt. or 112 pounds.

EXAMPLES.

1. If one pound cost 2 farthings, what cost 1cwt? Ans. 4s. 8d.

s. d.
2 · 4
2
Ans. 4s. 8d.

Dem.—The reason of our taking 2 shillings 4 pence for a multiplicand, is evident, when we recollect that in 112 farthings there are 2 shillings 4 pence, the price of 1cwt., at one farthing a pound; then it is plain, at 2 farthings a pound, it must be twice as much; consequently we multiply the price, at one farthing, by 2, which gives us the price of 1cwt., at 2 farthings per pound?

2. What will 1cwt. of rice cost, at $2\frac{1}{2}$d. per pound? Ans. £1 3s. 4d.

s. d.
2 4
10 farthings=$2\frac{1}{2}$d.
Ans. £1 3s. 4d.

3. What will 1cwt. cost, at $1\frac{1}{4}$d. per pound? Ans. 11s. 8d.
4. What will 1cwt. cost, at $1\frac{3}{4}$d. per pound? Ans. 16s. 4d.
5. What will 1cwt. come to, at 3 farthings per pound? Ans. 7s.
6. What will 1cwt. cost, at 4d. per pound? Ans. £1 17s. 4d.

s. d.
9 4
4
Ans. £1 17 4

Dem.—The reason of our setting down 9s. 4d. for a multiplicand is plain, because 9s. 4d. is the price of 1cwt. or 112 pounds, at 1 penny per pound, for 112 pence make 9s. and 4d. It is then plain, it must be four times as much, at four pence per pound, as at one penny per pound, consequently we multiply the price of 1cwt., at 1 penny per pound, by 4, which must give us the price of 1cwt., at 4d. per pound; because at 4d. per pound, 1cwt. must cost 4 times as much, as at one penny per pound.

7. What is the value of 1cwt., at 3 pence per pound? Ans. £1 8s.
8. What is the value of 1cwt., at 7 pence per pound? Ans. £3 5s. 4d.
9. At 9 pence per pound, what is the value of 1cwt.? Ans. £4 4s.
10. At 8 pence per pound, what is the value of 1cwt.? Ans. £3 14s. 8d.

PRACTICAL QUESTIONS.

1. What is the worth of 42 bushels of peas, at 4 shillings 6d pence per bushel? Ans. £9 9s.

2. What is the worth of 42 bushels of apples, at 2 shillings 6 pence per bushel? Ans. £5 5s.

3. What is the worth of 49 yards of broadcloth, at 18 shillings 4 pence per yard? Ans. £44 18s. 4d.

4. What is the weight of 8 hogsheads of sugar, each weighing 7cwt. 3qrs. 19lb.? Ans. 63cwt. 1qr. 12lb.

5. What is the weight of 6 chests of tea, each weighing 3cwt. 2qrs. 8lb.? Ans. 21cwt. 1qr. 20lb.

6. How many yards in 12 pieces of cloth, each containing 18 yards, 2qrs. 1na.? Ans. 222yds. 3qrs.

7. What is the weight of 2 dozen silver spoons, each weighing 2oz. 12pwts. 5grs.? Ans. 5lb. 2oz. 13grs.

8. How many cords of wood in 9 piles, each containing 26 cords, 98 feet? Ans. 240 cords, 114 feet.

9. In 35 pieces of cloth, each measuring 27¼ yards; how many yards in the whole? Ans. 953 yds. 3qrs.

10. How much land in 4 fields, each of which contains 12 acres, 2 roods, 16 rods? Ans. 50 acres, 1 rood, 24 rods.

11. What is the value of 20 cords of wood, at 9s. per cord? Ans. £9.

12. What are 18 yards of shirting worth, at 2 shillings 6 pence per yard? Ans. £2 5s.

13. What is the worth of 25 bushels of wheat, at 10 shillings per bushel? Ans. £12 10s.

COMPOUND DIVISION,

Is finding how often one number or sum is contained in another of different denominations, or how often it may be subtracted from another.

RULE.—Divide the left hand denomination the same as in simple division; and if any thing remains, reduce it to the next inferiour denomination, adding to the product whatever you have in the given sum of the next less denomination; then divide as before, and again reduce the remainder to the next inferiour denomination, and thus continue the work, till the whole is finished.

Proved by compound multiplication. Multiply the quotient by the divisor, and if the product equal the multiplicand the work is right.

CASE I.

1. Divide £32 16s. 8d. equally among 4 persons.
Ans. £8 4s. 2d. the share of each.

	£	s.	d.
	4)32	16	8
Ans.	8	4	2

DEM.—It is plain, if £32 16s. 8d. be divided into 4 parts or shares, that one part or share is a fourth part of the whole; and by dividing the whole sum by 4, our quotient must be a fourth part of the whole, which plainly appears from the quotient itself, because each quotient figure is one fourth part of a like denomination in the dividend.

2. If 5 bushels of clover seed cost £11 3s. and 4d.; what did it cost per bushel? Ans. £2 4s. 8d.

£	s.	d.	£	s.	d.
5)11	3	4	(2	4	8
10					5
1	Proof.		£11	3	4
20					
23					
20					
3					
12					
40					
40					
0					

DEM.—It is plain, that the quotient must be in each place of the same kind or denomination of that part of the dividend which produced it, hence by dividing the pounds of the dividend by the divisor (5,) our quotient is 2 pounds; and we have one pound remaining, which we multiply by 20, because it equals 20 shillings, and to the product, we add the 3 shillings of our dividend, and divide the 23 shillings by 5, the divisor, which gives 4 for a quotient; which is shillings, because the shillings of the dividend produced it; we now have 3 shillings for a remainder, which we reduce to pence, by multiplying by 12, and adding the four pence of the dividend to the product; we now have 40 pence, in which we find 5 contained 8 times, which gives us 8 pence in the quotient without a remainder.

NOTE.—The student will recollect, that compound division is exactly the reverse of compound multiplication. In compound multiplication, we have the price of one pound, one yard, &c. given to find the price of a quantity. In compound division, we have the price of a quantity given to obtain the price of one. And since this rule is exactly the reverse of multiplication, the student will discover, that our remainder, in each place, is produced by the carriage in multiplication; therefore, when any thing remains in pounds, it is as many times 20 in shillings, because in multiplication, we divide the shillings by 20 and carry the quotient to the pounds, which becomes the remainder in division; and for the same reason, when any thing remains in shillings, it is so many times 12 in pence, and so on.

3. A box containing 36 hats cost £48; how much was that per hat? Ans. £1 6s. 8d.

```
      £        £   s.  d.
36 ) 48  (    1   6   8
     36                6
     --      -----------
     12       8   0   0
      20               6
     ---     -----------
     240   £48   0   0
     216
     ---
      24
      12
     ---
     288
     288
     ---
       0
```

DEM.—It is plain, that one hat must cost one thirty-sixth part as much as 36, and when we divide the whole sum by 36, our quotient is one thirty-sixth part of the whole sum, as plainly appears by the proof; because when we repeat the quotient 36 times, we find that the product equals the dividend or whole cost. In the proof, it will be noticed, by multiplying by 6, and then that product by 6, is the same as directly multiplying by 36; because the two multipliers are the component parts of 36, for 6 times 6 are 36.

NOTE.—In addition to the illustration of this rule already given, the student has only to keep in mind one thing, that is, to obtain the price of one yard, one pound, &c.; we must divide the *price* of the quantity *by the quantity*, and the quotient will be the price of one yard, one pound, &c. Or if the weight of a number of hogsheads, bales, bags, or boxes, be given, to obtain the weight of one; divide the weight of the whole quantity or number by the number of hogsheads, bales, bags, or boxes, and the quotient will be the weight of one.

4. Three cows cost £22 3s. 9d.; what was the cost of each?
Ans. £7 7s. 11d.

5. If £12 9s. 8d. be divided equally among 4 men; how much will each receive?
Ans. £3 2s. 5d.

6. If 7 Ells cost £5 17 shillings 5 pence; what cost 1 ell?
Ans. 16s. 9¼d.

7. If 8 horses cost £185 17s. 6d.; what was the cost of one horse?
Ans. £23 4s. 8d. 1qr.

8. If 10 bushels of wheat cost £3 6s. 8d.; what cost one bushel?
Ans. 6s. 8d.

9. A man paid £2 10s. for 15 bushels of corn; what did he pay per bushel?
Ans. 3s. 4d.

10. A merchant paid £105 for 50 barrels of flour; what did he pay per barrel?
Ans. £2 2s.

CASE II.—*When the divisor exceeds 12, and is a composite number, it sometimes shortens the work to divide by the component parts of the divisor.*

RULE.—Divide first by one of the component parts of the divisor, and then that quotient by the other; and the last quotient will be the answer.

1. Divide £87 10s. by 21. Ans. £4 3s. 4d.

	£	s.	d.
7)	87	10	0
3)	12	10	0
Ans.	4	3	4

DEMONSTRATION.—Seven and three are the component parts of 21, because three times 7 are 21; and it is the same as dividing directly by 21, as the following example shows:

```
  £     s.   d.     £  s.  d.
21)87   10   0  (   4  3   4
   84
    3
   20
   70
   63
    7
   12
   84
   84
    0
```

Here we find that the same result is produced, and the reason is obvious from the principles of multiplication, which are exactly the reverse of those in division.

2. A man bought 36 bushels of apples for £2 14s.; what did he pay per bushel? Ans. 1s. 6d.

3. Bought 28 cords of wood for £16 16s.; how much was paid per cord? Ans. 12s.

4. Bought 72 bushels of wheat for £41 8s.; what was paid per bushel? Ans. 11s. 6d.

5. Sold 81 barrels of flour for £147 16s. 6d.; how much was that per barrel? Ans. £1 16s. 6d.

6. Bought 42 ploughs for £128 9s.; how much is that per plough? Ans. £3 1s. 2d.

EXAMPLES.

Of Weights, Measures, &c.

1. If 24 pieces of cloth contain 426yds.; how many yards in one piece? Ans. 17yds. 3qrs.

2. If 6 chests of tea weigh 21cwt. 1qr. 26lb.; what is the weight of 1 chest? Ans. 3cwt. 2qrs. 9lb.

3. If 7 hogsheads of sugar weigh 69cwt.; what is the weight of 1 hogshead? Ans. 9cwt. 3qrs. 12lb.

4. If 11 pieces of cloth contain 163yds. 2qrs. 2na.; how much in each, suppose they contain equal quantities? Ans. 14yds. 3qrs. 2na.

5. Divide 219 acres, 1 rood, 8 rods, into twelve equal parts. Ans. 18 acres, 1 rood, 4 poles.

6. If 174yds. 1qr. 2na. be divided equally among 5 persons, what will be the share of each? Ans. 34yds. 3qrs. 2na.

7. Divide eighteen gallons equally among one hundred and forty-four soldiers. Ans. 1 pint apiece.

PRACTICAL QUESTIONS

In Compound Multiplication and Division.

1. If one bushel of corn cost 45 cents; what will 8 bushels cost? Ans. $3,60 cents.

2. If 8 bushels of corn cost $3,60 cents; how much was that per bushel? Ans. $0,45 cents.

3. A man received $19,50 cents for 30 day's labour; how much was that per day? Ans. $0,65 cents.

4. A gentleman wishes to put 130 bushels of apples into barrels, containing 3 bushels and 1 peck each; how many barrels does he need? Ans. 40.

5. The Prince of Wales receives a salary of 150 thousand pounds a year; how much is that per day?
Ans. £410 19s. 2d.

6. A piece of calico containing 29 yards cost $8,70 cents; what was that per yard? Ans. $0,30 cents.

7. A privateer took a prize of $30,000, of which the owner took one third, and the officers one fourth; the remainder is equally divided among 125 seamen; how much must each seaman receive? Ans. $100.

8. One hundred and sixty-three men took a prize worth $1811,16 cents, of which the captain had four shares, the first lieutenant three shares, and the second lieutenant two shares; what was the share of each officer and each private?

Ans. Captain's share $42,12 cents; first Lieutenant's share $31,59 cents; second Lieutenant's share $21,06 cents; and a private's share $10,53 cents.

9. Divide £136 14 shillings 6 pence among two men and three women, and give each man three times as much as a woman; what will each man and each woman receive?

Ans. £45 11s. 6d. 1 man's share. £15 3s. 10d. 1 woman's share.

	£	s.	d.	
2 men, and				
3 women.	9)136	14	6	
6 equal to 6 w'n.	Ans. 15	3	10	one w's. share.
Add 3 women.			3	
9 the number of shares in the whole.	45	11	6	a man's share, or the 2 shares of the 3 w.
Divisor = 9 women.	91	3	0	the sh. of the 2 men.
	45	11	6	the sh. of the 3 w.
Proof.	136	14	6	

DEM.—It is evident from the preceding work, that a woman must receive one ninth of the whole sum; then it is plain that we obtain a man's share by multiplying a woman's share by 3, because each man receives three times as much as a woman.

10. Divide £60 8s. 8d. among 14 men and 14 women, and give the women three times as much as the men.

Ans. £1 1s. 7d. m's. share. £3 4s. 9d. w's share.

We have found, by division, where the price of a quantity is given, we obtain the price of one yard, one pound, &c. by dividing the price of the quantity by the quantity, and the quotient is the price of one yard, one pound, &c. And we have also found by multiplication, where the price of a unit or one is given, we obtain the price of a quantity by multiplying the price of one yard, one pound, &c. by the quantity, and the product is the price of the whole quantity.

Then from these two rules we may draw a very useful one in practical arithmetick, *where the price of a quantity is given to find the price of any other quantity.*

RULE.—Divide the price by the quantity of which it is the price, and the quotient will be the price of one; then multiply the price of one by the quantity of which you wish to obtain the price, and the product will be the price of the quantity required.

NOTE.—There is a double object in introducing this rule here: first, on account of its applying in one sum the principles of multiplication and division, and its importance in business; secondly, on account of its being a key to other rules which have been looked upon by the student as dark and intricate—without relation to any thing preceding them; but the student, from his knowledge of this rule, will be able to say, when he advances in other rules, that they are easy, because they are only a different manner of working and applying rules which he already understands; and also the relation which they bear to what has preceded them.

EXAMPLES.

1. If 8 yards of cloth cost 24 shillings; what will 4 yards cost? Ans. 12 shillings.

8)24
3s. the price of 1yd.
4
12s. the price of 4yds.

DEM.—It is plain, when we divide 24s. by 8, the number of yards of which it is the price, the quotient must be the price of 1 yard, because one yard must cost the eighth part of 8 yards, which gives us 3 shillings for 1 yard; then to obtain the price of 4 yards, it is plain, the price of one must be repeated 4 times, which gives us 12 shillings for the price of 4 yards.

2. If 4 yards cost 12 shillings; what will 8 yards cost? Ans. 24s.

3. If 4 yards of broadcloth cost 20 dollars; what will 9 yards cost? Ans. $45.

4. Suppose 30 yards of Irish linen cost $19,50 cents; what will 8 yards cost? Ans. $5,20cts.

5. If 3 cords of wood cost $4,35 cents; what will be the cost of 30 cords? Ans. $43,50 cents.

6. If 30 cords cost 43 dollars 50 cents; what will 3 cords cost? Ans. $4,35 cents.

7. If 3 hogsheads of sugar weigh 26cwt. 1qr. 12lbs.; what is the weight of 9 hogsheads? Ans. 79cwt. 0qr. 8lb.

8. Suppose 30 bushels of rye cost $18; what will 11 bushels cost? Ans. $6,60 cts.

9. If 3 bushels of apples cost 63 cents; what will 18 bushels cost? Ans. $3,78 cts.

10. Suppose 2½ yards of broadcloth cost $10; what will 9 yards cost? Ans. $36.

11. Divide 97deg. 55mi. 7fur. 35pol. 4ft. 2in. 1b. c. by 6.
Ans. 16deg. 20mi. 7fur. 12pol. 8ft. 11in. 1⅙b. c.

12. Divide 45deg. 10mi. 7fur. 37pol. 5yds. 1ft. 3in. and 2b. c. by 4.
Ans. 11deg. 20mi. 0fur. 39pol. 2yds. 2ft. 2in. 1¼b. c.

NOTE.—Let the student prove examples 11th and 12th. For the disposition of the fractions see note to example 3d, page 97, and demonstration to example 1st, page 97.

QUESTIONS ON THE COMPOUND RULES.

What is compound addition? A. Collecting in one sum two or more numbers of different denominations. How do you place your numbers for adding? A. Those of the same denomination exactly under each other. How do you proceed in adding? A. Add up the right hand denomination, and divide the amount by as many of that denomination as will make one of the next greater, and set down the remainder under the denomination added, and carry the quotient to the next greater, and so I proceed till I have come to the left hand denomination, which I add and set down, the same as in simple addition. In adding pounds, shillings, pence, and farthings; how do you carry, or by what numbers do you divide the several amounts to know what you must carry? A. Divide the amount in farthings by 4, and carry the quotient, because 4 farthings make a penny; and divide the pence by 12, and carry the quotient, because 12 pence make one shilling; and divide the amount in shillings by 20, because it takes 20 shillings to make one pound. What is Compound Subtraction? A. It is taking a *less* sum from a *greater* of different denominations. How do you place the two given numbers? A. The less sum under the greater, so that those of the same denomination may stand directly under each other. How do you proceed in subtracting? A. Take the figures in the subtrahend from those directly above them in the minuend, setting down the difference. When any denomination, in the subtrahend, exceeds that directly above it in the minuend, what do you do? A. Add as many to the upper denomination as will make a unit in the next higher, and from the amount subtract the figure or figures directly below in the subtrahend, and then add one to the next higher denomination in the subtrahend; or subtract from that number which is equal to a unit in the next higher, and to the difference add the figure or figures directly above in the minuend, and then add one to the next higher in the subtrahend. Why should this preserve the true difference between the subtrahend and minuend? A. Because it is adding equals to both the given sums, and adding equals to both, their difference must ever remain the same. If the pence in the subtrahend exceed the pence in the minuend, what must you do? A. Subtract the pence in the subtrahend from 12, and to the difference add the pence in the minuend, and then add one to the shillings in the subtrahend. How do you prove compound subtraction? A. By adding the difference to the subtrahend, and if the amount equals the minuend the work is right. Why should that prove it? A. Because the difference between two numbers added to the less must make it equal to the greater. If the difference be taken from the minuend, it must leave a number equal to the subtrahend, because if the difference between two numbers be taken from the greater, it must reduce it equal to the less. What is compound multiplication? A. It is repeating a given number of different denominations a certain proposed number of times. How do you proceed in the work? A. Place the multiplier under the right hand denomination of the multiplicand; then multiply the right hand denomination of the multiplicand by the multiplier, and divide the product by as many of that denomination as will make a unit in the next higher, placing the remainder under the denomination multiplied, and carrying the quotient to the product of the next at the left; and so I proceed till I come to the left hand denomination, and there set down the whole product the same as in simple work. Why do we divide the product in each place by the number which it takes to make a unit in the next higher, and carry the quotient? A. For the same reason that we divide and carry in compound addition, this being only a short way of performing compound addition. If the price of a unit is given to find the price of a quantity, how do you proceed? A. Multiply the price of one by the quantity, and the product will be the price of the quantity. If the price of one yard is 6 shillings, how would you obtain the price of five yards? A. Multiply 6 shillings, the price of one yard, by 5, the number of yards, and the product would be the price of 5 yards, because the price of 5 yards must be 5 times as much as the price of one yard. Where the price of one pound is given in farthings, how can you obtain the price of 1cwt. or 112lb.? A. By multiplying 2s. 4d. (the price of 112lb. at one farthing a pound) by the number of farthings which one pound cost. Why should that give it? A. Because 2 shillings 4 pence is the price of 1cwt., at one farthing a pound; and at two farthings it must be twice as much, at three farthings three times as much, and so on. When the price of 1lb. is given in pence, how may you obtain the price of 1cwt. or 112lb.? A. By multiplying 9 shillings 4 pence, the cost of 1cwt. or 112lb., at one penny per pound, by the number of pence which one pound cost; because if 1cwt. cost, at one penny per pound, 9 shillings 4 pence, at 2 pence per pound it must cost double, and at 3 pence per pound thrice as much, and so on. What is compound division? A. It shows how often one number is contained in another of different denominations, or how often one number may be subtracted from another of different denominations. How do you proceed in the work? A. Place the divisor at the left hand of the dividend, and divide the left hand denomination the same as a sum in simple division, and if any thing remains, reduce it, by reduction, to the next lower denomination, and to the product add the next lower denomination of the dividend, then divide as before, and so continue to do, till the whole dividend be divided; the quotient figures will then be the answer, in the same denominations as the dividend that produced them. Where the price of a quantity is given, how do you obtain the price of one yard, or one pound, &c.? A. Divide the price of the quantity by the quantity, and the quotient will be the price of a unit. Suppose the price of 4 yds. is given, how would you obtain the price of 1 yd.? A. By dividing the price of 4 yards by 4; the quotient will then be the price of one yard, because one yard must cost only one fourth part as much as 4 yards. Where you have the price of a quantity given, to find the price of some other quantity, how do you proceed? A. Divide the given price by the quantity of which it is the price, and the quotient will be the price of a unit, then multiply the price of one by the quantity which you wish to obtain the price of, and the product will be the price of the quantity required. How do you prove compound division? A. By compound multiplication; multiply the quotient by the divisor, and if the product equal the dividend the work is right.

NOTE.—The teacher should not wait to have the student complete the compound rules before he commences questioning him; but the questions should be asked as the student progresses in the rules. A little time should be spent every day by the teacher in questioning his class, and explaining the rules and their use in business.

VULGAR FRACTIONS

A *vulgar fraction* is a part, or the parts of a unit or integer, expressed by two numbers, one placed directly above the other with a line between; thus, $\frac{1}{4}$ signifies one fourth of 1, and $\frac{3}{8}$ signifies three eighths of one, &c. A vulgar fraction arises from unfinished division. The number above the line is called the *numerator*, and that below is called the *denominator*, thus,

$$\left\{\begin{array}{l} 3 \text{ Numerator} \\ \overline{8} \text{ Denominator} \end{array}\right.$$

The numerator, (which is the remainder in division,) shows the number of parts contained in the fraction; thus in the fraction $\frac{3}{8}$, the 8 shows that an integer is divided into 8 parts, and 3, the numerator, shows that the fraction contains 3 of those parts expressed in the denominator.

The denominator of a fraction, (which is the divisor in division,) shows into how many parts an integer or unit is divided.

A fraction is expressed in its least or lowest terms, when expressed by the least numbers possible, thus $\frac{12}{16}$, when reduced to its lowest terms, will be $\frac{3}{4}$, and $\frac{7}{21}$ is equal to one third, or $\frac{1}{3}$ is the fraction expressed in its lowest terms.

NOTE.—The history and theory of vulgar fractions will be given in some other part of this work; we only introduce sufficient of fractions here for our present purpose.

CASE I.—*To reduce fractions to their lowest terms.*

RULE.—Divide the numerator and denominator of the given fraction by any number that will divide them without a remainder, and the quotient again in the same manner, and so continue to do till it appears that there is no number greater than one that will divide them; and the last quotients will express the given fraction in its lowest terms.

EXAMPLES.

1. Reduce $\frac{48}{96}$ to its lowest terms. Ans. $\frac{1}{2}$.

$$\begin{array}{ll} \quad\ 4)\quad 2) & \qquad\qquad\quad 4) \\ 6)\frac{48}{96}=\frac{8}{16}=\frac{2}{4}=\frac{1}{2} \text{ Ans.} & \text{Or thus, } 12)\frac{48}{96}=\frac{4}{8}=\frac{1}{2} \text{ Ans.} \end{array}$$

DEM.—We first set down our fraction, and divide the terms of the fraction by 6, saying 6 in 48, eight times, setting down the 8 for the numerator of a new fraction; we then divide the denominator by 6, saying 6 in 9 once, and 3 over, then 6 in 36, six times, which gives us 16 for the denominator; we then divide the new fraction by 4, which gives us $\frac{2}{4}$, which we again divide by 2, which gives us $\frac{1}{2}$, which is the same in value of $\frac{48}{96}$. By this operation we do not alter the value of the fraction, because the numerator of the quotient bears the same proportion to the denominator of the quotient, in each place, that the numerator of the dividend bears to the denominator of the dividend, as our example plainly shows; neither does it make any difference what number we take for a divisor, if it will only divide the terms of our fraction without a remainder, which is also shown in our example; the same result being produced by different divisors.

2. Reduce $\frac{96}{544}$ to its lowest terms. Ans. $\frac{3}{17}$.

$$\begin{array}{l} \qquad\quad 4) \\ 8)\frac{96}{544}=\frac{12}{68}=\frac{3}{17} \end{array}$$ Ans.

3. Reduce $\frac{46}{184}$ to its lowest terms. Ans. $\frac{1}{4}$.
4. Reduce $\frac{216}{288}$ to its lowest terms. Ans. $\frac{3}{4}$.
5. Reduce $\frac{66}{72}$ to its lowest terms. Ans. $\frac{11}{12}$.
6. Reduce $\frac{57}{456}$ to its lowest terms. Ans. $\frac{1}{8}$.
7. Reduce $\frac{45}{315}$ to its lowest terms. Ans. $\frac{1}{7}$.
8. Reduce $\frac{5184}{6912}$ to its lowest terms. Ans. $\frac{3}{4}$.
9. Reduce $\frac{50000}{100000}$ to its lowest terms. Ans. $\frac{1}{2}$.
10. Abbreviate $\frac{784}{952}$ as much as possible. Ans. $\frac{14}{17}$.
11. Express $\frac{144}{240}$ by the least number of figures possible. Ans. $\frac{3}{5}$.
12. Reduce $\frac{288}{480}$ to its lowest expression. Ans. $\frac{3}{5}$.

CASE II.—*To find the value or quantity of a fraction in the known parts of an integer, that is, in the inferiour denomination of the integer.*

RULE.—Multiply the numerator by the common parts of the integer, (the same as in reduction descending,) and divide the product by the denominator, and if you have a remainder, reduce it to the next inferiour denomination, and again divide the product as before, and so on, till the work is finished, or till you have reduced it to the lowest denomination.

EXAMPLES.

1. What is the value of $\frac{3}{5}$ of a pound sterling? Ans. 12s.

DEM.—The denominator, 5, shows that a pound is divided into 5 parts, and the numerator, 3, shows how many of those parts our fraction contains. It is evident, if £1 or 20s. is divided into 5 parts, that one

```
   £
   3
  20
  ──
5)60(12
   5
  ──
  10
  10
  ──
   0
```

part must be 4 shillings because 4 shillings is the fifth part of £1 or 20s.; but our numerator expresses three parts, consequently the value of our fraction must be 12 shillings, because three times 4 shillings make 12 shillings. Again, to instruct the student so that he shall find no difficulty in this rule, we give him our reasoning in a manner, if possible, more plain than in the preceding demonstration. The student must bear in mind that 3, the numerator, arises from unfinished division, where it had the name of remainder, and the dividend, of which the 3 is a part, must have been pounds, and 5, the denominator, was the divisor; it is then plain, when we wish to continue the division down, and express this fraction in the inferiour denomination, we must reduce the remainder to the next inferiour denomination, and divide by our divisor which is the denominator of our fraction, because a fraction is formed by placing the divisor below the remainder.

2. What is the value of $\frac{3}{4}$ of a pound sterling? Ans. 15s.

```
£3
 20
 ──
4)60(15s. Ans.
  4
 ──
 20
 20
 ──
  0
```

Demonstration.—Here our denominator shows that £1 is divided into 4 parts, each part then must be 5 shillings; and our numerator shows, that our fraction contains three of those parts, which must be 15 shillings, because three times 5 shillings make 15 shillings.

3. What is the value of $\frac{9}{15}$ of a pound sterling? Ans. 12s.

4. What is the value of $\frac{3}{8}$ of a shilling? Ans. $4\frac{1}{2}$d.

5. What is the weight of $\frac{3}{4}$ of a pound Troy? Ans. 9oz.

6. What is the weight of $\frac{3}{4}$ of a pound Avoirdupois? Ans. 12oz.

7. Reduce $\frac{7}{9}$ of a cwt. to its proper quantity. Ans. 3qr. 3lb. 1oz. $12\frac{4}{9}$dr.

8. Reduce $\frac{5}{6}$ of a mile to its proper distance, in inferiour denominations. Ans. 6fur. 26pol. 3yds. 2ft.

9. Express $\frac{5}{8}$ of an acre in the inferiour denominations of the integer. Ans. 2 roods, 20 poles.

10. Express $\frac{6}{7}$ of a hogshead in an inferiour denomination, or inferiour denominations of the integer. Ans. 54 gallons.

11. Reduce $\frac{3}{4}$ of a tun to its proper quantity. Ans. 15cwt.

12. Reduce $\frac{29}{80}$ of a pound Sterling to its proper value. Ans. 7s. 3d.

13. Express $\frac{9}{13}$ of a day in the inferiour denominations of a day. Ans. 16h. 36m. $55\frac{5}{13}$sec.

CASE III.—*To reduce inferiour denominations to the fraction of some superiour denomination retaining the same value.*

RULE.—Reduce the given sum to the lowest denomination mentioned for a numerator; then reduce the unit to the same denomination, for a denominator, which will be the fraction required.

EXAMPLES.

1. Reduce 3s. 4d. to the fraction of a pound? Ans. $\frac{40}{240}$ or £$\frac{1}{6}$.

s. d.
3 4 the given sum.
12
40 numerator.

£
1 a unit or integral part.
20
20
12
240 Denominator.

$$8)\overset{5)}{\frac{40}{240}}=\frac{5}{30}=£\frac{1}{6}.$$

DEM.—We first reduce 3s. 4d. to pence for a numerator, we then reduce £1 or 20 shillings to pence for a denominator; and we find that a pound reduced to pence is divided into 240 parts, and our numerator contains forty similar parts, because both numerator and denominator express pence, but the denominator expresses the pence in a pound, and when we reduce the terms of the fraction to its lowest terms, it stands one sixth of a pound, and it is plain, that it should be one sixth, because if we multiply 40, the numerator, by six, the product will be 240, a number equal to the denominator; or if we multiply 3 shillings 4 pence by 6, the product will be 20 shillings, a sum equal to our denominator; and our denominator, 240, shows that a unit or, £1, is divided into 240 parts, and our numerator shows that our fraction contains 40 of those parts.

2. Reduce 13s. and 4d. to the fraction of a pound. Ans. £$\frac{2}{3}$.

3. Reduce 6 furlongs, 16 poles, to the fraction of a mile. Ans. $\frac{4}{5}$.

4. Reduce 4½ pence to the fraction of a shilling. Ans. $\frac{3}{8}$.

5. Reduce 1 rood, 30pol., to the fraction of an acre. Ans. $\frac{7}{16}$.

6. Reduce 10s. 6d. to the fraction of a pound. Ans. $\frac{21}{40}$.

7. Reduce 7oz. 4pwt. to the fraction of a pound. Ans. $\frac{3}{5}$.

8. Reduce 4 quarters, 1½ nail, to the fraction of an ell English. Ans. $\frac{7}{8}$.

9. Reduce 6 furlongs, 26 poles, 11 feet, to the fraction of a mile. Ans. $\frac{5}{6}$.

10. Reduce 2qrs., $2\frac{2}{3}$na. to the fraction of a yard. Ans. $\frac{2}{3}$.

11. What part of a hogshead is 54 gallons. Ans. $\frac{6}{7}$.

QUESTIONS ON VULGAR FRACTIONS.

What is a vulgar fraction? A. A broken number, or unfinished division, of which the numerator is the remainder, and the denominator, the divisor. How is a vulgar fraction set down? A. By two numbers, one directly above the other, with a line between. What is the number called, above the line? A. Numerator, because it numbers or declares the number of parts contained in the fraction. What is the number called, below the line? A. Denominator, because it denotes or declares into how many parts a unit is divided. How is a fraction reduced to its lowest terms or expressed in the least number of figures? A. By dividing the numerator and denominator by any number that will divide them without a remainder, and, then the quotients in the same manner, till no number will divide the terms of the fraction without a remainder; the last result will be the fraction in its lowest terms. How do you find the value of a fraction in known parts of the integer? A. By multiplying the numerator of the fraction by the parts or inferiour denominations of an integer, and dividing the product after each multiplication by the denominator, and the quotient will express the value of the fraction in the inferiour denominations of the integer? How do you reduce inferiour denominations to the fraction of some superiour denomination retaining the same value? A. By reducing the given sum to the lowest denomination mentioned for a numerator, and reducing an integer to the same denomination for a denominator, then write the fraction down with the numerator above the denominator, and then reduce the fraction to its lowest terms for the answer required. In reducing a fraction to its lowest terms, do your quotients, after the division, express the same value of your dividends? A. They do, because the numerator, in the new fraction, bears the same proportion to the denominator of the fraction as the numerator of the dividend bears to the denominator of the dividend, consequently the value of the fraction is not altered.

DECIMAL FRACTIONS.

A decimal fraction is a fraction whose denominator is a unit with as many ciphers annexed as the numerator has decimal places or figures. A decimal fraction is usually expressed by writing the numerator only, with a comma or point prefixed at the left hand of the fraction; thus ,5 tenths is the same as $\frac{5}{10}$; and ,75 hundredths is the same as $\frac{75}{100}$, &c.—In reading a decimal, we always express the number of parts in the fraction, also the number required to make a unit; thus when we read ,5 tenths, the five shows the number of parts in the fraction, and the tenths in the expression, show that it requires ten of those parts to make a unit.

Whole numbers and decimals may be written in the same line, with a point between them, called the separatrix; thus, $35\frac{3}{10}$ is written 35,3; and $3\frac{47}{100}$ is written 3,47. The denominator, the student will recollect, is repeated in the expression when it is not written, for we say, 3 tenths, and, 47 hundredths, and so on, without having a denominator expressed in the fraction. Decimals decrease in a tenfold proportion, counting from the left to the right; thus ,5 is only one tenth the value it would express in the place of units, by taking away the decimal point; and ,05 is only one tenth as much as ,5; so it will be perceived that they diminish in a tenfold proportion as they recede from the place of units.

NOTE.—Ciphers prefixed to decimals at the right, neither increase nor diminish their value, thus ,5 ,50, ,500 being $\frac{5}{10}$, $\frac{50}{100}$, $\frac{500}{1000}$ are all equal in value; because whenever we annex a cipher to the decimal, the denominator which is understood, assumes one, so that it is multiplying the numerator and denominator by the same number, consequently the proportion between them must remain the same. But ciphers prefixed at the left diminish the value of the decimal in a tenfold proportion, thus ,5 ,05, ,005 are the same as $\frac{5}{10}$, $\frac{5}{100}$, $\frac{5}{1000}$ in value, for in the first example ,5 shows that a unit is divided into 10 parts, and that the fraction contains 5 of those parts; and the second example ,05 shows that one is divided into 100 parts, and the fraction contains only 5 of those parts, &c. This will appear very plain to the student from what has been said of *Federal Money*, which is purely decimal money, of which the dollar may be considered as the unit, and the inferiour denominations the decimal parts; thus, 6 dollars and 4 dimes are expressed $\$6,4$ or $\$6\frac{4}{10}$. The learner must know that it takes 10 tenths or 10 dimes to make a unit in dollars, and 8 dollars and 45 cents are expressed $\$8,45$, or $\$8\frac{45}{100}$; in our first example $\$6,4$, it will be discovered that our decimal ,4 tenths, shows that a unit is divided into 10 parts, and the fraction contains 4 of those parts; and when a dollar is divided into ten parts, the parts must be dimes; and in the last example, $\$8,45$, the decimal (,45) shows that it takes 100 parts, to make a unit, and when a dollar is divided into 100 parts, the parts are cents, consequently the ,45 hundredths are cents, of which it takes 100 to make a unit, and so of mills it takes 1000 to make a unit. Such being the nature of *Federal Money*, it is plain that tenths represent dimes; hundredths, cents; and thousandths, mills; but we commonly express the decimals where the unit is a dollar, in cents and mills; or taken together they represent thousandths of a dollar. Considering decimals in this light, the student must understand them, because he can see and handle the inferiour denominations of a dollar, that is, the decimal parts of a dollar.

TABLE.

		$705\frac{1}{1000000}$	$36\frac{5}{1000}$	$6\frac{3}{100000}$	$\frac{4873}{10000}$	$\frac{75}{1000}$	$\frac{5}{100}$	$\frac{5}{10}$	
Whole Numbers.	3d place.	7	=	=	=	=	=	=	Hundreds.
	2d place.	0	3						Tens.
	1st place.	5	6	6					Units.
		.	.	.	.	.	.	.	
Decimal parts.	1st place.	0	0	0	4	0	0	5	Tenths.
	2d place.	0	0	0	8	7	5		Hundredths.
	3d place.	0	5	0	7	5			Thousandths.
	4th place.	0		0	3				Ten Thousandths.
	5th place.	0		3					Hundred Thousandths.
	6th place.	1							Millionths.
		705, and 1 Millionth.	36, and 5 Thousandths.	6, and 3 Hundred Thousandths.	4873 Ten Thousandths.	75 Thousandths.	5 Hundredths.	read 5 Tenths.	

Thirty-six, and eight tenths=$36\frac{8}{10}$=36,8 the decimal expression.

In like manner write the following sums:

Fifteen, and three tenths.
Eighteen, and seventy-five hundredths.
Five, and five thousandths.
One, and one millionth.
Seventy-five, and seventy-five hundredths.
Eleven, and seven thousandths.
Eleven millionths.
Three, and seven tenths.
Four, and four hundredths.

ADDITION OF DECIMALS.

RULE.—Place the given numbers according to the value of their places, whether mixed or pure decimals; so that tenths may stand under tenths, and hundredths under hundredths, &c. Add the same as in whole numbers, and point off as many places for decimals, at the right hand, as shall equal the greatest number of decimal places in any of the given numbers. Or set the decimal point in the amount, exactly under those in the given numbers.

EXAMPLES.

1. Add ,3 dimes and ,9 dimes together, or 3 tenths and 9 tenths. Ans. $1,2di. or 1,2.

di.		
,3		,3
,9	or	,9
$1,2 Ans.		1,2 Ans.

DEM.—It is plain, that 3 tenths and 9 tenths make 12 tenths, or one unit and two tenths, because ten tenths are equal to a unit or one, and the reason of our pointing off directly below the given points, is manifest, because whenever our tenths exceed 9 tenths, they must equal something in units; and the number of figures must increase, and that increase must be units; for when we suppose these decimals to be dimes, (which correspond with tenths,) ten of which make a dollar or unit, we then have $1 or 1 unit, and 2 dimes, or two tenths.

2. Add 1,7, 3,45, 6,75, 1,705, ,50, ,05 together. Ans. 14,155.

```
      1,7
      3,45
      6,75
      1,705
       ,50
       ,05
     ------
Ans. 14,155
```

DEM.—The learner will perceive, that we add the same as in whole numbers, and the reason is plain, for as the parts diminish in a tenfold proportion from the left to the right, so they must increase in a tenfold proportion from the right to the left, which the student may perceive from *Federal Money*, in which it takes ten mills, which occupy the place of thousandths, to make a cent; ten cents, which occupy the place of hundredths, to make a dime; and ten dimes, which occupy the place of tenths, to make a dollar.

3.	4.	5.	6.
,12	381,31	6,124	67
,467	19,70	1,734	,156
,345	149,000	9,360	7,46
,639	705,15	4,0005	3,05
,438	610,0005	6,7001	1,01
2,009			

7. Add three hundredths, five tenths, forty-five hundredths, eleven thousandths, three ten-thousandths, and four millionths together. Ans. ,991304.

8. Add together the following sums, viz.: forty-five thousandths, four tenths, four hundredths, and five thousandths. Ans. ,49.

9. Add six tenths, six hundredths, six thousandths, six ten thousandths, five hundredths, and eleven thousandths. Ans. ,7276.

10. Add 105,7, 19,4, 1119,05, 648,006, and 19,041, together. Ans. 1911,197.

11. Add one thousand and one thousandth, three hundred and eleven thousandths. Ans. 1300,012.

SUBTRACTION OF DECIMALS.

RULE.—Place the given numbers the same as in addition of decimals, with the less under the greater, and subtract the same as in whole numbers. Set the decimal point in the remainder directly under those in the given numbers.

EXAMPLES.

1. From ,65 take ,32, or from 6 dimes and 5 cents, take 3 dimes and 2 cents. Ans. ,33, or 3 dimes and 3 cents.

		dimes.	cents.	
,65		,6	5	
,32	or	,3	2	
,33 Ans.		,3	3	Ans.

Dem.—It is plain, if we take 2 hundredths from 5 hundredths, 3 hundredths will remain; and if we take 3 tenths from 6 tenths, 3 tenths must remain, consequently the difference between the given numbers, is 3 tenths, and 3 hundredths, or as we express it, 33 hundredths. When we consider the decimal to be *Federal Money*, with which it agrees exactly in principle, it will appear still more plain to the young student; for if we take 2 cents from 5 cents, 3 cents will remain; and if we take 3 dimes from 6 dimes, 3 dimes will remain; the difference then between the given numbers, is 3 dimes and 3 cents, or as we commonly express the decimal of a dollar, it stands 33 cents.

	2.	3.	4.	5.
From	134,63	189,6145	841,101	671,617
Take	101,1409	10,151	84,509	116,1
Difference.	33,4891			
Proof.	134,6300			

6. From 463,05 take 17,0613. Ans. 445,9887.

7. From 412,63402, take 17,1. Ans. 395,53402.

8. From 19, take 14673. Ans. 18,85327.

9. From sixty-five and eight tenths, take forty-nine and seventy-five hundredths. Ans. 16,05.

10. From one, take one millionth. Ans. ,999999.

11. From one hundred, take one tenth. Ans. 99,9.

12. From nine tenths, take seventy-five hundredths. Ans. ,15.

Practical Exercises in Addition and Subtraction.

1. A man bought cloth at several times as follows, viz.: 20,3 yards, 41,5 yards, 21,7 yards, 9,025 yards, and 5 yards; how much did he buy in all? Ans. 97,525 yards.

2. From 4 dollars, take 3 mills. Ans. $3,99,7.

3. From five dollars and twenty-five cents, take eight cents and four mills. Ans. $5,16,6.

4. From ninety yards and three tenths, take forty-seven yards and eight tenths. Ans. 42,5.

5. A man pays eleven dollars and eighty cents, on his note of forty-five dollars; what remains due? Ans. $33,20 cents.

6. If you take three tenths from one and eight tenths; how much will remain? Ans. 1 and 5 tenths.

7. What will be the amount if one millionth be added to one thousandth? Ans. ,001001.

8. From 9 tenths of a gallon, take 25 hundredths of a gallon. Ans. ,65 of a gallon.

9. If from 3 yards, you take 75 hundredths of a yard; what will remain? Ans. 2,25 yards.

10. If from a tenth you take a thousandth; what will remain? Ans. ,099.

MULTIPLICATION OF DECIMALS.

RULE.—Multiply the same as in whole numbers, and point off from the right hand of the product as many figures for decimals as there are decimal places in both the factors; if there are not figures enough in the product, supply the deficiency by annexing ciphers to the left hand of the product.

EXAMPLES.

1. Multiply 20,8 by ,5. **Product, or Ans. 10,4.**

20,8
,5
Ans. 10,40

Dem.—Here we multiply by 5 tenths, which is equal to ½, consequently we are taking one half of the multiplicand; because when our multiplier is 1, we take the multiplicand once; when it is 2, we take the multiplicand twice, &c.; when it is ¼, we take one fourth part of the multiplicand; when ⅓, we take one third part of the multiplicand. So it will be perceived that multiplying by a pure decimal, is only taking a part of the multiplicand, as plainly appears from the example, for our multiplier is only the half of a unit, and our product is only one half of the multiplicand.

Note.—In the multiplication of decimals, the student should keep this truth constantly in mind; that when he multiplies by a pure decimal, that is, by something less than a unit, the product must be less than the multiplicand; and that the product bears the same proportion to the multiplicand that the multiplier bears to a unit; as may be seen from our first example, when thus arranged: as 10,40 is to 20,8, so is 5 tenths to a unit or 1.

2. Multiply 4 by 5 tenths. Ans. 2,.

4
,5
2,0

Dem.—Our multiplier is one half of a unit, and it is plain, that we have taken one half of our multiplicand, because our product is 2, and our multiplicand 4. The reason of our pointing off is also evident, because our multiplier, 5 tenths, is only one tenth part the value it would be, standing in the place of units, consequently the product can have only 1 tenth part the value it would have, multiplied by 5 units; and we give it one tenth the value by pointing off one figure from the right of the product; for without pointing the product would stand 20; but by pointing it is only 2, which is one tenth part of 20; the same reasoning will hold good when we have any number of decimals.

3. Multiply 67,924 by ,003. Ans. ,203772.
4. Multiply ,0007 by ,003. Ans. ,0000021.
5. Multiply 44 by ,4. Ans. 17,6.
6. Multiply 10 by ,1. Ans. 1,.
7. Multiply 100 by one tenth. Ans. 10,.
8. Multiply 7 dollars, 4 dimes, 6 cents, and 3 mills by 6. Ans. $44,77,8.
9. Multiply 46,5 by 37,9. Ans. 1762,35.

10. Multiply 7,1 by 8,2. Ans. 58,22.

11. Multiply 4 dollars 30 cents by 12 cents.
Ans. $0,5160=51 cents 6 mills.

12. Multiply 10 dollars by 10 cents. Ans. $1.

13. Multiply 100 by 1 cent. Ans. $1.

Note.—It may, at first, seem strange to the learner, that multiplying a number should diminish it, but the difficulty vanishes when he considers the value of our multiplier; because when our multiplier is 1, the product is exactly the same as the multiplicand; now the student must understand that a less multiplier must produce a less product than a greater multiplier, and by noticing the 13th example, where we multiply 100 dollars by one cent, our product is only 1 dollar, because our multiplier is only the one hundredth part of a unit or dollar, consequently our product, 1 dollar, is only the one hundredth part of the multiplicand, 100 dollars; if multiplying by a decimal did not give a product less than the multiplicand, a less multiplier would give a product equal to a greater multiplier, which is absurd.

14. What cost 6,5 yards of cloth, at 2 dollars, 35 cents, 2 mills per yard? Ans. $15,28cts. 8 mills.

15. What will 20,75 yards cost at $3,50 cents per yard?
Ans. $72,62 cts. 5 mills.

16. What is the worth of 10,5 gallons of whiskey, at 25 cents per gallon? Ans. $2,62 cts. 5 mills.

17. If rice is worth 5 cents per pound; what is the worth of 8,75 pounds? Ans. $043 cts. 7,5 mills.

18. Multiply 15,4 by one thousandth. Ans. ,0154.

19. Multiply 35,6 by fifteen tenths. Ans. 53,4.

20. Multiply seventeen tenths by twelve tenths.
Ans. 2,04.

To multiply by 10, 100, 1000, *&c.*

Remove the separatrix as many places to the right hand as the multiplier has ciphers, thus ,256 multiplied by ten stands 2,56, because removing the separatrix one figure to the right gives it ten times the value; and to multiply by 100, remove the separatrix two figures to the right, thus ,256 multiplied by 100 becomes 25,6, &c.

DIVISION OF DECIMALS.

Division of decimals is exactly the reverse of multiplication, and they consequently prove each other. Multiplying by a decimal, is taking a part of the multiplicand as many times as the multiplier contains like portions of an integer, and consequently the product must be less than the multiplicand. When we divide by a pure decimal which is less than an integer or one, the quotient must be greater than the dividend, because the quotient shows how many times the divisor may be subtracted from the dividend; and a pure decimal, which is less than a unit, can evidently be subtracted oftener than a unit.

RULE.—Divide as in whole numbers, and from the right hand of the quotient, point off as many figures for decimals as the decimal places in the dividend exceed those in the divisor; if there be not enough in the quotient, prefix ciphers to the left hand of the quotient, to supply the deficiency.

EXAMPLES.

1. Divide 10,5 by 5 tenths. Ans. 21,.

,5)10,5(21, Quo.
10 ,5
5 10.5 Proof.
5
0

DEM.—It is evident, if we should divide the dividend by a unit or 1, that the quotient would be the same as the dividend; then when we divide by 5 tenths, which is ½ of a unit, it is evident that our quotient must be double our dividend, because the dividend must contain a half or 5 tenths twice as often as a whole; the quotient then shows that five tenths, or one half a unit, can be subtracted from the dividend, 10,5, twenty-one times.—The reason of pointing off in the quotient is evident from the principles of multiplication, because in multiplication we point off as many places for decimals in the product as there are in both the factors; the dividend in division is that product, and the divisor and quotient are the factors; consequently what the divisor falls short must be made up from the quotient, by counting from the right to the left. Again it is evident, because our divisor is only 1 tenth part of five units, and consequently is contained in the dividend ten times oftener than five units, and our quotient, by pointing off according to the rule, has ten times the value it would have, divided by five units, because our quotient is now 21, and if our divisor had been five units, it is evident, that our quotient must have been two and one tenth.

NOTE.—When the dividend has not as many decimal places as the divisor, or will not contain the divisor, annex ciphers to the right hand of the dividend to supply the deficiency, and if there be a remainder after all the figures of the dividend are brought down, annex ciphers to the remainders till your quotient shall at least contain two or three decimal places, and these ciphers which you annex to the remainders are counted as belonging to the dividend.

2. Divide 5 by 25 hundredths. Ans. 20.

,25)5,00(20,
50
0

DEM.—It is plain, that ¼ or 25 hundredths can be subtracted from the dividend four times as often as a unit; for it is plain, that we can subtract four times as many quarters as whole ones; and as our quotient shows, it is plain that ¼ or 25 hundredths is contained in five, twenty times, because in five there are 20 quarters.

3. Divide 24,4 by 2,2. Ans. 11,0909.

```
2,2)24,4(11,0909.
    22
    --
     24
     22
     --
      200
      198
      ---
        200
        198
        ---
          2 Rem.
```

Here we have five decimal places in the dividend, and but one in the divisor, then we count four figures from the right of the quotient to make up the deficiency.

4. Divide 206,79 by 2,46. Ans. 84,06.
5. Divide 6,344 by ,54 Ans. 11,748
6. Divide 44 by 2 tenths. Ans. 220.
7. Divide 15 by 25 hundredths. Ans. 60.
8. Divide 10 by 1 tenth. Ans. 100.
9. Divide 10 by 2 tenths. Ans. 50.
10. Divide 100 by 1 tenth. Ans. 1000.
11. Divide 20 by 40. Ans. ,5.

```
40)20,0(,5 Quotient.
   20 0
   ----
      0
```

DEM.—The learner may, at first thought, think it impossible to divide a less number or quantity by a greater; but how soon the difficulty vanishes when he learns the nature of notation; the divisor, 40, we see is not contained in 20, but when we annex a cipher to our dividend it is then reduced to tenths, and then the divisor is contained in 200 tenths 5 times, and the 5 is tenths, because the dividend that produced it is tenths. The principle is the same as if our dividend had been dollars; we know that the divisor, considered as $40, is not contained in 20 dollars; but if we annex to the $20 a cipher, the dividend is then dimes, and the divisor, $40, is contained in the dimes 5 times, and the quotient is dimes, because the dividend which produced it is dimes, or tenths of a unit.

12. Divide 25 by 100. Ans. ,25.
13. Divide one by one thousand. Ans. ,001.
14. Divide 25 by 1000. Ans. ,025.

NOTE.—When decimals or whole numbers are to be divided by 10, 100, 1000, &c., that is, unity with any number of ciphers; it is performed by placing the decimal point in the dividend as many places towards the left hand as there are ciphers in the divisor.

EXAMPLES.

687 divided by	10,	the quotient is	68,7.
	100,	....	6,87.
	1000,	....	,687.

REDUCTION OF DECIMALS.

Case I.—*To reduce a Vulgar Fraction to its equivalent Decimal.*

RULE.—Divide the numerator by the denominator the same as in Division of Decimals, annexing ciphers to the numerator as far as necessary; and the quotient will be the decimal required.

EXAMPLES.

1. Reduce $\frac{1}{2}$ to a decimal. Ans. ,5.

```
2)1,0(,5
  1 0
  ———
    0
```

Dem.—The reason of this operation is plain, because our numerator arises from a remainder in division, and our denominator is the divisor in division; we then only continue the division down below a unit, by reducing the remainder to tenths, hundredths, thousandths, &c., and finding how often the divisor is contained in the inferiour denominations, if they may be so called.—And that we preserve the true value of the fraction, may be seen by the quotient, because 5 tenths bear the same proportion to a unit, that the numerator does to the denominator.

2. Reduce $\frac{3}{4}$ and $\frac{1}{4}$ to decimals. Ans. ,75 and ,25.

3. Reduce $\frac{1}{5}$ to a decimal. Ans. ,2.

4. Reduce $\frac{5}{8}$ to a decimal. Ans. ,625.

5. Reduce $\frac{1}{8}$ to a decimal. Ans. ,125.

6. Reduce $\frac{5}{26}$ to a decimal. Ans. ,192307,

7. Reduce $\frac{7}{375}$ to a decimal. Ans. ,0186.

Case II.—*To find the value of a Decimal in terms of the inferiour denominations.*

RULE.—Multiply the given decimal by the number of parts in the next inferiour denomination; and cut off as many places at the right for a remainder, as there are decimal places in the given sum or decimal. Then multiply the remainder by the parts in the next lower denomination, and cut off as before; and so proceed through all the parts of the integer; then the several denominations standing on the left hand will be the answer.

Note.—The operation here is the same as in Reduction Descending in whole numbers.

EXAMPLES.

1. Find the value of ,75 of a pound sterling. Ans. 15s.

```
   £
  ,75
   20
s. 15,00
```

Dem.—As decimals increase in a tenfold proportion, it is plain, all the figures above the given number of decimals, must be integers of the next inferiour denomination, and so on, because they arise from the carriage in the place of tenths.

2. What is the value of ,775 of a pound? Ans. 15s. 6d.

3. What is the value of ,765 of a pound Troy?
Ans. 9oz. 3pwt. 14gr.

4. What is the value of ,617 of a cwt.?
Ans. 2qrs. 13lb. 1oz. 10dr.

5. What is the value of ,617 of a pound Avoirdupois?
Ans. 9oz. $13\frac{952}{1000}$dr.

6. What is the value of ,981 of a dollar?
Ans. 98 cents, 1 mill.

7. What is the value of ,3 of a year? Ans. 109d. 12h.

8. What is the value of ,367 of a yard?
Ans. 1qr. 1,872na.

9. What is the value of ,84 of 5oz. Troy?
Ans. 4oz. 4pwt.

10. What is the value of ,8635 of a pound?
Ans. 17s. 3,24d.

11. What is the value of ,3375 of an acre?
Ans. 1 rood, 14 poles.

12. What is the value of ,0375 of a pound? Ans. 9d.

13. What is the value of ,645 of a day?
Ans. 15h. 28m. 48sec.

NOTE.—Addition and subtraction of Decimals of different denominations, may be performed, after the decimals are reduced to their proper quantities.

14. What is the sum of ,75 of a pound, and ,285 of a shilling? Ans. 15s. 3d. 1,68qrs.

15. What is the sum of ,19 of a pound Troy, and ,85 of an ounce? Ans. 3oz. 2,6pwt.

16. What is the difference between ,45 of a pound, and ,45 of a shilling? Ans. 8s. 6d. 2,4qrs.

17. What is the difference between ,48 of a day, and ,35 of an hour? Ans. 11h. 10m. 12s.

CASE III.—*To reduce inferiour denominations to the decimal of any superiour denomination.*

RULE.—Divide the given sum reduced to the lowest denomination mentioned, by the proposed integer, reduced to the same denomination, and the quotient will be the answer or decimal required. Or, set down the given numbers, from the least to the greatest, in a perpendicular column, and divide each denomination by such a number as will reduce it to the next superiour denomination; in each place annexing the quotient to the right hand of the next higher denomination, and the last quotient will be the decimal required.

EXAMPLES.

1. Reduce 15s. 7d. 2qrs. to the decimal of a pound.

Ans. ,78125=15s. 7d. 2qrs.

£
1
20
20
12
240
4
qrs. 960

s. d. qrs.
15 7 2
12
187
4
960)750,0(,78125 the decimal required.
6720
7800
7680
1200
96
2400
1920
4800
4800
0

Or thus, which is the better way.

4|2
12|7,5
20|15,625
,78125 Ans.
20
s. 15,62500
12
d. 7,50000
4
qrs. 2,00000
s. d. qrs.
15 7 2

Proof.

DEM.—The reason of the first operation is obvious, because the given sum reduced to the lowest denomination mentioned, stands as the numerator of the integer reduced to the same denomination for a denominator; the decimal or quotient must then bear the same proportion to the integer, as the given sum bears to the integer. In the last operation it is plain, that the quotient, 5 tenths, after the first division, bears the same proportion to a penny that two farthings bear to a penny, and so of the rest, consequently the principle is the same as in the first operation.

2. Reduce 9d. 3qrs. to the decimal of a shilling.

Ans. ,8125.

3. Reduce 10s. 6d. to the decimal of a pound. Ans. ,525.

4. Reduce 2qrs. 3na. to the decimal of a yard.
Ans. ,6875.

5. Reduce 10cwt. 3qrs. 14lb. to the decimal of a tun.
Ans. ,54375.

6. Reduce £14 12s. to a decimal expression. Ans. £14,6.

7. Reduce £47 15s. 6d. to a decimal expression.
Ans. £47,775.

8. Reduce 3qrs. and 2na. to the decimal of an Ell English.
Ans. ,7 E. E.

9. Reduce 109 days, 12 hours, to the decimal of a year.
Ans. ,3 of a year.

10. Reduce 3qrs, 12lb. 5oz. 1,92dr. to the decimal of a cwt.
Ans. ,86.

11. Reduce 5fur. 16pol. to the decimal of a mile.
Ans. ,675 of a mile.

12. Reduce 7½ calendar months, to the decimal of a year.
Ans. ,625 of a year.

QUESTIONS ON DECIMAL FRACTIONS.

What is a decimal fraction? A. It is a fraction that has for its denominator a unit, 1, with as many ciphers annexed as the numerator has figures, and is usually expressed by setting down the numerator only, with a point before it. What is a mixed number? A. It is made up of a whole number with some decimal fraction, one being separated from the other by a point. What is a pure decimal? A. Something less than a unit, 1, with a point at the left hand. Would a pure decimal become a unit by annexing significant figures at the right? A. It would not, a pure decimal, like certain mathematical lines which always approach each other, but never meet, may continually approach towards a unit, yet never become a unit, thus 9 tenths want one tenth of a unit, ,99 want one hundredth of being a unit, and ,999 want one thousandth of becoming a unit; so we might carry the decimal to any number of figures whatever: it would want something of being a unit, 1, and yet continually approaching towards a unit. How do you set down decimals for adding? A. According to the value of their places; tenths under tenths, hundredths under hundredths, &c. How do you point off, in addition of decimals? A. Directly below the given decimal points. How are numbers set down in subtraction of decimals? A. The same as in addition, and point off from the difference the same for decimals. How do you point off in multiplication of decimals? A. As many figures from the right hand of the product as there are decimal places in both the factors. When you multiply by a pure decimal, is the product greater or less than the multiplicand? A. Less, because it is repeating the multiplicand by something less than a unit. What proportion does the product bear to the multiplicand? A. The same proportion that the multiplier bears to a unit. How do you point off in division of decimals? A. As many figures from the right hand of the quotient, as the decimal places in the dividend exceed those in the divisor. When you divide by a pure decimal, is the quotient greater or less than the dividend? A. Greater. Why should it be greater? A. Because the divisor is less than a unit, and it is evident, that the dividend will contain a part oftener than a whole. What proportion does the dividend bear to the quotient? A. The same proportion as the divisor bears to a unit. If you divide 10 by one tenth, what will be the quotient? A. 100. If you multiply 10 by one tenth, what will be the product? A. One. How do you reduce a vulgar fraction to a decimal? A. By dividing the numerator by the denominator. How do you reduce inferiour denominations to the decimal of a superiour denomination? A. I reduce the given sum to the lowest denomination mentioned for a dividend, and then reduce an integer to the same denomination for a divisor, and the quotient will be the decimal required. How do you find the value of a decimal in the inferiour denominations of the integer? A. By multiplying the decimal by the inferiour denominations of the integer, the same as in reduction descending, and pointing off from the products in each place as many figures for decimals as will equal the decimals in the given number; the figures at the left of the decimal points will be the value of the fraction in the inferiour denominations of the integer.

REDUCTION OF CURRENCIES.

Reduction of Currencies is finding the value of the coin or currency of one state or country in that of another. Although the same denominations and coin are generally used in the different countries and states, yet the standard value frequently differs in each.

Thus, a dollar is reckoned in

New-York, Ohio, and North Carolina,	8s. called *New-York* currency.
The New England States, Virginia, Kentucky, and Tennessee,	6s. called *New England* currency.
South Carolina, and Georgia,	4s. 8d. called *Georgia* currency.
New Jersey, Pennsylvania, Delaware, and Maryland,	7s. 6d. called *Pennsylvania* currency.
England,	4. 6d. called *English* or *Sterling* money.
Canada, and Nova Scotia,	5s. called *Canada* or *Halifax* currency.

To reduce the Currency of each State to Federal Money.

RULE.—Divide the given sum, reduced to shillings, to sixpences, or to pence, by the number of shillings, sixpences, or pence in a dollar as it is reckoned in each state.

EXAMPLES.

1. Reduce £96 New-York currency to federal money.
Ans. $240.

£
96
20
8)1920
$ 240 Ans.

DEM.—It is plain, if we reduce the given sum, and a dollar, to the same denomination, and divide by the number of shillings in a dollar, the quotient will be the answer; because it shows how often 8s. or 1 dollar can be subtracted from the given sum, and so on, when it is reduced to the inferiour denomination.

2. Reduce £109 3s. 8d. New England currency to federal money. Ans. $363,94cts. 4 mills.

$
1
6s. in $1.
6
12
$1=72d.

£. s. d.
109 3 8
20
2183
12
72)26204(363,94,4
216
460
432
carried up.

brought up.
284
216
680
648
320
288
320
288
32

Note.—A dollar in New England is the same as a dollar in New-York, but, a shilling or a pound in New England is more than a shilling or pound in New-York; four shillings and 6d. are a dollar in England, consequently a shilling in England is more than a shilling in New-York, and £1 in England more than a pound in New-York.

3. Reduce £103 New-York currency to federal money.
Ans. $257,50cts.

4. Reduce £103 New England currency to federal money.
Ans. $343,33cts. 3 mills.

5. Reduce £127 10s. 5d. New England currency to federal money. Ans. $425,06cts. 9 mills.

6. Change £104 12s. New-York currency to federal money. Ans. $261,50cts.

7. Change £380 New-York currency to federal money.
Ans. $950.

8. Change £37 10s. Pennsylvania currency to federal money. Ans. $100.

9. Reduce £125 5s. Pennsylvania currency to federal money. Ans. $334.

10. Reduce £73 16s. 4d. Georgia currency to federal money. Ans. $316,35cts. 7 mills.

11. How many dollars are equal to £44 1s. 4d. Georgia currency? Ans. $188,85cts. 7 mills.

12. How many dollars are equal to £35 6s. Georgia currency? Ans. $151,28cts. 5m.

13. Reduce £5 9s. 0d. 1 farthing English currency to federal money. Ans. $24.22cts. 6m.

14. Change £22 10s. English or Sterling money to federal money. Ans. $100.

15. Change £5 6s. 6d. 1qr. Canada currency to federal money. Ans. $21,30cts. 4m.

16. How many dollars are equal to £270 17s. 6d. Canada currency. Ans. $1083,50cts.

17. Change £25 Canada currency to federal money. Ans. $100.

To change Federal Money to the currency of each State.

RULE.—Multiply the given sum in cents, by the number of pence in a dollar, and cut off two figures to the right of the product; those at the left hand, will be the answer in pence; and if the figures thus cut off at the right hand, be multiplied by 4, and two figures again cut off, those at the left hand will be farthings.

EXAMPLES.

1. Change $135,50 cents, to New-York currency. Ans. £54 4s.

$ *cts.*
135,50
96
81300
121950
12)13008,00
2|0)108|4
Ans. £54 4s.

DEM.—It is plain, that every hundred cents in New-York currency, must be diminished four in number to change them into pence, because 96 pence are equal to 100 cents; and multiplying by 96, and cutting off two of the right hand figures, lessens the given number 4 of every 100, though it retains the same value; cutting off two figures is dividing the product by 100.

NOTE.—When there are mills in the given sum, multiply the same as when the given sum is in dollars and cents, and cut off three figures at the right for a decimal; then proceed exactly as in the first and second examples.

2. Change 155 dollars, 9 cents, to New England currency. Ans. £46 10s. 6d. 2qr. nearly.

```
  $155,09
       72
  -------
    31018
  108563          ,48
  -------         ---
12) 11166,48.       4
  -------         ---
 2|0)93|0 6d.     1,92
Ans. £46 10s. 6d. 1,92far.
```

3. Reduce 116 dollars, 31cts. 7 m. to New-York currency.
Ans. £46 10s. $6\frac{1}{4}$d.

4. Reduce $120,25cts. 5 mills to New-York currency. Ans. £48 2s. 0d. 2qrs. nearly.

5. Reduce ,75cts. to New-York currency. Ans. 6s.

6. Reduce ,75 cents, to New England currency.
Ans. 4s. 6d.

7. Reduce $720 to New England currency.
Ans. £216.

8. Reduce $701,34cts. to Pennsylvania currency.
Ans. £263 0s. 0d. 2qrs.

9. Reduce $525,40cts. to Georgia currency.
Ans. £122 11s. 10d. 1qr.

10. Reduce $80,43cts. to English or Sterling money.
Ans. £18 1s. 11d.

11. Reduce 1918 dollars, 50cts. to Canada currency.
Ans. £749 12s. 6d.

12. Reduce 86 dollars, 25cts. to New England currency.
Ans. £25 17s. 6d.

13. Reduce 820 dollars, to New-York currency.
Ans. £328.

PROMISCUOUS QUESTIONS.

1. Change £120 2s. New-York currency, to federal money.
Ans. $300,25cts.

2. Change £120 2s. New England currency, to federal money. Ans. 400,$33\frac{1}{3}$cts.

3. Reduce £205 4s. New England currency, to federal money. Ans. $684.

4. Reduce 684 dollars, to New England currency.
Ans. £205 4s.

5. Reduce $6\frac{3}{4}$d. Pennsylvania currency to federal money.
Ans. 7cts. 5 mills.

6. Reduce 7cts. 5 mills, to Pennsylvania currency.
Ans. $6\frac{3}{4}$d.

7. Reduce £251 12s., New York currency, to federal money. Ans. $629.

8. Reduce 629 dollars to New York currency. Ans. £251 12s.

9. Reduce £527 12s. 6d., Canada currency, to federal money. Ans. $2114,50 cts.

10. Reduce $2110,50cts. to Canada currency. Ans. £527 12s. 6d.

QUESTIONS

On Reduction of Currencies.

What is reduction of currencies? A. Changing the money or currency of one state or country to that of another. How do you reduce the currency of the several states to federal money? A. Divide the given sum, reduced to shillings, to sixpences, or to pence, by the number of shillings, sixpences, or pence in a dollar, as it is reckoned in each state. How would you reduce pounds, N. Y. currency, to Federal Money? A. Multiply the pounds by 20, and divide the product by 8, the number of shillings in a dollar. How do you reduce pounds, N. E. currency, to Federal Money? A. Multiply the pounds by 20, and divide the product by 6, the number of shillings in a dollar in New England. Where the given sum is in pounds, shillings, and pence, N. Y. currency, how do you reduce it to Federal Money? A. Reduce the given sum to pence, and divide by 96, and the quotient will be the answer in Federal Money. How do you reduce Federal Money to the currency of each state? A. Multiply the given sum in cents by the number of pence in a dollar, and cut off two figures from the right hand of the product, and the figures at the left hand will be the answer in pence; and if the figures at the right be multiplied by 4, and cut off as before, the figure at the left will be farthings. If there are mills in the given sum, how many figures must be cut off? A. Three, at the right hand. After you have obtained the answer in pence, how do you proceed to get it in pounds? A. By reduction *ascending;* dividing by twelve and by twenty. How many shillings make a dollar in N. Y.? A. 8s. How many a dollar in N. E.? A. 6s. How many a dollar in Georgia? A. 4s. 8d. How many a dollar in Pennsylvania? A. 7s. 6d. How many a dollar in England? A. 4s. 6d. How many a dollar in Canada? A. 5s. How much then is a shilling in Canada currency equal to in New York? A. 20 cents; but 1 dollar in New York is equal to 1 dollar in Canada.

INTEREST.

INTEREST is a certain per cent allowed for the use or forbearance of money. The money *lent* or *forborne* is called the principal. The *rate* is the per cent agreed on between the parties, or what is established by law. The *amount* is the sum of the principal and interest added. Interest is of two kinds, *simple* and *compound.*

SIMPLE INTEREST,

Is that which is allowed on the principal only. And the principles of the rule are equally applicable to Commission, Brokerage, Ensurance, purchasing Stocks, Duties, or any thing else, rated at a certain per cent.

In the New England states, the law allows the lender to take 6 per cent from the borrower for the use of money, that is, 6 cents for the use of 100 cents, or $6 for the use or forbearance of $100 per year. In New York, the *per cent* established by law is 7, that is, 7 cents for the use of 100 cents for the term of one year, or 7 dollars for the use of 100 dollars for one year, and in like proportion for a greater or less sum or for a longer or shorter time. Five per cent is what is allowed in England.

The courts of the United States allow interest according to the law of the state where the suit was commenced. *Per cent*, or *per centum*, signifies so much by the hundred, or for the use of a hundred.

NOTE.—Legal or lawful interest is that which is allowed by the laws of the state, and is different under different governments and in different states. The lender is not permitted to receive a greater per cent than is allowed by the laws of his state.

RULE.—Multiply the principal by the rate per cent, and divide the product by 100; but to divide by 100, you need only remove the separatrix two figures from its natural place towards the left hand, and you will have the interest for one year.

EXAMPLES.

1. What is the interest of 200 dollars for one year, at 7 per cent? Ans. 14 dollars.

$200
7
$14,00

DEM.—We multiply the principal, 200 dollars, by 7 dollars, the interest of one hundred dollars; consequently our product must be 100 times too much, and for that reason the product is divided by 100 for the true interest. This will appear plain to the learner, when he is informed that interest is only an application of two rules which he already understands, that is, multiplication and division; for the interest of one dollar, or 100 dollars, is always given to obtain the interest of some other sum; then it is plain, by the rules laid down in multiplication, if we multiply the interest of one dollar by the number of dollars, our product must be the interest of the whole number of dollars, as the following work shows.

,07 cts. the interest of $1, for 1 year.
200 the number of dollars in the whole sum.
$14,00 the interest of 200 dollars.

Now it must appear evident, that the interest of $200 is 200 times as much as the interest of $1; and multiplying 7 cents, the interest of 1 dollar, by 200, it is plain that our product is two hundred times as much as our multiplicand. And it is also plain, that it can make no difference whether we multiply 200 dollars by 7 cents, or 7 cents by 200 dollars, the result must be the same.

2. What is the interest of 2 dollars for one year, at 7 per cent? Ans, 14 cents.

$2
7
Ans. ,14 cents.

DEM.—For convenience of the operation, we generally multiply the sum by the interest of 100 dollars for one year, and consequently produce a product 100 times too much: for our multiplier is 100 times greater than when we multiply by 7 cents, and our product being 100 times too great, we take one hundredth part by removing the decimal point two figures further to the left than its usual place, which is dividing the product by 100, for our true answer. Or the student may, if he choose, consider the multiplier to be 7 cents, that is, ,07 of a dollar; he will then discover that we point off two figures further to the left than the usual place of pointing off decimals, because the multiplier is, in fact, 7 cents, or 7 hundredths of a dollar, instead of 7 dollars, and may properly be considered as a contraction for the expression ,07. The same must be produced if we multiply the interest of $1, by the number of dollars, thus:

,07 cents.
2
,14 the answer as before.

It is plain, if a man receives 7 cents for the use of $1, for a year, he should receive 14cts. for the use of $2, 21 cents for $3, and 28 cents for $4, and so on.

3. What is the interest of $5 for one year, at 7 per cent? Ans. ,35cts.

4. What is the interest of $6 for one year, at 7 per cent? Ans. ,42cts.

5. What is the interest of $20 for one year, at 7 per cent? Ans. $1,40cts.

6. What is the interest of $55 for one year, at 7 per cent? Ans. $3,85cts.

7. What is the interest on $300 for 1 year, at 7 per cent? Ans. $21.

8. What is the interest of $300 for 2 years, at 7 per cent? Ans. $42.

$300
7
$21|00 interest for one year.
2
$42 interest for two years.

DEM.—First, it is plain, if $100 gain for the lender 7 dollars, that 300 dollars must $21 in the same time, because $300 dollars must gain the lender three times as much as $100, and three times 7 are 21, that is, $21, for one year. It is likewise evident, that 2 years must produce twice the interest of 1 year, and twice $21 are $42, the interest of $300 for two years. Then we may draw from this the following rule:

RULE.—Multiply the interest of one year by the number of years, and the product will be the interest for the whole number of years, because if the interest of 100 dollars for one year is $7, for two years it must be 14 dollars, and for three years 21 dollars; and if 7 dollars be multiplied by two years the product will be 14 dollars, and if multiplied by three years the product will be 21 dollars.

9. What is the interest of 400 dollars for three years, at 7 per cent? Ans. $84.

10. What must a man receive for the use of $500 for four years, at 7 per cent? Ans. $140.

11. What is the interest of 40 dollars for two years, at 6 per cent? Ans. $4,80cts.

$40
6
2,40
2
Ans. $4,80

Here, it will seem that the use of 100 dollars is considered to be only worth 6 dollars, consequently the principle is the same as in the preceding examples; and whatever the per cent may be we observe the same rule, to multiply by the rate per cent, and point off as the rule directs.

12. What is the interest of $50,45 cents, at 6 per cent, for two years? Ans. $6,05cts. 4m.

13. What is the interest of $49,50 cents for three years, at 6 per cent? Ans. $8,91 cents.

14. What is the interest of 350 dollars for two years, at 5 per cent? Ans. $35.

15. What is the interest of £150 10s. 6d. for two years, at 6 per cent? Ans. £18 1s. 3d.

£ *s.* *d.*
150 10 6×6
£9|03 3 0
20
s. 0|63
12
d. 7|56
4
qr. 2|24 Rem.

£ *s.* *d.* *qr.*
9 0 7 2 interest for one year.
2
Ans. 18 1 3 0 interest for two years.

DEM.—Here, we first multiply as in Compound Multiplication, because the given sum is in pounds, shillings, and pence, and we divide the pounds by 100, because we have multiplied by the interest of £100, and consequently the product is 100 times too much, till we cut off the two right hand figures in pounds, which is dividing the product by 100. Whatever stands at the left of the line or mark which divides the product by 100, is units of the same kind of the units in the principal; consequently it is pounds in this place. And then to obtain the excess of interest, we multiply as in Reduction Descending, the figures cut off at the right, and adding to the product the next inferiour denomination, and cut off as before, and so proceed till the work is finished. The several figures cut off at the left, will be the interest of the given sum for one year; then if it is required for a number of years, you may multiply the interest of 1 year by the number of years.

16. What is the interest of £155 for one year, at 6 per cent? Ans. £9 6s.

17. What is the interest of £300 for two years, at 7 per cent? Ans. £42.

18. What is the interest of £500 for five years, at 5 per cent? Ans. £125.

19. What is the interest of £200 10s. 4d. for three years, at 7 per cent? Ans. £42 2s. 2d.

£ *s.* *d.*
200 10 4
7
1403 12 4
3
£ 42) 10 17 0
20
s. 2) 17
12
d. 2) 04
4
qr. 0) 16

It is frequently the better way, after multiplying by the per cent, to multiply by the time before the division is performed, because there may be some small loss by means of the remainder. If we first divide by 100, and then find the interest in the inferiour denominations, it is plain, if there is a loss on one year's interest, the same loss will be sustained on each year, where we first find the interest of one year, in all the inferiour denominations, before we multiply by the time.

20. What is the interest of £150 8s. 3d. for 5 years, at 6 per cent? Ans. £45 2s. 5d. 2qr.

21. What is the amount of 100 dollars for 1 year, at 7 per cent? Ans. $107.

$100
7
7|00
100 The principal added.
$ 107 Ans.

Dem.—It is plain, if a man lets out 100 dollars, at 7 per cent, for one year, that, at the end of the year, the interest due is seven dollars, and the principal or money lent is also due, which is 100 dollars, consequently the amount due is the interest and principal added together, which make 107 dollars due, at the year's end.

Note.—The learner may now discover, that casting interest on 100 dollars or any other sum for one year, does not differ from finding the value of so many yards of cloth. For, to find the value of 100 yards of cloth, at 7 cents per yard, we may repeat 7 cents, the price of one yard, by 100, the number of yards, and the product will be the price of the whole quantity. So we may multiply 7 cents, the interest or price of one dollar for one year, by 100, the number of dollars, and the product will be the interest or price of one hundred dollars for one year; but, letting a certain number of dollars for a year or any other time, for so many cents on the dollar, differs from selling so many yards of cloth for the same number of cents per yard, because the buyer acquires a perfect right to the cloth purchased for the price which he pays, which is never to be returned; but he who receives money from the lender must not only return to the lender the per cent for the use of the money, but the principal or money received, must be paid back to the lender.

22. What is the amount of 200 dollars for one year, at 7 per cent? Ans. $214.

23. What is the amount of 300 dollars for 3 years, at 7 per cent? Ans. $363.

24. What is the amount of 350 dollars for 4 years, at 6 per cent? Ans. $434.

25. What is the amount of $3,75 cents for 2 years, at 7 per cent? Ans. $4,27cts. 5 mills.

26. What is the amount of $75,60cts. 5 mills, for 5 years; at 7 per cent? Ans. $102,06cts. 6$\frac{75}{100}$ mills.

27. What is the interest of 1600 dollars for 1 month, at 6 per cent? Ans. $8, one month's interest.

```
   $1600
       6
12)96,00 the interest for 1 year.
   $8,00 the interest for 1 month.
```

DEM.—Here, we find by multiplying the principal by the rate per cent, that the interest of 1600 dollars for 1 year, is 96 dollars; it is then evident, that one month's interest must be one twelfth part of 96 dollars, a year's interest, because 1 month is the twelfth part of a year, and it is plain, that by dividing 96 dollars by 12, the number of months in a year, the quotient, 8 dollars, must be the interest of one month.

28. What is the interest of 3600 dollars for 2 months, at 6 per cent? Ans. $36.

```
   $3600
       6
12)216|00
  $ 18|00
      2
  $ 36  for 2 months.
```

DEM.—We first obtain the interest for 1 month, as in the 27th example, we then multiply the interest of one month by 2, the number of months, which must give the interest for two months, because it is evident, that the interest for two months, must be twice as much as the interest for one month. Hence we derive the following

RULE.—Divide the interest of one year by 12, and the quotient will be the interest for one month; then multiply one month's interest by the number of months, and the product will be the interest for the number of months required.

29. What is the interest of 196 dollars for 3 months, at 7 per cent? Ans. $3,43cts.

30. What is the interest of 396 dollars for 4 months, at 7 per cent? Ans. $9,24cts.

31. What is the amount of 396 dollars for 4 months, at 7 per cent? Ans. $405,24cts.

32. What is the amount of ,86 cents for 5 months, at 7 per cent? Ans. 88 cents, 5 mills.

33. What is the amount of 25 dollars and fifty cents for 10 months, at 7 per cent? Ans. $26,98cts. 7$\frac{5}{10}$m.

34. What is the interest of 79 dollars for 15 months, at 7 per cent? Ans. $6,91cts. 2m.

35. What is the amount of fifty-four dollars for 9 months, at 7 per cent? Ans. $56,83cts. 5m.

36. What is the interest of 360 dollars for 27 days, at 7 per cent? Ans. $1,89cts.

```
     $360
        7
 12)25,20
   30)2,10(,07 one day's int.
      2,10 27
         0 49
           14
Ans. $1,89 in. for 27 days.
```

DEM.—The interest for one month, is obtained the same as in the preceding examples; we then divide the interest of one month by 30, to obtain one day's interest. We divide by 30, because one day is considered to be the one thirtieth part of a month; we then multiply 7 cents, one day's interest, by 27, the number of days, which gives us the interest for 27 days, because the interest for 27 days must be 27 times more than the interest for one day. Or, which would come nearer the true interest, we might divide the interest of a year by 365, the number of days in a year, and the quotient would be the interest for one day, and that multiplied by 27, would give the interest for 27 days.

NOTE.—In obtaining the interest for one day by dividing by 12 and by 30, we go on the supposition, that 12 and 30, the divisors, form the component parts of a year in days, which is not strictly true; they only form the component parts of 360, which is 5 days less than a year; consequently our divisors being too small, it follows that the quotient must be too great, because our quotient is one three hundred and sixtieth part of a year's interest, which is a little too much, and the interest of 27 days thus obtained, must consequently be too much, although it is sufficiently accurate for common business.

37. What is the interest of 150 dollars for 19 days, at 7 per cent? Ans. $0,55cts. 4m.

38. What is the amount of 200 dollars for 20 days, at 7 per cent? Ans. $200,77cts. 7m.

39. What is the amount of 96 dollars for 21 days, at 7 per cent? Ans. $96,39cts. 2m.

When there are certain parts of a year in the time, as quarters, months, or days, the interest may be obtained by taking the aliquot parts of the interest of a year, for the months, and the aliquot parts of a month, for the days, and add the several quotients together, and the sum will be the interest for the whole time.

NOTE.—By taking aliquot parts, is perhaps the better way of casting interest; the whole work is then together, and by a little practice the arner will find no difficulty in taking parts. The teacher should frequently exercise his students in taking parts from 1 month up to 12 months, and from 1 day to 30 days.

40. What is the interest of $372 for 1 year, 10 months, and 15 days, at 7 per cent? Ans. $48,82cts. 5m.

	$372	
	7	
6 mo. =$\frac{1}{2}$)	26,04	one year's int.
3 --- =$\frac{1}{2}$)	13,02	int. for 6 mo.
1 --- =$\frac{1}{3}$)	6,51	int. for 3 mo.
15ds.$\frac{1}{2}$)	2,17	int. for 1 mo.
10m. 15ds.	1,08,5	int. for 15d.
Ans.	$48,82,5	in. for 1yr. 10m. 15d.

NOTE.—Where it is more convenient, the student can take parts of parts, as in this example; and in taking parts for days, he should reckon the month thirty days. It only wants a little exercise of the mind to determine that one month is $\frac{1}{12}$ of a year; 2 months the $\frac{1}{6}$ of a year; three months the $\frac{1}{4}$ of a year; 4 months the $\frac{1}{3}$ of a year; 5 months the $\frac{1}{3}$ and the $\frac{1}{12}$ of a year; 6 months the $\frac{1}{2}$ of a year; 7 months the $\frac{1}{2}$ and $\frac{1}{12}$ of a year; 8 months the $\frac{1}{2}$ and the $\frac{1}{6}$ of a year; 9 months the $\frac{1}{2}$ and the $\frac{1}{4}$ of a year; 10 months the $\frac{1}{2}$ and the $\frac{1}{3}$ of a year; 11 months the $\frac{1}{2}$, the $\frac{1}{3}$ and the $\frac{1}{12}$ of a year. And so of days, take the parts of 30 days or one month, thus, 1 day is $\frac{1}{30}$ of a month; two days $\frac{1}{15}$; 3 days $\frac{1}{10}$; 5 days $\frac{1}{6}$; 6 days $\frac{1}{5}$; 10 days $\frac{1}{3}$; 15 days $\frac{1}{2}$, &c.; but where it is more convenient, take parts of parts.

41. What is the interest of 960 dollars for 7 months and 12 days, at 7 per cent? Ans. $41,43cts. 9m.

$960
7
6m. -------- =$\frac{1}{2}$)67,20
1 -------- =$\frac{1}{6}$)33,60 int. for 6mo.
10ds. =$\frac{1}{3}$) 5,60 int. for 1mo.
2ds. =$\frac{1}{5}$) 1,86,6 int. for 10ds.
7m. 12ds. 37,3 int. for 2ds.
Ans. $41,43,9 int. for 7m. 12d.

DEM.—It is plain that the interest for 6 months is $\frac{1}{2}$ the interest for 1 year, and 1 month's interest is $\frac{1}{6}$ part of the interest for 6 months, and the interest for 10 days is $\frac{1}{3}$ of the interest for one month, or 30 days, and 2 day's interest is $\frac{1}{5}$ part of the interest for 10 days; it is then evident, when these sums are added, the total sum must be the interest for the whole time.

42. What is the interest of $221,75 cents for 3 years, 7 months, and 6 days, at 7 per cent? Ans. $55,88cts.

$221,75
7
6mo. -------- =$\frac{1}{2}$)15,5225
3
46,5675 int. for 3 years.
1 -------- =$\frac{1}{6}$) 7,7612 int. for 6mo.
6ds. =$\frac{1}{5}$) 1,2935 int. for 1mo.
,2587 int. for 6ds.
Ans. $55,88,09 in. for 3y. 7m. 6d.

NOTE.—The shortest method of casting interest for days, at 7 per cent, is to multiply the principal by the number of days and divide the product by 52, the quotient will be the interest of the given number of days.

43. What is the amount of 950 dollars, at 7 per cent, for 3 years and 6 months? Ans. $1182,75cts.

44. What is the amount of 950 dollars, at 6 per cent, for 3 years and 6 months? Ans. $1149,50cts.

45. What is the amount of $350,50 cents for 2 years and 10 months, at 7 per cent? Ans. $420,01ct. 5m.

46. What is the amount of $35,12 cents for 4 years, 11 months, and 10 days, at 7 per cent? Ans. $47,27cts. 5m.

47. What is the amount of $1000 for 3 years, $10\frac{1}{2}$ months, at 7 per cent? Ans. $1271,25cts.

When there is a fraction, as, $\frac{1}{4}$, $\frac{1}{2}$, $\frac{3}{4}$, &c. in the rate per cent:

RULE.—Multiply the principal by the rate per cent, then to the product add $\frac{1}{4}$, $\frac{1}{2}$, or $\frac{3}{4}$ of the principal, and divide the sum by 100, as in the preceding examples, and the quotient will be the interest for one year.

48. What is the amount of $250 dollars, at 5½ per cent, for one year? Ans. $263 75cts.

2)250 dollars.
5½
12,50 interest for one year at 5 per cent.
1,25 interest for one year at ½ per cent.
13,75 interest for one year at 5½ per cent.
250
Ans. $263,75 cents.

49. What is the amount of 1000 dollars for two years, at 6¼ per cent? Ans. $1125.

50. What is the amount of 500 dollars for four years, at 5¾ per cent? Ans. $615.

51. What is the amount of 750 dollars for 5 years, at 6½ per cent? Ans. $993,75cts.

To calculate interest on notes, bonds, &c. on which partial payments have been made.

The following Rule is established for the practice of the Courts in the State of New-York:

RULE I.—"The *Rule* for casting Interest, where partial payments have been made, is to apply the payment, in the first place, to the discharge of the interest then due. If the payment exceeds the interest, the surplus goes towards discharging the principal, and the subsequent interest is to be computed on the balance of principal remaining due. If the payment be less than the interest, the surplus of interest must not be taken to augment the principal, but interest continues on the former principal until the period when the payments, taken together, exceed the interest due, and then the surplus is to be applied towards discharging the principal; and interest is to be computed on the balance of principal as aforesaid."

(Johnson's Chancery Reports, Vol. I. page 17.)

This Rule is founded on the following principles:

1st. That interest is due whenever a payment is made, if the payment is equal to the interest to that time. If the payment is not equal to the interest, the payment is to be added to the following payment or payments, until their sum equals or exceeds the interest accrued, and then the balance forms a new principal.

2d. Interest is not allowed on endorsements, because payments first go to the discharge of the interest due, and the surplus towards the extinction of the debt.

3d. Interest cannot be taken on interest, nor go to augment the principal, because the interest is due when the payment or payments are made; and our courts will not sustain an action for the payment of interest from year to year, unless the express condition is contained in the obligation, that the interest shall be *paid annually*.

EXAMPLES.

1. Suppose you have a bond against B for 1000 dollars, dated May 15th, 1821, upon which you find the following endorsements, viz.:

			y.	*m.*	*d.*
1. September 20th, 1822.	Rec'd	$150,60cts.	1	4	5
2. October 25th, 1824.	-----	200,90	2	1	5
3. July 11th, 1826.	-----	75,20	1	8	16
4. September 20th, 1827.	-----	112,11	1	2	9
5. December 25th, 1828.	-----	105,00	1	3	5
				4	25

What remained due upon the bond May 20th, 1829? Interest to be cast at 7 per cent. Ans. $869,19cts. 9m.

The periods of time from one date to another may be found by subtracting the date of the bond or note from the time when the first payment was made, and so on, subtracting the time of one payment from the next, till the time of settlement. In setting down the date to obtain the time, first set down the year, then the months that have fully passed in that year, and then the days that have passed in the month named; then in the operation reckon 30 days to the month, and 12 months to the year; thus,

y.	*mo.*	*d.*	
1822	8	20	time of first payment.
1821	4	15	date of bond.
1	4	5	time up to the first end'd't.

y.	*mo.*	*d.*
1824	9	25
1822	8	20
2	1	5

y.	*mo.*	*d.*
1826	6	11
1824	9	25
1	8	16

y.	*mo.*	*d.*
1827	8	20
1826	6	11
1	2	9

y.	*mo.*	*d.*
1828	11	25
1827	8	20
1	3	5

y.	*mo.*	*d.*
1829	4	20
1828	11	25
	4	25

		y.	m.	d.
$1000	Principal.			
94,30,5	Int. to the first endorsement, being	1	4	5
1093,30,5	Am't. due, at the time the 1st pay't. was made.			
150,60	1st payment deducted.			
943,70,5	Principal remaining due, Sept. 20, 1822.			
138,54,1	Interest to Oct. 25, 1824.			
1082,24,6	Am't. at the time of the 2d payment.			
200,90	2d payment, Oct. 25, 1824.			
881,34,6	Principal remaining due, Oct. 25, 1824.			
	$105,56,5 Int. exceeds the pay't. July 11, 1826.			
179,08,3	73,51,8 Int. to Sept. 20, 1827, upon the same principal.			
	179,08,3 sum of the Int's.			
1060,42,9	Amount.			
75,20				
112,11				
187,31	Sum of the third and fourth payments deducted Sept. 20, 1827.			
873,11,9	Principal remaining due, Sept. 20, 1827.			
77,24,6	Interest to Dec. 25th, 1828.			
950,36,5	Amount to Dec. 25th, 1828.			
105,	5th payment deducted.			
845,36,5	Principal remaining due, Dec. 25, 1828.			
23,83,4	Interest to May 20, 1829.			
$869,19,9	Balance due, May 20, 1829.			

2. A note was given, April 20th, 1825, of $800,50cts.; May 25th, 1826, $250 were endorsed; and Dec. 20th, 1828, $300 were endorsed: what was due on the note, June 20th, 1829; interest at 7 per cent? Ans. $436,82cts. 4m.

3. A note was given April 15th, 1824, of $1200, upon which the following payments were made, viz.: April 15th, 1826, Rec'd. $400; Sept. 20th, 1827, Rec'd. $350,20cts. What amount remained due on the note, Sept. 20, 1828; interest 7 per cent? Ans. $764,76cts. 4m.

4. Suppose a bond or note dated July 10, 1825, was given for $450, interest at 7 per cent, on which was the following endorsement: Nov. 20th, 1827, Rec'd. $200; what remained due Sept. 10th, 1829? Ans. $365,37cts. 2m.

5. A note was given for $600, dated Jan. 15, 1826, and

there were payments endorsed upon it as follows, viz.: April 20th, 1827, Rec'd. 300 dollars; May 15th, 1828, Received 200 dollars; what remained due May 10th, 1829; interest at 7 per cent? Ans. $191,90cts.

The following Rule was established by the Superiour Court of Connecticut, in 1784.

RULE II.—"Compute the interest to the time of the first payment, if that be one year or more from the time the interest commenced, add it to the principal, and deduct the payment from the sum total. If there be after payments made, compute the interest on the balance due to the next payment, and then deduct the payment as above; and in like manner from one payment to another, till all the payments are absorbed; *provided*, the time between one payment and another be one year or more. But if any payment be made before one year's interest hath accrued, then compute the interest on the principal sum due on the obligation for one year, add it to the principal; and compute the interest on the sum paid, from the time it was paid, up to the end of the year; add it to the sum paid, and deduct that sum from the principal and interest added as above. If any payment be made of a less sum than the interest arisen at the time of such payment, no interest is to be computed, but only on the principal sum, for any period." —*Kirby's Reports, page* 49.

This Rule is founded on the following principles:

1. That, when obligations have run on interest for 1 or more years, interest is not due at intervals of time less than one year.

2. If an endorsement be made after one year's interest has accrued, but which is less than the interest at the time of such endorsement, then the endorsement shall not bear interest like endorsements made before a year's interest has become due.

3. "Interest is allowed on all endorsements from the time of their payment, until the year has elapsed, or until an endorsement is made beyond the year, which with the preceding endorsement or endorsements, and its or their interest, equals or exceeds the interest due at the time of such endorsement."

NOTE.—Legal interest in the New England states is 6 per cent.

1. Suppose a note was given on demand for 1000 dollars, dated February 1st, 1825; on which were the following endorsements:

1.	April 1, 1826, Received,	$80.
2.	August 1, 1826,	30.
3.	October 1, 1827,	10.

4. December 1, 1827, ... 600.
5. May 1, 1828. .... 200.

What was the balance due on the note, October 1, 1828, at 6 per cent? Ans. $263,93cts.

$1000	Principal.
70	Int. to the 1st end't, being 1 year and 2 months.
1070	Amount.
80	1st Endorsement.
990	Principal remaining due April 1, 1826.
99	Int. to the 4th end't, being 1 year and 8 months.
1089	Amount to the 4th endorsement.
30	2d Endorsement.
2,40	Interest to the 4th endorsement.
32,40	
10,	3d Endorsement, which does not bear interest, because 1 year's int. has accrued on new principal.
600,	4th Endorsement.
642,40	Deduct from 1089 dollars.
446,60	Principal remaining due, Dec. 1, 1827.
22,33	Int. to Oct. 1, 1828, being 10 months.
468,93	Amount to October 1, 1828.
200,	5th Endorsement.
5,	Interest to Oct. 1, 1828, being 5 months.
205,	Amount—deduct from $468,93 cents.

$263,93 cents. Balance due October 1, 1828.

2. Suppose a bond or note, dated May 10, 1824, was given for 2000 dollars, interest at 6 per cent, upon which were the following endorsements, viz.:

1. March 10, 1825, Received, ... $800.
2. May 10, 1826, ... 400.
3. September 10, 1827, ... 300.

What remained due January 10, 1829?

Ans. $821,78 cents 7 mills.

The following Rule was established by the Superiour Court of the state of Massachusetts in 1821.

RULE III.—"Compute the interest on the principal sum, from the time when the interest commenced, to the first time when a payment was made, which exceeds, either alone or in conjunction with the preceding payments, if any, the interest at that time due: add that interest to the principal, and from the sum subtract the payment made at that time, together with the preceding payments, if any; and the remainder forms a new principal, on which compute interest, as upon the first principal, and from the amount subtract the payment; and proceed in this manner, to the time of the judgment or settlement."—*Mass. Reports, Vol.* 17, *page* 418.

This Rule involves the same principles as the Rule FIRST *established for the practice of the Courts in the State of New York.*

1. That interest is due at any time when a payment is made; and that the payment first goes to discharge the interest, and then to reduce the principal; but interest is never allowed to form a part of the principal, so as to carry interest; for the effect in such case would be to give Compound Interest, which the law does not allow.

2. Where the interest exceeds the payment, interest is continued on the same principal, till the amount of the payments exceed the interest.

EXAMPLES.

1. Suppose a note was given on demand for $1000, dated Feb. 1, 1825, on which were the following endorsements.

1. April 1, 1826, Received	$80.
2. August 1, 1826,	30.
3. October 1, 1827,	10.
4. December 1, 1827,	600.
5. May 1, 1828,	200.

What remained due on the note, October 1, 1828, at 6 per cent? Ans. $265,87cts. 2m.

$1000	Principal.
70	Interest to the 1st end't., being 1 year 2mo.
1070	Amount to the first endorsement.
80	1st Endorsement.
990	Principal remaining due, April 1, 1826.
19,80	Int. to the 2d endorsement, being 4mo.
1009,80	Amount to 2d endorsement.
30,	2d Payment.
979 80	Principal remaining due August 1, 1826.

Carried over.

Brought over.

979,80	Principal remaining due August 1, 1826.
68,58,6	Interest to the 3d endorsement.
9,79,8	Int. to the 4th endorsement on the same principal; the interest exceeding the payment.
1058,18,4	Amount to the 4th endorsement.
10	3d Endorsement.
600	4th Endorsement.
610	Sum of the 3d and 4th end'ts. deducted.
448,18,4	Principal remaining due Dec. 1, 1827.
11,20,4	Interest to the 5th payment.
459,38,8	Amount to the 5th payment.
200,	5th Endorsement.
259,38,8	Principal remaining due May 1, 1828.
6,48,4	Interest to October 1, 1828.
$265,87,2	Balance due on note, October 1, 1828.
263,93	Balance due according to Rule II.
1,94,2	Difference in the two methods of work.

2. Suppose a bond or note was given for 1200 dollars, at 6 per cent interest, dated October 15th, 1826, on which were the following payments, viz.: Oct. 15th, 1827, Received $1000; April 15th, 1828, Rec'd $200. What remained due October 15th, 1828? Ans. $82,56cts. 4m.

Some of the *States* have adopted the following Rule: 1st. Find the amount of the principal and interest for the whole time; then find the amount of each endorsement, from the time it was made to the time of settlement; then from the first amount deduct the sum of the amounts of the several endorsements, and the remainder will be the balance due.

This Rule is founded on the following principle:

That interest is not due until the obligation is paid, and consequently interest must be allowed on endorsements from the time they were severally made, until the time of settlement.

As this *Rule* involves an absurdity, we therefore do not think it necessary to advert to it further than to show its fallacy. Suppose a man gives his note on demand for $100, and it runs for 22 [illegible], he, at the end of each year, paying 7 dollars, which is the lawful interest computed at 7 per cent, and justly due to the lender at the end of each year for the use of his money, brings the lender in his debt six dollars and nineteen cents, without paying him one cent of the principal; because 100 dollars, in 22 years, amounts to 254 dollars; and the 7 dollars endorsed at the end of the first year running on interest according to this rule, 21 years amounts to 17 dollars and 29 cents; and the endorsement, 7 dollars, made at the end of the second year, amounts to 16 dollars and 80 cents, and so on, casting the interest on the $7 paid at the end of each year to the time of settlement, or to the twenty-second year from the date of the note, and adding the several amounts, we find the total amount to be 260 dollars 19 cents, which is $6,19 cents more than the amount of the note, so that it is evident, according to this method of computing interest, that the borrower would get the lender in debt in 22 years, by barely paying him, at the end of each year, the interest justly due on the note; the rule is therefore absurd.

NOTE.—The difference of opinion relative to the computation of lawful interest, arises from the principle assumed in respect to the *time* when interest becomes due.

There is another method of casting Interest, which frequently shortens the work, especially where the rate is six per cent.

RULE.—Take half the greatest even number of months for a multiplier; and if there be an odd month, reckon it 30 days, and to the 30 add the given days, if any; then divide the days by 6, placing the quotient at the right of half the greatest even number of months, for a decimal, and take the whole for a multiplier, by which multiply the principal; pointing off from the product two figures further to the left hand than the usual place for decimals, and the product will be the interest, for the given time, at 6 per cent.

Should there be a remainder after taking one sixth of the days, reduce it to a vulgar fraction, that is, to its proper part or parts of 6 days, for which take aliquot parts of the multiplicand, thus:

If the remainder be 1=$\frac{1}{6}$, divide the multiplicand by 6
If ---------------- 2=$\frac{1}{3}$, ------------------------ 3
If ---------------- 3=$\frac{1}{2}$, ------------------------ 2
If ---------------- 4=$\frac{2}{3}$, ---------------- by 3 twice.
If ---------------- 5=$\frac{1}{2}$, and $\frac{1}{3}$, -------- by 2 and 3.

The quotients arising must be added to the product of the principal, multiplied by half the months, &c. The product will then be the interest required.

Or, instead of taking aliquot parts of the principal, for the remaining days, you may continue the division when dividing the days by 6, to *two* decimal places, which will give you a multiplier sufficiently accurate in business.

Should the rate be any other than 6 per cent, first find the interest at 6 per cent, then divide the interest, already found, by such parts as the interest at the rate required, *exceeds* or *falls short* of the interest at 6 per cent, and the quotient *added* or *subtracted* from the interest at 6 per cent, as the case may require, will give the interest.

O

EXAMPLES.

1. What is the interest of 500 dollars for 1 year 3m. and 15 days, at 6 and 7 per cent?

Ans. { $38,75 cts. 0m. interest at 6 per c
     { $45,20 cts. 8m. interest at 7 per c

```
   y. mo. d.
    1  3  15
   12
 2)15  m,
    7  1 over=30ds.
                    15
                  6)45 days.        6)45
                     7  3ds.   Or,    75  for a decima
3ds.=½)$500                          $500
         7,7                           7,75
        3500  or thus,                 2500
       3500                           3500
         250                         3500
    6)38,75,0--interest at        6)38,75,00 interest at
       6,45,8--6 per cent.          6,45,08 6 per cent.
Ans. $45,20,8--in't. at 7    Ans. $45,20,08 interest at 7 per
                 per cent.                  cent.
```

DEM.—It is plain, when we multiply by half the even number of months, we do the same as multiplying by the per cent, that is, 6 per cent; for suppose the number of months were 12, then half the number would be six, the same as the per cent, that is, 6 per cent, and suppose we call the odd month 30 days, and divide by 6, the quotient would stand as 5 tenths of two months; hence the reason of the rule is manifest.

NOTE I.—If the interest of 500 dollars, in the preceding example, had been required at 5 per cent, then the $[illegible]45cts. and 8 mills, should have been deducted from $38,75 cents; the remainder would have been the interest at 5 per cent.

NOTE II.—Where there are days, of a number less than 6, so that 6 cannot be contained in them, put a cipher in the place of the decimal, at the right hand of the months in the multiplier, so that the quotients, when the multiplicand is divided for the days, may stand in their proper place.

2. What is the interest of $300,50 cents for 2 years, 5 months, and 9 days, at 7 per cent? Ans. $51,36cts.

COMMISSION, OR FACTORAGE,

Is an allowance of a certain per cent, to a *Factor* or *Person* engaged in buying and selling goods for his employer. A *Factor* is an agent employed to transact business for another.

The work is precisely the same as casting the interest on the same sum for one year; but the *Factor* can only receive for his services from his employer, the *per cent* on the sales or purchases which he has made; thus, if the *Factor* buys or sells goods to the amount of one hundred dollars, at three per cent, he receives only *three dollars* for his services, and the remaining part of the gain, if any, goes to his employer.

EXAMPLES.

1. What may a factor demand from his employer, when he sells goods to the amount of $1000, and is allowed 3½ per cent for his services? Ans. $35.

$1000
3½
3000
500
Ans. $35,00

The learner will perceive, that the operation of the work here, is exactly the same as computing *interest* on one thousand dollars for one year, at 3½ per cent, and 35 dollars is what the factor receives for his services for selling the goods for the benefit of his employer.

2. What will the commission of 500 dollars come to at 5½ per cent? Ans. $27,50cts.

NOTE.—*Commission* here means the premium arising from the per cent on the given sum, which is the reward for the agent's services.

3. Required the commission of fifteen hundred dollars, at 3 per cent. Ans. $45.

4. What may a factor demand for laying out $1200, commission at 2½ per cent? Ans. $30.

BROKERAGE,

Is an allowance of a certain per cent, to persons assisting merchants or factors, in their purchases and sales.

The work is performed the same as in Simple Interest, where interest is to be computed for one year; and the *Broker* receives his per cent for his services; thus, if he assists his employer in buying or selling goods to the amount of one hundred dollars at 3½ per cent, he receives three dollars and fifty cents for his services.

EXAMPLES.

1. What is the brokerage upon 1500 dollars, at 1½ per cent? Ans. $22 50cts.

2)1500
$1\frac{1}{2}$
1500
750
Ans. $22,50cts.

NOTE.—A *Broker* is one that assists another in buying or selling goods. Brokerage is the allowance or premium given to the broker for his services.

2. A broker sold goods to the amount of $6000, at 1 per cent; what was his demand against his employer? Ans. $60.

3. A broker made sale of goods to the amount of $3000, at $\frac{1}{2}$ per cent, what was his demand? Ans. $15.

4. A broker bought goods to the amount of $1000, at $1\frac{3}{4}$ per cent; what must he receive for his services?
Ans. $17,50 cents.

ENSURANCE,

Is a premium, at a certain per cent, allowed to persons and offices, for indemnifying against loss of property of any description, such as houses, ships, merchandise, &c. which may happen from fire, storms, &c.

Policy is the name given to a writing by which the contract of indemnity is binding upon the parties.

If the person having his property ensured suffer damage or loss not exceeding 5 per cent, he must bear it himself, and cannot call on the *ensurers*, the average loss being 5 per cent.

The premium is paid to the ensurers, by the ensured, who receives the amount of his ensurance in case of loss from fire, from storms, &c. unless he has broken his policy or indemnity, by his own wrong.

The method of operation is the same as in Simple Interest for one year.

EXAMPLES.

1. What is the premium on 1500 dollars, at 8 per cent?
Ans. $120.

$1500
8
$120,00

The learner will understand, that if the ensurers indemnify against loss to the amount of 1500 dollars, in case of loss they must pay the person having his property ensured, the 1500 dollars, and the ensurers receive 120 dollars, whether there be loss or not.

2. A man's house, valued at 4000 dollars, was ensured against fire for $1\frac{1}{2}$ per cent a year; what ensurance did he pay annually? Ans. $60.

3. What is the ensurance of a vessel and cargo, valued at $90146,46cts. at $8\frac{1}{2}$ per cent? Ans. $7662,44cts. 9m.

BUYING AND SELLING STOCK.

Stock is the name of the capital of trading companies, or of a fund established by government; the value of which frequently varies.

The operation of the work is the same as in Simple Interest; thus, if bank or other stock be worth 75 per cent, 100 dollars stock would be worth 75 dollars; and 1000 dollars stock would be worth 750 dollars, which is obtained by multiplying the stock by the per cent; the product is the worth of the stock.

EXAMPLES.

1. Required the value of 1500 dollars of bank stock, worth 75 per cent? Ans. $1125.

```
        $                           1500
50=½)1500 dollars.                   ,75
25=½) 750            or thus,      7500
      375                         10500
    $1125                      $1125,00
```

Note.—In the last operation, we multiply by ,75 cents, because $1 is worth only ,75 cents, and in the first operation we take parts.

2. What is the value of 500 dollars of stock, at 95 per cent? Ans. $475.

3. What is the value of $200 dollars of stock, at 107 per cent? Ans. $214.

4. What is the value of 1500 dollars of bank stock, at 60 per cent? Ans. $900.

COMPOUND INTEREST.

Is that which arises from the *principal* and *interest* added together, as the interest becomes due; and for this reason it is called Compound Interest.

RULE.—Find the amount of the given sum for the first year the same as in *Simple Interest*, which will be the principal for the second year; then find the amount of that principal for the second year, and that will be the principal for the third year; and so on for any number of years. From the last amount subtract the given principal, the remainder will be the compound interest.

This rule is founded on the following principle:

That interest becomes due at the end of each year, at which time it becomes a part of the principal, if it remains in the hands of the borrower; this is not, however, allowed by law; yet in purchasing annuities, pensions, or leases in reversion, it is usual to allow Compound Interest to the purchaser for his ready money.

Note.—In our rule we have considered the interest to be due annually, and then becoming a part of the principal; but other periods of time are sometimes reckoned, as half-yearly, quarterly, &c. and then the interest is computed for every six months, or for every quarter of a year, and at the end of each period becomes a part of the principal, carrying interest for the next period.

EXAMPLES.

1. What is the compound interest of $1000, for 3 years, at 7 per cent? Ans. $225,04cts. 3m.

$1000	Principal.
7	Per cent.
70,00	Int. for 1 year.
1000	
1070	Am't. at the end of [the 1st year.
7	
74,90	
1070	
1144,90	Amount at the end of the 2d year.
7	
80,14,30	
1144,90	
1225,04,30	Amount at the end of the 3d year.
1000	Principal deducted.
$225,04,30	Compound Interest of $1000 for 3 years.

NOTE.—The learner will perceive, that compound interest is casting interest, at the end of each year, on the amount.

2. What is the compound interest of $750 for 4 years, at 6 per cent? Ans. $196,85cts. 7m.

3. What will $250 amount to in 3 years, at 7 per cent, compound interest? Ans. $306,26cts.

4. What will £50 amount to in 5 years, at 5 per cent per annum, compound interest? Ans. £63 16s. 3½d.

QUESTIONS ON INTEREST.

What is interest? A. An allowance or premium of a certain per cent for the use of money. What is the principal? A. The money lent. What is the amount? A. The principal and interest added together. What is the rate? A. The per cent paid by the borrower to the lender for the use of money. What is meant by *per cent?* A. It means a certain number of dollars for the use of a hundred dollars, or a certain number of cents for the use of one dollar, or one hundred cents. What is the per cent in the New England States? A. Six. What is the per cent allowed by law in New York? A. Seven. How many kinds of interest are there? A. Two, Simple and Compound. What is Simple Interest? A. That which arises from the principal daily? How do you cast the interest on any sum for one year? A. Multiply the interest for one dollar by the number of dollars, and the product will be the interest for one year; or, multiply the given sum by the interest of one dollar, and the product will be the interest for one year. Why should that give the interest for one year? A. Because it is plain, that the interest of one dollar repeated by the number of dollars, must give the interest on the whole number of dollars. Is this the most convenient way of casting interest? A. It is not; the best method of casting interest is, to multiply the given principal by the interest of one hundred dollars, and divide the product by one hundred. Why divide the product by 100? A. Because the product is one hundred times too much, the multiplier being the interest of one hundred dollars instead of the interest of one dollar. When you have computed the interest for one year, how do you compute it for a number of years? A. Multiply the interest of one year by the number of years, and the product will be the interest for the whole number of years. What general rule may be given to cast interest on a sum for any number of months? A. First multiply by the per cent, and the product will be the interest for one year, then divide the interest of one year by 12, and the quotient will be the interest for one month, and the interest for one month multiplied by the number of months, gives the interest for the whole number of months; but the better way is to take aliquot parts of the interest of a year for the months. What general rule may be given to compute interest for any number of days? A. Divide the interest for one year by 12, and the quotient will be the interest for one month; then divide the interest for one month by 30, and the quotient will be the interest for one day, and one day's interest multiplied by the number of days, will give the interest for the whole number of days; but the better way is to take aliquot parts of a month for the days. How is the amount obtained? A. By adding the principal to the interest. How do you cast interest on bonds in the state of New-York? A. Compute interest to the time of the first payment, and from the amount subtract the payment, and the remainder will be a new principal, upon which cast interest till the time of the next payment, and from the amount again subtract the payment, and so on, till the time of settlement; but if the payment be less than the interest, at the time it is made, cast again on the same principal, and so continue to do, till the amount of payments exceeds the interest, and then subtract the sum of the payments from the amount of the note or bond at that time. On what principle is the computation of interest founded in the state of New York when partial payments are made? A. On the principle that interest is due whenever a payment is made, and that the payment must first go to pay the interest, and then to discharge the principal. What is *Commission* or *Factorage?* A. It a premium or an allowance of a certain per cent, to a factor engaged in buying and selling goods for his employer. How is the work performed? A. The same as in Simple Interest. What is *Brokerage?* A. It is a premium allowed, at a certain per cent, to persons assisting merchants or factors in sales and purchases. How is the work performed? A. The same as in Simple Interest. What is *Ensurance?* A. It is a premium of a certain per cent allowed to persons and offices indemnifying against hazard or losses of different kinds. What is *Compound Interest?* A. It is reckoning interest upon interest; or at certain periods of time, the interest is incorporated with the principal, and draws interest. How do you proceed in the operation of the work? A. Compute interest on the given sum, the same as in Simple Interest, from time to time, as the interest becomes due, each time making the amount a new principal for the next period of time. Is compound interest allowed by law? A. It is not; yet it is usual to allow compound interest in purchasing annuities, pensions, or leases in reversion.

THE SINGLE RULE OF THREE.

The Single Rule of Three is properly an application of *Multiplication* and *Division;* for on those two rules it principally depends. It is generally divided into two parts, viz. the *Rule of Three Direct* and the *Rule of Three Inverse* or *Indirect.* It is sometimes called, and very properly too, the *Rule of Proportion, or the Golden Rule of Proportion,* on account of its extensive usefulness, in the transaction of business, and in the solution of almost every mathematical inquiry. The rule is founded on this obvious principle; that the magnitude or result of any effect, varies constantly in proportion to the varying part of the cause; thus the quantity of articles purchased, is in proportion to the money laid out; and the space gone over by a uniform motion, is in proportion to the time.

This is the sign of proportion, : :: : which is placed between numbers thus, 4 : 8 :: 5 : 10, and read thus, as 4 is to 8, so is 5 to 10.

The Rule of Three is so called, because three terms or numbers are given to find a fourth, which in the Rule of Three Direct, shall bear the same proportion to the third, as the second bears to the first, thus, 4 is : to 8 :: as 5 is : to 10.

It is obvious that 10, the fourth term, bears the same proportion to 5, the third term, that 8, the second term, bears to 4, the first term.

Two of the given numbers are called *terms of supposition,* and the other the *term of demand.* The terms of supposition may generally be known by being preceded, in most cases, by words like these, *if, suppose, &c.* The term of demand is generally preceded by words like these, *How far? What cost? What will? How many? How much? &c.*

RULE OF THREE DIRECT.

The Rule of Three Direct is far more useful in business than the Rule of Three Inverse, and may be distinguished from it by the conditions of the question. When the third term is greater than the first, and requires the fourth term, or answer, to be greater than the second, it belongs to the Rule of Three Direct; or when the third term is less than the first, and requires the fourth term, or answer, to be less than the second, it belongs to *this rule.*

RULE FOR STATING.

Write down for the first term, that term of supposition which is of the same name or kind with the demanding term. Place the remaining term of supposition in the second place, which must be of the same name or kind with the answer. Then set the demanding term in the third place, which must be of the same name or quality with the first term.

RULE FOR WORKING.

If the first and third terms are of different denominations, reduce them to the lowest denomination mentioned in either. If the second term stands in different denominations, reduce it to the lowest denomination mentioned in that term: Then multiply the second and third terms together, and divide their product by the first term, and the quotient will be the fourth term or answer in the same denomination of the second term, in whatever denomination it stands or has been reduced; then if the answer does not stand in the highest denomination, it should be brought to the highest by Reduction.

NOTE 1.—All the following rules strictly belong to the Rule of Three Direct, viz: the *Double Rule of Three, Exchange, Interest, Practice, Single and Double Fellowship, Tare and Tret, Barter, Loss and Gain, Alligation, Discount and Annuities.*

These *Rules* have acquired different names from the business to which they are applied; and in treating of them separately, it will be shown that they belong to this rule, and may, like Direct Proportion, or the Rule of Three Direct, be reduced back to the fundamental or Simple Rules.

The Double Rule of Three, or Compound Proportion, will in some cases embrace both the Rule of Three Direct and Inverse.

Interest, on account of its importance in business, and its evident relation to the Simple Rules, has been placed before the Rule of Three, though in all cases may be worked by the Rule of Three.

NOTE 2.—As the chief difficulty which learners experience in this rule, is in stating questions, teachers should exercise their pupils in stating fifteen or twenty questions for a few days in succession, and the difficulty will then vanish. They should be in the daily habit of demonstrating to their pupils the rules, and explaining the principles upon which they are founded, tracing them back to the simple rules on which they immediately depend. Instructors by pursuing this course, will present to the young mind this important science free from all its obscurity, and students will soon learn that the whole science of Arithmetick, is nothing more than a proper application of the fundamental or Simple Rules.

EXAMPLES.

1. If 2 yds. of calico cost 4s. what will 4 cost? Ans. 8s.

yds. *s.* *yds.*
2 : 4 :: 4
4
2)16
Ans. 8 shillings.

DEM.—When we multiply the second term by the third, we are multiplying the price of a quantity, consequently the product must be as many times too great as the first term exceeds a unit; therefore by dividing the product by the first term, the quotient must be the answer, or fourth term because the second term is double the price of a unit. N. B. The learner will perceive, that 8, the fourth term or answer, bears the same proportion to 4, the third term, that 4, the second term, bears to 2, the first term; because the fourth term is double the third, and the second term is double the first. The 1st and 4th terms are called *extremes*, and the second and third terms, *the means;* and the celebrated property of proportional numbers is, *that the product of the extremes, is equal to the product of the means;* thus, 2 : 4 :: 4 : 8, then 2×8=4×4=16. The first term of two proportional numbers, is also called the antecedent, and the other, to which it bears proportion, is called its consequent; thus, in the example given, 2, the first term, is the antecedent, and 4, the second term, the consequent; that is, two yards have produced the 4 shillings; and the third term, 4 yards, is also the antecedent which produced 8 shillings its consequent. Now it is evident, that the first antecedent bears the same proportion to its consequent, that the second antecedent bears to *its* consequent; that is, 2 yards bear the same proportion to the cost, 4 shillings, that 4 yards bear to the cost, 8 shillings; because in each case, the money bears the same proportion to the quantity of cloth purchased.

This rule may be proved by inverting the terms of the question. Or by multiplying the means, and if the product equal the product of the extremes, the work is right; thus, if 4 yds. cost 8s. what will 2 cost? Ans. 4s.

4*yds.* : 8*s.* :: 2*yds.*
2
4)16
4s.

This sum serves as a proof of the first example; here we find that 2 yards cost 4s. which agrees with the first supposition.

In order to render this excellent rule as plain as possible to the learner, the same example is again repeated, and reduced back to the simple rules on which it depends.

If 2 yards of calico cost 4 shillings; what will 4 yards cost? Ans. 8s.

2*yds.* : 4*s.* :: 4*yds.*
4
2)16
Ans. 8s.

Reduced to the simple rules by dividing the second term by the first, and multiplying the quotient by the third term; thus,

2)4s.
2s. price of 1 yard.
4
8s. price of 4 yards.

DEM.—We have learned by division, that dividing the price of a quantity by the quantity gives the price of a unit or 1; then when we divide 4 shillings, the price of 2 yards, by 2, the quotient, 2 shillings, is the price of 1 yard. And we have found by multiplication, that multiplying the price of 1 yard, by the number of yards, gives the price of the whole number; then when we multiply 2s., the price of 1 yard, by 4, the number of yards, the product 8s. must be the price of 4 yds. Hence it is plain, that *Direct Proportion, or the Rule of Three Direct,* is nothing more than an application of Multiplication and Division, and from Multiplication and Division it might be reduced back to Addition and Subtraction, as those two rules have been reduced back, in their proper places. And it is the better way to divide the second term by the first, and multiply that quotient by the third term, whenever the first term will divide the second without a remainder.

2. If one yard of cloth cost four dollars; what cost eight yards? Ans. $32.

yd. $ *yds.*
1 : 4 :: 8
8
$32

DEM.—It is plain, when the first term is a unit, that the work is performed by multiplication, because it is only repeating the price of a unit or one, by the whole number; and dividing that product by the first term, when it is a unit, can evidently make no difference.

3. If eight yards cost $32; what cost one yard? Ans. $4.

yds. $ *yd.*
8 : 32 :: 1
1
8)32
Ans. $4

DEM.—It is plain, when the 3d term is a unit, that the work is performed by division merely, for multiplying by one can make no difference; it is only bringing down the middle or second term for the convenience of dividing by the first term, and it is evident, if the price of a quantity be divided by the quantity, that the quotient must be the price of a unit.

4. If 6 yards of cloth cost $24; what cost 12 yards? Ans. $48.

yds. $ *yds.*
6 : 24 :: 12
12
6)288
Ans. $48 the price of 12 yds.

Or thus,

$
6)24 the price of 6 yds.
4 the price of 1 yd.
12
A. $48 the price of 12 yds.

DEM.—It is evident from the reasoning under the first example, that when we multiply 24 dollars, the price of 6 yards, by 12, the product is 6 times too much; then when we divide the product by 6, the first term, the quotient must be the price of 12 yards. And it is also plain, when we divide $24, the price of 6 yards, by 6, that the quotient must be the price of one yard; and the price of one yard multiplied by 12, evidently gives the price of 12 yards.

5. If six pounds of tea cost $3,75cts.; what cost eighteen pounds? Ans. $11,25cts.

heat cost $12; what *cost 36 bush-*
Ans. $48.

48; what will 9 bush. *cost? Ans. $12.*

lax seed cost 45 dollars; what will 1
Ans. ,75cts.

ost 75 cents; what will 60 *bushels*
Ans. $45.

ll buy 12 yards of cloth; how many
ry? Ans. 9 yards.

road cloth be worth 144 dollars; what
Ans. $36.

butter cost $2,52 cents; what will 7
Ans. ,63cts.

f oats cost $20,16cts.; what will 12
Ans. $3,84cts.

ad; what will 40 sheep cost?
Ans. $50.

l buy 40 sheep; how many sheep will
ts buy? Ans. 90.

acres of land cost, at ten dollars and
Ans. $3150.

land cost 3150 dollars; what cost 1
Ans. $10,50cts.

beef cost $8,40cts.; what is the value
Ans. $18,80cts.

r. *lb.*
l 12

).

$ *cts.*
(18,80 Ans.

When the 1st and third terms are of different denominations, the better way is to state the question, and then perform the reduction, because the whole work will then show itself together.

NOTE.—The learner must be careful in stating questions, that the first and third terms be alike; that is, if the first term be weight, the third term must be weight; if the first term be money, the third must be money; and the middle or second term must be the same kind of the answer sought; that is, if weight be required for the answer, the second term must be weight, if money be required for the answer, the second term must be money.

19. If a man's yearly income be 438 dollars; what is that per day? Ans. $1,20cts.

20. If a man's wages be 162 dollars a year; what is that per calendar month? Ans. $13,50cts.

21. If 4yds. 2qrs. cost 19 dollars; what will 3yds. 3qrs. cost? Ans. $15,83cts. 3m.

22. If 2yds. 3qrs. cost $6,60cts.; what will 11 yards cost? Ans. $26,40cts.

23. If 11 yards cost 26 dollars 40cts.; what will 2 yards 3qrs. cost? Ans. $6,60cts.

24. If a man spend 1 shilling and 6 pence a day; how long will £27 7s. 6d. last him? Ans. 1 year, or 365d.

25. A merchant failing in trade, compounds with his creditors for 56 cents on the dollar, and at that rate, pays 2840 dollars 12 cents; what was his debt? Ans. $5071,64,2m.

26. Suppose the clothing of a regiment of 750 men amounts to £2381 5 shillings; what will the clothing of a body of 3400 men amount to? Ans. £10795.

```
men.   £     s.   men.
 750 : 2381  5 :: 3400
         20
       -----
       47625
        3400
     ---------
     19050000
     142875    2|0
     ---------
750|161925000(21590|0
    1500       ------
    ----       £10795
     1192
      750
     ----
      4425
      3750
      ----
       6750
       6750
       ----
        000
```

The learner will recollect, that when the second term is expressed in different denominations, it must be reduced to the lowest denomination mentioned, and the quotient or answer will be of the same name of the second term, to whatever it is reduced.

NOTE.—Whenever there is a remainder, after dividing by the first term, and the second term has not been reduced to the lowest denomination, the remainder must be reduced to the next inferiour denomination, and the division continued, the same as in Compound Division.

27. What will be the yearly income from 178 acres, 2 roods, 14 poles of land, at the rate of £1 7s. 8d. per acre?

Ans. £240 2s. $7\frac{1}{20}$d.

28. If 9cwt. of sugar cost £54; what will 7cwt, 3qrs. 14lb. cost? Ans. £47. 5s.

29. A owes B $350, but B compounds with him for 65 cents on the dollar; what must B receive for his debt?

Ans. $227,50cts.

30. A bankrupt owes 1250 dollars, and his money and effects amount to $750,50 cents; what is a creditor entitled to, whose demand is 40 dollars? Ans. $24,01ct. 6m.

31. If a person's income be $1877,50cts. a year; how much may he spend each day, to save, at the year's end, $600? Ans. $3,50cts.

32. If a man receive 7 dollars interest a year, for the use of 100 dollars; what should he receive for the use of 375 dollars, the same time? Ans. $26,25cts.

33. If the interest of 375 dollars for one year, be $26,25cts.; what will it be for 19 days? Ans. $1,36cts. 6m.

34. Bought 10 pieces of cloth, each piece containing $25\frac{3}{4}$ yards, at $2,50cts. per yard; what did the whole come to?

Ans. $643,75cts.

35. Bought 120 gallons of brandy for $187,50cts.; how much water must be added to the brandy, to reduce the first cost to $1,25cts.? Ans. 30gal.

NOTE.—When the given number is in barrels, bales, packages, or pieces, each containing an equal quantity, reduce the quantity contained in one, to its lowest denomination, and then multiply by the given number of packages or pieces. If the given number of barrels, bales, pieces, &c. be of unequal content, put the separate content of each under one another, as in Compound Addition, then add them, and their sum will be the whole quantity.

36. Bought 3 pieces of cloth, containing 36, $34\frac{1}{2}$, and $28\frac{1}{2}$ yards, at $2,50 per yard; how much did the whole amount to? Ans. $247,50cts.

37. Bought 3 hogsheads of rum, containing $60\frac{1}{2}$, $62\frac{1}{2}$, and 61 gallons, at 90 cents per gallon; what did the whole come to? Ans. $165,60cts.

38. *Suppose a freight boat leave Utica for Albany, 12 hours before a packet boat, and go at the rate of $2\frac{1}{2}$ miles an

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* This sum will require two statements; one to ascertain the time, and the other the distance.

hour, the packet follows at the rate of 4 miles an hour; in what time, and at what distance will the packet overtake the freight boat?

Ans. { The packet will overtake the freight boat, in 20h, and at the distance of 80 miles from Utica.

39. A, leaves the city of New York, to go to Montreal, in Lower Canada, and travels at the rate of 35 miles a day; B, at the same time, leaves Montreal to go to New York, and travels the same road, at the rate of 30 miles a day; how far from the city of New York will they meet, allowing the distance to be 390 miles? Ans. 210 miles

40. As I was hunting on the forest grounds,
Up starts a hare, before my two grey-hounds;
The dogs, being light of foot, did fairly run,
Unto her fifteen rods, just twenty-one.
The distance that she started up before
Was fourscore and sixteen rods, just, and no more;
Now this I'd have you unto me declare,
How far they ran before they caught the hare?
Ans. 336 rods.

41. If 80 gallons of water, in one hour, be conducted into a cistern which will hold 400 gallons, and a conductor convey 30 gallons from the cistern in an hour; in what time will it be filled? Ans. 8 hours.

42. The valuation of the property in a certain town, according to the town's inventory, is $610,000, and the tax levied on that town, is $3050; what is A's tax, whose estate according to the inventory is valued at $1500?
Ans. $7,50cts.

NOTE.—As the valuation of the whole property in the town, is to the whole tax, so is the valuation of each man's estate, to his tax.

43. If a tax of $650 be laid on a certain school district, for erecting a school house, and the inventory of all the estates, in that district, amount to $130,000; what must A pay, whose estate is $2500? Ans. $12,50cts.

44. The time made out by the inhabitants of a certain school district, amounts to 3040 days, during the winter, and the teacher is to have $75 for his services; what must A pay, who has sent 80 days to the school? Ans. $1,97cts. 3m.

45. How many yards of carpeting that is half a yard wide, will cover a room 18ft. wide, and 20ft. long? Ans. 80yds.

NOTE.—*To obtain the surface of a plane, as a floor, field, &c.* Multiply the length by the breadth, and the product will be the surface; then as the number of square feet in a yard of the carpeting which is 4,5 feet, is to one yard, so is the number of square feet in the floor, to the number of yards required.

46. How many bricks that are 4 inches wide, and 8 inches long, will lay a floor sixteen feet wide, and twenty feet long? Ans. 1440 bricks.

47. If a staff six feet in length cast a shadow of four feet, what is the height of a tree, whose shadow at the same time measures 125 feet? Ans. 187 feet 6 inches.

48. A merchant bought 8 hogsheads of brandy, for 950 dollars; he paid for the freight $100, for the duties and other charges $50; and he wishes to gain $100; how must he sell it per hogshead? Ans. $150.

NOTE.—It is just the same as if the merchant had, at first, paid out for the brandy the amount of the freight, duties, and other charges, together with the gain and the first cost; and then should wish to know what it cost him per hogshead.

49. A merchant bought 16 hogsheads of wine, at the rate of £87 13s. 6d. per hogshead, New-York currency; how many dollars will pay for it, in federal money? Ans. $3507.

50. Suppose a merchant pay 12s. 6d. New England currency, a gallon, for wine; what would 1000 gallons amount to, at that rate, in federal money? Ans. $2083,33cts. 3m.

51. Bought 300 Ells English, for 500 dollars, but being damaged, I am willing to lose 50 dollars by the sale; how must I sell it per yard? Ans. $1,20cts.

52. If I buy 9lb. of coffee for $1,08 cents; how many pounds can I buy for 9 dollars? Ans. 75lb.

53. A gentleman being asked what hour of the forenoon it was, answered, it is between 8 and 9, and the minute and hour hands are together; what was the exact time?

Ans. $43\frac{7}{11}$ minutes past 8.

NOTE.—The motion of the minute hand, to that of the hour hand, is as 12 to 1, therefore 12—1=11; then as 11 is to 8, so is 12 to the answer.

THE RULE OF THREE INVERSE.

This Rule, like the Rule of Three Direct, is properly an application of Compound Multiplication and Compound Division, and may like Direct Proportion be reduced back to those rules. This rule is sometimes called Indirect or Inverse Proportion.

RULE.—State the questions, and reduce the terms, the same as in *the Rule of Three Direct;* then multiply the first and second terms together, and divide the product by the third; and the quotient will be the answer, in a denomination of the same name, to which the second term was reduced.

In this rule the fourth term or answer bears the same proportion to the second term, as the first term bears to the third.

If more require less, or less require more; the question belongs to Inverse Proportion.

More requiring less, is when the third term is greater than the first, and requires the fourth term to be less than the second.

Less requiring more, is when the third term is less than the first, and requires the fourth term to be greater than the second.

NOTE.—The principal difficulty which the student will experience in this rule, will be to distinguish Inverse or Indirect Proportion from Direct Proportion, but as the rule given for stating the questions is the same in both, the student can easily decide to which it belongs, by considering, whether *more requires less, or less requires more.*

EXAMPLES.

1. There was a certain dam erected across a river, in 60 days, by 12 workmen; but the same being swept away, it is required to rebuild it in 30 days; how many men must be employed about it, working at the same rate? Ans, 24 men.

d. *men.* *d.*
60 : 12 :: 30
60
3|0)72|0
24 Ans.

DEM.—From the condition of this question, it is plain, that it belongs to Inverse Proportion, because it is evident, the less the time, the more men must be employed to finish the work.—Here our third term is less than the first, and requires the fourth term or answer to be greater than the second term; and it is evident from the conditions of the question, that the fourth term or answer should be double the second term, because it must require double the number of men to perform the same work in 30 days, that it would in 60 days. And it will be seen, that the fourth term or answer, 24 men, bears the same proportion to the second term, 12 men, that the first term, 60 days, bears to the third term, 30 days; thus, 24 : 12 :: 60 : 30. To reduce this sum back to Compound Multiplication and Division, divide the first term by the third, and multiply the quotient by the second term; thus,

3|0)6|0d.
2
12
24 The answer as before.

2. If a footman perform a journey in 6 days, when the days are 16 hours long; how many days will he require to go the same journey, when the days are 12 hours long? Ans. 8 days.

```
 h.   d.   h.
16 : 6 :: 12
     16
    ---
     36
     6
    ---
  12)96
    ---
Ans.  8
```

Dem.—From this example the learner can plainly see, that the footman travelled 96 hours; then when he travelled 16 hours in a day, it is evident, that it took him a less term of time in days, than when he travelled only 12 hours in a day; and it is plain, that every 12 hours are equal to a day, when he travelled only that time in a day; then as often as 12 is contained in 96, the whole number of hours employed in travelling, so many days he must have been employed in travelling, which proves to be 8 days.

3. If 3 teams can plough a field in 12 days; in what time will 9 teams do it? Ans. 4 days.

4. How many yards of shalloon that is 3 quarters wide will line 18yds. of cloth that is 5qrs. wide? Ans. 30yds.

5. If I lend a friend 200 dollars, for 90 days; how long ought he to lend me 450 dollars, to return the favour? Ans. 40 days.

6. If a board be 8 inches wide; how much in length will make a square foot? Ans. 18 inches.

7. How many yards of paper, 3 quarters wide, will paper a room that is 26 yards round, and 3 yards high? Ans. 104 yards.

8. How much land, at $3,75 cents per acre, should be given in exchange for 300 acres, at $6,50 cents per acre? Ans. 520 acres.

9. How many men must be employed to do a piece of work in five days, which 8 men can do in fifteen days? Ans. 24 men.

10. If 4 men will set 250 apple trees in six days; how many men will set the same number in two days? Ans. 12 men.

EXAMPLES,

Promiscuously placed, in Direct and Inverse Proportion.

1. If $\frac{5}{9}$ of a farm be worth 1250 dollars, what is the whole farm worth? Ans. $2250.

```
Shares.  $   Shares.
  5 : 1250 :: 9
             9
     --------
     5)11250
     --------
Ans.  $2250
```

Dem.—Here it is plain, that the farm is divided into 9 equal shares, or 9 parts; then it is evident, that as 5 shares, are to the worth of 5 shares, that is, $1250, so are 9 sh., the whole farm, to the worth of the whole.

2. If a farm be worth 2250 dollars, what is $\frac{5}{9}$ of the farm worth? Ans. $1250.

3. If a man receive $52,50cts. for the use of a certain sum, for the term of one year; what should he receive on the same sum, for 25 days? Ans. $3,59cts. 5m.

4. If 100 dollars gain 7 dollars interest in 12 months; what principal will gain the same interest in 4 months? Ans. $300.

5. If a man, worth 4500 dollars, pay 12 dollars tax; how much should he pay, who is worth 1500? Ans. $4.

6. If an ounce of silver be worth 63 cents, what is the price of 9 ingots, each weighing 4lb. 3oz.? Ans. $289,17.

7. If 60 bushels of corn, at 50 cents per bushel, will pay a debt, how many bushels, at 75 cents per bushel, will pay the same? Ans. 40 bushels.

8. A man is 1600 dollars in debt, and his whole estate amounts to but 992 dollars; how much can he pay on the dollar? Ans. 62 cents.

9. If a staff, 4ft. cast a shade 5 feet; what is the height of that steeple whose shade, at the same time, measures 175 feet? Ans. 140 feet.

QUESTIONS ON THE RULE OF THREE DIRECT.

What is the Single Rule of Three? A. It is properly an application of Multiplication and Division. Why is it called the Rule of Three? A. Because three terms or numbers are given to find a fourth term or answer to the question. How may the first term be known? A. By being preceded, generally, by words like these, *if*, *suppose*, *&c.*, and being of the same name or kind with the demanding term. How is the second term known? A. By being a term of supposition, and of the same name or quality with the answer. How is the third term known? A. By being the demanding term, and generally preceded by words like these, *What cost? How far? How much? What will? &c.*—On what principle is the Rule of Three founded? A. On the obvious principle, that the magnitude or result of any effect, varies constantly in proportion to the varying part of the cause; thus, the quantity of articles purchased, is in proportion to the money laid out. How is the Single Rule of Three divided? A. Into two parts, the Rule of Three Direct, and the Rule of Three Inverse or Indirect. It is sometimes distinguished by the name of *Proportion*, Direct or Inverse. What is meant by Proportion? A. That two numbers have the same relation to each other, that two other numbers have to each other; thus 4 bears the same proportion to 8, that 6 bears to 12. What are the two first terms in the statement called? A. Terms of supposition. What is the remaining term, (which is given,) called? A. The demanding term. If the first and third terms are of different denominations, what must be done? A. They must be reduced to the same denomination. If the second term is of different denominations, what must be done?. A. It must be reduced to the lowest denomination mentioned in that term. What must be done, after the terms are reduced? A. The second and third terms must be multiplied together, and the product divided by the first term; then the quotient will be the answer in the same name of the middle term, if the question be in the Rule of Three Direct. How is the Rule of Three Direct distinguished from the Rule of Three Inverse? A. If the third term be greater than the first, and require the fourth to be greater than the second; or if the third term be less than the first, and require the fourth term to be less than the second; the question belongs to the Rule of Three Direct. How can the Rule of Three Direct be reduced back to multiplication and division? A. By dividing the second term by the first, and multiplying that quotient by the third term; the quotient will be the answer, because we, in division, divide the price of a quantity, by the quantity, and the quotient is the price of a unit; and the price of a unit multiplied by the quantity, gives the price of the whole quantity. What rules strictly belong to the Rule of Three? A. Interest, Practice, Exchange, Single and Double Fellowship, Tare and Tret, Barter, Loss and Gain, Alligation, Discount, Annuities, Position, and every other rule where *proportion or ratio* exists, may be considered as belonging to this rule. These rules have acquired different names principally on account of the business to which they are applied.

QUESTIONS ON THE RULE OF THREE INVERSE.

How is the Rule of Three Inverse distinguished from the Rule of Three Direct? A. When the third term is greater than the first, and requires an answer to be less than the second term; or when the third term is less than the first, and requires the answer or fourth term to be greater than the second, the question belongs to the Rule of Three Inverse. Are the questions stated the same in Inverse and Direct Proportion? A. They are, and reduced in the same manner. What terms are multiplied together? A. The first and second; and the product divided by the third. What proportion do the terms bear to each other, in Inverse Proportion? A. The fourth term, or answer, bears the same proportion to the second, as the first bears to the third. In Direct Proportion, what proportion does the fourth term or answer bear to the third? A. The same as the second bears to the first

PRACTICE,

Is a contraction of the Rule of Three Direct, when the first term is a unit or one. It has acquired its name from its daily use among merchants, being an easy method of working, where the price of a unit or one is given, to find the price of a quantity. This rule was once of great use to tradesmen and merchants, when the price of one was given in sterling money, to find the price of a quantity; but it is now rendered almost useless, as reckoning in federal money has almost become universal; and when the price of a unit or one is given in federal money it is much easier to work by multiplication.

Proved by the Single Rule of Three, Compound Multiplication, or by varying the parts.

CASE I.—*When the price of one pound, one yard, &c. is given in farthings.*

RULE.—Suppose the given number of yards or pounds to be so many pence, and take aliquot parts of a penny; thus, if it be at one farthing per yard, take one fourth; if at two farthings, take one half; if at three farthings, take one half, and one fourth, and add the quotients; if there be 1 quarter of a yard, call it 1 farthing, if ½ of a yard, call it 2 farthings; if ¾ of a yard, call it 3 farthings, in setting down the given sum.

EXAMPLES.

1. What will 364 yards of tape come to, at two farthings per yard? Ans. 15s. 2d.

qrs. *d.*
2=½)364
12)182
Ans. 15s. 2d.

DEM.—It is plain that at two farthings per yard, one half the number of yards, would be the value of the tape in pence; therefore we suppose the number of yards to be so many pence, and take one half of them, and divide the quotient by 12, to bring the pence into shillings.

By Multiplication.

yds.
364
2
4)728
12)182
Ans. 15s. 2d.

By the Rule of Three Direct.

yd. *qrs.* *yds.*
1 : 2 :: 364
2
4)728
12)182
Ans. 15s. 2d.

This operation, by Multiplication, and by the Rule of Three, is too plain to need demonstration.

2. What will 396½ yards come to, at three farthings per yard? Ans. £1 4s. 9d. 1qr.

qrs. *d.* *qrs.*
2=½)396 2
1=½)198 1 at 2 qrs.
99 0 at 1 qr.
12)297 1 at 3qrs.
2|0)2|4 9d.
Ans. £1 4s. 9d. 1qr.

It will be seen, that there is a small loss in working according to this rule, when there is a remainder in the lowest denomination. Where it is more convenient we take parts of parts; thus, at 1qr. it is ½ of the price of the whole quantity, at 2qrs. per yard.

3. What will 34692 yards come to, at one farthing per yard? Ans. £36 2s. 9d.

4. What will 34964½ yards come to, at two farthings per yard? Ans. £72 16s. 10¼d.

Case II.—*When the price is given in pence, or in pence and farthings.*

RULE.—Suppose the yards, pounds, &c. to be so many shillings; and if there be ¼ of a yard, call it 3 pence; if there be ½ of a yard, call it 6 pence; and ¾ of a yard, call it 9 pence. Then take aliquot parts of a shilling or the price, at one shilling a yard for the given price.

EXAMPLES.

1. What will 396½ yards come to at 9 pence per yard? Ans. £14 17s. 4½d.

d. *s.* *d.*
6=½)396 6
3=½)198 3 price at 6d. per yd.
99 1½ price at 3d. per yd.
2|0)29|7 4½ price at 9d. per yd.
Ans. £14 17s. 4d. 2qrs.

2. What will 864yds. come to, at 4 pence per yard? Ans. £14 8s.

3. What will 3484¼ yds. cost, at 3d. per yd.? Ans. £43 11s. 0d. 3qrs.

4. What will 3050 yards come to, at 8 pence per yard? Ans. £101 13s. 4d.

Case III.—*When the price of one yard, or one pound, is shillings, or shillings and pence.*

RULE.—Suppose the number of yards, or pounds, to be so many pounds in money; and if there be ¼ of a yard, call it 5 shillings; if ½ of a yard, call it 10 shillings; if ¾ of a yard, call it 15 shillings; then as it stands, at £1 per yard, take aliquot parts for the given price per yard.

1. What will 396½ yards cost, at 5 shillings per yard? Ans. £99 2s. 6d.

s. £ *s.*
5=¼)396 10
Ans. £99 2s. 6d.

2. What will 980 yards come to, at 15s. per yard? Ans. £735.

3. What will 384 yards come to, at 12s. per yard? Ans. £230 8s.

4. What will 980 yards come to, at £2 5 shillings per yard? Ans. £2205.

Note.—When the price of one yard exceeds £1; multiply the given quantity standing at £1 per yard, by the number of pounds per yard; and then take, as in the other examples, parts for the shillings and pence.

QUESTIONS ON PRACTICE.

What is Practice? A. A contraction of the Rule of Three Direct.
How would the questions be stated in the Rule of Three? A. As 1lb.

or 1 yard, is to the price of 1 lb. or one yard, so is the given number of pounds or yards, to the price of the whole quantity. Can it be performed by multiplication? A. It can, by multiplying the price of one yard or one pound by the whole quantity; or the whole quantity multiplied by the price of one yard or one pound, will give the price of the quantity. When the price is given in farthings, what do you suppose the yards to be? A. So many pence, and then take aliquot parts. When the price of one yard or one pound is given in pence, what do you suppose the given quantity to be? A. So many shillings, and take aliquot parts. When the price of one yard or one pound is given in shillings, what do you suppose the given number of yards or pounds to be? A. So many pounds sterling, and then take aliquot parts of the price, at £1 per yard or pound.

SINGLE FELLOWSHIP,

Is only another name given to the *Rule of Three Direct*, when applied to the business of merchants, and others trading in company. Its principles are applied in finding each man's share of the gain or loss in trade, in proportion to his stock. In Single Fellowship, partners continue their stock in trade an equal term of time.

RULE.—As the whole stock is to the whole gain or loss, so is each man's stock to his share of the gain or loss.

NOTE.—This rule may be applied in adjusting a bankrupt's estate among his creditors; for as the sum of the debts due to the creditors is to the bankrupt's estate, so is each creditor's demand to his share of the bankrupt's estate. But it would reduce it to greater simplicity, to work by Multiplication and Division; first dividing the whole estate by the sum of the debts, and the quotient will be the sum to be received on a dollar, and this quotient multiplied by each creditor's demand, will give, separately, each creditor's share of the estate. It will in the same manner apply in making out a tax list; for as the amount of property is to the tax to be levied, so is each man's estate to his share of the tax. A school bill may be made out in the same way; for as the sum of the days sent is to the teacher's wages, so is each person's number of days to his bill; but the better way to make out a tax list or school bill, is to reduce the work to Multiplication and Division.

EXAMPLES.

1. Two merchants trading in company, gained $800; A's stock was $1800, B's $1400; what was each man's share of the gain? Ans. { $450 A's share. / $350 B's share.

A's stock, $1800
B's stock, $1400

$$\begin{array}{l}\quad\$\qquad\qquad\quad\$\\ \underline{3200} : 800 :: \begin{cases}1800\\ \underline{1400}\end{cases} : \text{Ans.} \begin{cases}\$450 \text{ A's share.}\\ \underline{\$350 \text{ B's share.}}\\ 800 \text{ PROOF.}\end{cases}\end{array}$$

By adding the several shares of gain or loss.

DEM.—It is evident, that the operation here, is as in the Rule of Three Direct, and that each man's share of the gain, is in proportion to his stock.

```
$1800       Same example by Division and Multiplication.
$1400  $
-----
3200)800,00 ( ,25 cents on a dollar.     ,25
     640,0     1800                      1400
    ------    -----                     -----
     160,00   20000                     10000
     160,00   25                        25
    ------    -----                     -----
          0   $450,00 A's share.       $350,00 B's share.
```

2. A, B, and C companied; A put in $400, B 600, C 800 dollars; they gained 540 dollars; what must each have of the gain?
Ans. A's share 120, B's 180, C's $240.

3. Divide 742 dollars between A, B, and C, so that B shall have twice as much as A, and C twice as much as B; what will be the share of each?
Ans. A's 106, B's 212, C's $424.

4. A captain, mate, and 20 seamen, took a prize worth 4674 dollars; of which the captain is to have 12 shares, and the mate 6 shares; the remainder is to be divided equally among the seamen; how much is each person's share?
Ans. { The captain's share is $1476, the mate's $738, each seaman's share $123.

QUESTIONS ON SINGLE FELLOWSHIP.

What is Single Fellowship? A. the Rule of Three Direct applied to the business of partners in trade, to ascertain each man's share of the gain or loss. Why does this belong to the Rule of Three? A. Because it is evident that each man's share of gain or portion of loss, must be in proportion to his stock. Can Fellowship be worked by Multiplication and Division? A. It can, by dividing the whole gain by the whole stock; the quotient will be the gain on a dollar, and that repeated by each man's stock will give the gain of each.

DOUBLE FELLOWSHIP.

In *Double Fellowship*, or Fellowship with time the stocks are continued in Trade unequal times. This *rule*, as well as Single Fellowship, belongs strictly to the *Rule of Three Direct.*

RULE.—Multiply each man's stock by the time it was continued in trade; then, as the whole sum of the products, is to the whole gain, or loss, so is each man's product to his share of the gain or loss.

EXAMPLES.

1. Three persons traded in company; A put in 400 dollars for 2 years, B 400 dollars for 4 years, and C 200 dol-

lars for 8 years; they gained 600 dollars; what must each have of the gain? Ans.

$$\left.\begin{array}{l} \text{A}\$400\times2=\ \ 800\\ \text{B}\ \ 400\times4=1600\\ \text{C}\ \ 200\times8=1600\\ \quad\text{Sum, }\overline{4000}\end{array}\right\}\text{As }4000:600::\left\{\begin{array}{l}\ \ 800:\$120\text{ A's sh.}\\ 1600:\ \ 240\text{ B's sh.}\\ 1600:\ \ \underline{240}\text{ C's sh.}\\ \text{Proof, }\$600\end{array}\right.$$

DEM.—From the example given, it is evident that this is strictly the Rule of Three Direct, and each person's share of gain or loss, must be in proportion to his product of money and time. A's share of gain is only one half as much as B's, because his money is invested in trade only half as long; and C's is equal to B's, because he has half as much money as B, invested in trade double the time; and it is plain, that 200 dollars must gain as much in 8 years as 400 dollars in 4 years.—This can be reduced back to Multiplication and Division, by dividing the whole gain, 600 dollars, by the sum of the products; the quotient, which would be the gain on a dollar, must then be multiplied separately by each man's product for the gain of each.

2. Two partners gained by trading, 800 dollars; A's stock was 1200 dollars for 8 months, and B's $800 for 10 months; what is each partner's share of the gain?

Ans. A's share $436,36cts. 3m., B's $363,63cts. 6m.

3. A, B, and C, entered into partnership; A, put in $400 for 6mo.; B, $250 for 8mo.; C, $360 for 5½mo.; on adjusting their accounts they find their loss to be $638; what is each man's loss? Ans. A's loss $240, B's $200, C's $198

QUESTIONS ON DOUBLE FELLOWSHIP.

Wherein does Double Fellowship differ from Single Fellowship? A. In Double Fellowship, the stocks are continued in trade unequal times, which is not the case in Single Fellowship. Does Double Fellowship depend on the Rule of Three Direct the same as Single Fellowship? A. Just the same, and may be reduced back to division and multiplication. How is the work prepared for a statement, in the Rule of Three? A. By multiplying each man's stock by the time it was continued in trade, and then adding the several products; then say, as the sum of the products, is to the whole gain or loss, so is each man's product, to his share of the gain or loss.

BARTER,

Is exchanging one commodity for another, according to the price or value agreed upon by the parties concerned. It is merely an application of the *Rule of Three Direct* to the exchange of specifick articles, so that neither party shall sustain a loss. The operation is frequently easier performed by multiplication and division.

RULE.—Find the value of the commodity, whose quantity is given, then find what quantity of the other, at the proposed rate, can be purchased for the same money, and it will give the answer.

EXAMPLES.

1. How much corn, at 40cts. a bushel, must be given for 60 bushels of wheat, at 80cts. a bushel? Ans. 120 bushels.

By Multiplication and Division.

```
bu. cts.  bu.            cts.
 1 :,80 : : 60            ,80
      60                   60
  ---------           ---------
,40)48,00(120 Ans.   ,40)48,00(120 Ans.
    40                   40
    ---                  ---
     80                   80
     80                   80
     ---                  ---
      0                    0
```

DEM.—It is plain, when we repeat the price of 1 bushel of wheat, by the number of bushels, that the product is the price of the whole quantity, then, when we divide the whole cost of the wheat, by the price of 1 bushel of corn, the quotient must be the quantity of corn; because as often as the sum of 40 cents is contained in the whole money, so often a bushel of corn can be had in exchange for the wheat.

2. What quantity of butter, at 10 cents a pound, must a lady give in barter for 8 yards of satin, at 1 dollar 25 cents per yard? Ans. 100 pounds.

When the quantities of both commodities are given, and the difference of their values is to be paid in money; find the value of both, and the difference of their values will be the answer.

When one commodity is rated above cash price, to find the bartering price of the other, say, as the cash price of the one, is to its bartering price, so is the cash price of the other, to its bartering price.

3. A, lets B, have 50 bushels of rye, at 45 cents a bushel, for which he receives 60 bushels of oats, at 25 cents a bushel, and the balance in money, how much money must A receive? Ans. $7,50 cents.

4. A, has calico worth 18cts. a yd., ready money, but in barter he will have 25 cents; B, has broadcloth worth $3,75cts. per yard, ready money; at what price ought the broadcloth to be rated at in barter? Ans. $5,20 cents, 8 mills.

QUESTIONS ON BARTER.

What is barter? A. It is strictly the Rule of Three Direct applied to the exchange of one commodity for another, so that neither party shall sustain loss. When you have the price of one quantity or commodity, how do you find the quantity of the other commodity, which may be received for the money? A. Divide the money by the price of a unit of the required quantity. When the price and quantity of both commodities are given, how do you find the balance which is to be paid in money? A. Subtract what one commodity comes to, from the amount of the other. When the ready money price of one commodity, is less than its bartering price, how do you find the bartering price of the other commodity? A. As the ready money price of the one, is to its bartering price, so is the ready money price of the other, to *its* bartering price.

LOSS AND GAIN,

Is only a particular application of the Rule of Three Direct, by which merchants and traders discover their profits, or loss per cent, or by the gross: It also teaches them to raise or fall on the price of their goods, so as to gain or lose so much per cent, &c.

Case I.—*To know what is gained or lost per cent.*

RULE.—First see what the gain or loss is, by subtraction; then, as the price it cost, is to the gain or loss, so is $100, or £100, to the gain or loss per cent.

EXAMPLES.

1. If a merchant buy calico, at 24 cents a yard, and sell it, at 36cts. per yard; what does he gain per cent, or what does he gain in laying out $100? Ans. $50, or 50 per cent.

cts.
,36
,24
Gain ,12 per yd.

cts. *cts.* $
24 ; ,12 :: 100,00
12 $
,24) 1200,00($50,00
120
000

Dem.—it is evident, that the merchant on the sale of one yard gained half what it cost him, consequently he must have gained $50, in laying out $100; because the gain on 100 dollars must be in the same proportion as the gain on twenty-four cents.

2. Bought butter at 9cts. a pound; and sold it at 12cts. a pound; what was gained per cent? Ans. 33$\frac{1}{3}$ per cent.

3. Bought 30 yards of broadcloth at $4,50 cents per yard, and sold it for 110 dollars; what was gained or lost per cent? Ans. $18,51cts. 8m. loss per cent.

Case II.—*To know how a commodity must be sold, to gain or lose so much per cent.*

RULE.—As $100, or £100, is to the price it cost, so is $100 or £100, with the profit added, or the loss subtracted, to the gaining or losing price.

EXAMPLES.

1. If a merchant buy sugar, at 9cts. a pound, how must he sell it per pound, to gain 25 per cent? Ans. 11cts. 2m.

$ *cts.* $
As 100 : ,09 : : 125
,09
1|00),11|25

DEM.—As the third term is increased by the per cent above the first, so the fourth term or answer, must increase above the second term. To gain 25 per cent, is adding one fourth the given sum or cost to itself.

2. If a man buy corn, at 40 cents per bushel, how must he sell it, to gain 30 per cent? Ans. 52 cents per bushel.

CASE III.—*When there is gain or loss per cent, to know what the commodity cost.*

RULE.—As $100, or £100, with the gain per cent added, or the loss per ct. subtracted, is to the price, so is 100 dollars, or 100 pounds, to the first cost.

EXAMPLES.

1. If a yard of cloth be sold at $4,50cts., and there is gained $25,50cts. per cent, what did the yard cost? Ans. $3,58 cents 5 mills.

$25,50
100 $ *cts.* $
125,50 : 4,50 : : 100,00 : Ans. $3,58cts. 5m.

2. By selling wheat, at 75 cents per bushel, I lose at the rate of 30 per cent, what was the first cost of the wheat per bushel? Ans. $1,07 cents.

CASE IV.—*If by wares sold, at a given rate, there is so much gained or lost per cent, to know what would be gained or lost per cent, if sold at another rate.*

RULE.—As the first price is to $100 or £100, with the profit per cent added, or the loss per cent subtracted, so is the other price, to the gain or loss per cent, at the other price.

NOTE.—If the answer exceed 100, the excess is gain per cent, but if it be less than 100, the deficiency is loss per cent.

EXAMPLES.

1. If brandy, sold at $1,12 cents per gallon, be 20 per cent profit, what gain or loss per cent shall I have, if I sell the same at 95cts. per gallon? Ans. Gain $1,78,5m. pr. ct.

$1,12 : 120 : : 95 : 101,78,5

2. If I sell cloth at 60cts. per yard, and thereby gain 25 per cent, what shall I gain per cent, if I sell it at 70cts. per yard? Ans. I shall gain $45,83cts. 3m. per cent.

3. If I sell wheat at $1,25 per bushel, and thereby gain 10 per cent, what shall I gain or lose per cent, if I sell it at 90cts. per bushel? Ans. Lose $20,80cts. per cent.

$1,25 : 110 :: 90 : 79,20

QUESTIONS ON LOSS AND GAIN.

What is Loss and Gain? A. It is a particular application of the Rule of Three, by which merchants are able to discover their profit or loss per cent; it also teaches them to raise or fall on their goods so as to gain or lose so much per cent. How do you find what is gained or lost per cent? A. First find what is gained or lost on a yard or pound by subtraction; then say, as the price it cost is to the given gain or loss, so is 100 dollars to the gain or loss on 100 dollars; because it is plain, that the gain or loss on 100 dollars is in proportion to the gain or loss on the price of one yard or one pound. How do you find how a commodity must be sold to gain or lose a certain per cent? A. As 100 dollars is to the price it cost, so is 100 dollars, with the profit added, or the loss subtracted, to the gaining or losing price; for as the third is increased by the gain, or lessened by the loss subtracted, so the fourth term or answer will become increased, or diminished, in proportion to the second term.

DISCOUNT,

Is an allowance made for the payment of money before it becomes due, or upon advancing ready money on notes, obligations, &c. which are payable at some future period. What remains after deducting the discount, is called the present worth, or such a sum as, if put to interest, would, in the given time, and at the given rate per cent, amount to the sum or debt then due. This rule is only an application of the Rule of Three Direct, as may be seen from the first example.

RULE.—As the amount of 100 dollars, or 100 pounds, for the given rate and time, is to 100 dollars, or 100 pounds, so is the given sum or debt to the present worth. Subtract the present worth from the given sum, and the remainder will be the discount.

PROOF.—Find the amount of the present worth, for the time and rate on the given sum, which must equal the given sum or debt.

EXAMPLES.

1. What is the present worth, and what is the discount of 100 dollars, payable in one year, at 7 per cent a year?

Ans. { $93,45cts. 7m. present worth.
$6,54cts. 3m. discount.

$ $ $
107 : 100 :: 100
100
107)10000(93,45cts. 7m. Present worth.
963
370
321
490
428
620
535
850
749
Rem. 101

$100,
93,45,7
$6,54;3 Discount.

Proof.
93,45,7
7
6,54199
93,457
99,99,899

Dem.—It is plain that 100 dollars, the second term, is the present worth of 107 dollars, due a year hence, because 100 dollars put to interest at 7 per cent, in one year, amounts to $107. And it is evident that the fourth term, or answer, bears the same proportion to 100 dollars, the third term, that 100 dollars, the second term, bears to 107 dollars, the first term. The truth of this rule is also evident from the nature of Simple Interest; for as the debt may be considered as the amount of some principal, (called the present worth,) at a certain rate per cent, and for the given time, that amount must be in the same proportion to its principal or present worth, as the amount of any other sum, to its principal or present worth.

Note.—In discount, money is supposed to bear no interest till after it becomes due; and that a discount should be made for the payment of such obligations, before they become due, is very reasonable, because the debtor by retaining the money till it becomes due, may put it to interest for the time; but by paying it before it becomes due, he gives that benefit to another. Some have very erroneously supposed, that the interest on the given sum for the given rate and time, was the discount, and this interest taken from the principal, gave the present worth; but our example proves it to be untrue, because, according to that, the discount on 100 dollars would be 7 dollars, and the present worth of 100 dollars, due a year hence, at 7 per cent, would be $93; but 93 dollars put to interest for one year, at 7 per cent, will not amount to 100 dollars, consequently they labour under a mistake who suppose the interest to be the discount.

2. What is the present worth of $500 due 2 years hence, at 7 per cent per annum? Ans. $436,59cts. 6m.

3. What is the discount on 1000 dollars, due four years hence, at 7 per cent? Ans. $218,75cts.

4. What is the present worth of 1500 dollars, due 90 days hence, at 7 per cent? Ans. $1474,55cts.

Note.—When a debt is to be discharged by several payments, to be made at different times; find the present worth of each payment by itself, and the sum of these will be the present worth of the whole.

5. What is the present worth of 800 dollars, the one half payable in one year, and the other half in two years, at 7 per cent? Ans. $724,70cts. 8m.

QUESTIONS ON DISCOUNT.

What is discount? A. It is an allowance made for the payment of money before it becomes due. On what rule is discount depending? A. On the Rule of Three Direct. What rule do you observe in stating the sums in Discount? A. The sums are stated in the Rule of Three Direct; by saying, as the amount of 100 dollars, for the given rate and time, is to 100 dollars, so is the given sum to the present worth; and the present worth subtracted from the given sum, gives the discount. What is the present worth? A. It is such a sum as, if put to interest, for the given time and rate, will amount to the given sum. Is the discount on a given sum less than the interest on the given sum for the given time and rate? A. It is; because taking the interest from the given sum, for the given time, and at the given rate, the remainder put at interest for the given time and rate, would not amount to the given sum.

TARE AND TRET.

Tare and *Tret* are allowances made to the buyer, on the weight of some particular commodities.

Tare is an allowance made for the weight of the barrel, box, bag, or whatever contains the articles or goods.

Tret is an allowance of 4lb. on every 104lb. for waste, dust, &c.

Cloff is an allowance, on some commodities, of 2lb. on every 3cwt. to turn the scale, or to make the weight hold out, when goods are reweighed, and is claimed chiefly, or only, by the merchants of London.

Suttle is what remains after a part of the allowance is deducted from the *gross* weight.

Neat weight is what remains after all allowances are made.

All the questions in this rule may be worked by the Rule of Three Direct, and, like the Rule of Three, reduced back to Multiplication and Division; but it is frequently more convenient to work by taking aliquot parts, as in Practice.

CASE I.—*When the tare is given on the whole gross weight.*

RULE.—Subtract the tare from the gross, and the remainder will be the neat weight.

EXAMPLES.

1. What is the neat weight of 112cwt. 3qrs. 12lb. of tobacco, tare on the whole, 6cwt. 3qrs. 20lb.? Ans. 105cwt. 3qrs. 20lb.

	cwt.	*qrs.*	*lb.*	
	112	3	12	By Compound Subtraction.
	6	3	20	
Ans.	105	3	20	neat weight.

2. What is the neat weight of 85cwt. 2qrs. 18lb. gross; tare 2cwt. 3qrs. 12lb.? Ans. 82cwt. 3qrs. 6lb.

3. What is the neat weight of 4 hogsheads of sugar, the gross weight and tare, as follows? Ans. 39cwt. 0qr. 16lb.

	cwt.	qrs.	lb.		lb.
1st *hhd.*	9	3	18	Tare	104
2d	10	1	12		98
3d	11	3	9		90
4th	10	2	11		106
Gross weight	42	2	22	Whole tare 398lb.	
398lb.=	3	2	6		
Ans.	39	0	16	neat weight.	

4. What is the neat weight of 3 casks of indigo, each weighing 4cwt. 2qrs. 14lb. gross; tare on each cask 1cwt. 0qr. 12lb.? Ans. 10cwt. 2qrs. 6lb.

Case II.—*When the tare is so much per hundred weight.*

RULE.—Find, by Practice, the tare, by taking aliquot parts of a cwt.; then deduct the tare from the gross, the remainder will be the neat weight. Or, by the Rule of Three; as 112 is to the tare per cwt., so is the given weight to the tare. Then deduct the tare from the gross weight, and the remainder will be the neat weight.

EXAMPLES.

1. What is the neat weight of 20 hogsheads of tobacco, weighing in all 246cwt. 3qrs. 7lb. gross; tare 16 lb. per cwt.? Ans. 211cwt. 2qrs. 6lb.

lb.	*cwt.*	*qrs.*	*lb.*	
$16=\frac{1}{7}$)	246	3	7	
	35	1	1	tare deducted.
Ans.	211	2	6	neat weight.

By the Rule of Three.

lb.	*lb.*	*cwt.*	*qr.*	*lb.*
112 :	16 ::	246	3	7
		4		
		987		
		28		
		7903		
		1974		
		27643		
	Carried up.	16		

```
   165858              Brought up.
    27643     28)   4)
112)442288 ( 3949 ( 141
    336       28        35cwt. 1qr. 1lb. Tare.
    1062     114
    1008     112              cwt.  qr.  lb.
     548      29              246    3    7
     448      28               35    1    1
     1008      1lb.    Ans.   211    2    6  neat weight.
     1008
```

DEM.—It is plain, that the tare, in the fourth term, bears the same proportion to the gross weight, in the 3d term, as the tare, in the 2d term, bears to the gross weight, in the first term; and this tare subtracted must leave the neat weight.

2. What is the neat weight of 123cwt. 1qr. 20lb. gross; tare 14lb. per cwt.? Ans. 108cwt.

NOTE.—*If the tare be 14lb per cwt.*, deduct $\frac{1}{8}$ of the gross weight for the tare; the remainder will be the neat weight, 14lb. being $\frac{1}{8}$ of a cwt. or 112lb., which is shown thus, $14)\frac{14}{112}=\frac{1}{8}$ reduced to its lowest terms. *If the tare be 16lb. per cwt.*, deduct $\frac{1}{7}$ of the gross weight for the tare; the remainder will be the neat weight, 16lb. being $\frac{1}{7}$ of a cwt. or 112lb. which is shown, thus, $16)\frac{16}{112}=\frac{1}{7}$. *If the tare be any other quantity than 14lb. or 16lb. per cwt.*, first obtain the tare for 14lb., or 16lb. per cwt. and then take aliquot parts of the quotient.

3. What is the neat weight of 18 hogsheads of sugar, weighing 167cwt. 2qrs. gross, tare 20lb. per cwt.?

```
  lb.    cwt.  qrs.  lb.
  16=1/7)167    2     0
   4=1/4) 23    3    20
  20lb.    5    3    26
          29    3    18 Tare.
 Ans.    137    2    10 Neat weight.
```

4. In 115cwt. 1qr. 20lb. gross, tare 21lb. per cwt.; what is the neat weight? Ans. 93cwt. 3qrs. 4lb.

5. In 115cwt. 1qr. 20lb. gross, tare 7lb. per cwt., what is the neat weight? Ans. 108cwt. 0qr. 24lb.

CASE III.—*When Tret is allowed with Tare.*

RULE.—Find the tare as in the preceding case, which subtract from the gross, and call the remainder suttle. Divide the suttle by 26, and the quotient will be the tret, which subtract from the suttle, and the remainder will be the neat weight.

EXAMPLES.

1. What is the neat weight of 32cwt. 3qrs. 20lb. gross, tare 14lb. per cwt., tret 4lb. per 104?

	cwt.	*qrs.*	*lb.*	
$14=\frac{1}{8}$)	32	3	20	
	4	0	13	tare.
$4=\frac{1}{26}$)	28	3	7	suttle.
	1	0	12	tret.
Ans.	27	2	23	neat.

Statements in the Rule of Three.

1st. For the tare.

lb. *lb.* *cwt.* *qrs.* *lb.*
112 : 14 : : 32 3 20 :
Ans. 4cwt. 0qr. 13lb. tare.

2d. For the tret.

lb. *lb.* *cwt.* *qrs.* *lb.*
104 : 4 : : 28 3 7 :
Ans. 1cwt. 0qr. 12lb. tret.

When the remainder is less than $\frac{1}{4}$lb. it is not reckoned.

The reason of dividing by 26, is to shorten the work; 4lb. being $\frac{1}{26}$ part of 104, $4)\frac{4}{104}=\frac{1}{26}$.

2. What is the neat weight of 495cwt. 1qr. 2lb. gross, tare 28lb. per cwt., and tret 4lb. for every 104lb.?
Ans. 357cwt. 0qr. 18$\frac{1}{2}$lb.

3. What is the neat weight of 4hhds. of sugar, weighing 33cwt. 1qr. 12lb. gross, tare 14lb. per cwt., tret 4lb. per 104, and cloff 2lb. per 3cwt.? Ans. 27cwt. 3qr. 16lb. 10oz.

NOTE.—When cloff is allowed, deduct the tare and tret as usual, then divide what remains by 168, and subtract the quotient, the remainder will be the neat weight, 2lb., the cloff, being $\frac{1}{168}$ part of 3cwt. $2)\frac{2}{336}=\frac{1}{168}$.

QUESTIONS ON TARE AND TRET.

What is Tare and Tret? A. An allowance made to the buyer, on the weight of goods. What is gross weight? A. It is the weight of the goods together with the box, bag, or whatever contains them. What is tare? A. An allowance made to the buyer for the weight of the box, cask, bag, or whatever contains the goods. What is tret? A. An allowance of 4 pounds on every 104 pounds for waste, dust, &c. What is cloff? A. An allowance of 2 pounds upon every 3cwt. What is suttle? A. It is what remains after one or two allowances have been deducted. When the tare, on the whole quantity, is given; how do you find the neat weight? A. By subtracting the tare from the gross weight. When the tare is so much per cwt., how do you find the neat weight? A. By taking aliquot parts of a cwt. for the tare, or find the tare by the Rule of Three, and deduct the tare from the gross weight. When tare and tret are allowed, how do you find the neat weight? A. First deduct the tare as usual, then divide the suttle by 26, for the tret, which subtract, the remainder will be the neat weight. When tare, tret, and cloff are allowed; how do you find the neat weight? A. First, find and deduct the tare and tret as usual; then divide what remains by 168, and the quotient will be the cloff, which subtract; and the remainder will be the neat weight. What is the use of finding the neat weight? A. The use is, that the buyer only pays for the neat weight, at the price agreed on per cwt. Does the buyer pay for the freight of his goods, by the gross or neat weight? A. Gross weight is reckoned, except where otherwise provided for by statute.

ANNUITIES.

An *Annuity* is a sum of money payable at regular periods, generally every year, for a certain time or for ever. The annuity sometimes depends on some contingency, as the life or death of a person and it is then said to be *contingent*. When the annuity does not commence until a certain number of years has elapsed, it is then said to be in *reversion*. The annuity is said to be in *arrear*, when the debtor keeps it beyond the time of payment. The *present worth* of an annuity is such a sum as being now put out to interest, would exactly pay the annuity when it becomes due, and such a sum as must be given for the annuity, if it be paid at the commencement. The *amount* is the sum of the annuities for the time it has been forborne, with the interest due on each payment or annuity.

The rules in annuity are only particular applications of the Rule of Three.

Case I.—*To find the amount of an annuity at Simple Interest.*

RULE.—First, find the interest of the given annuity for one year; and then for 2, 3, and so on, up to the given number of years, less 1. Then multiply the annuity by the given number of years, and add the product to the whole interest, and the sum will be the amount sought.

EXAMPLES.

1. What is the amount of an annuity of $100 for four years, computing interest at 7 per cent?

The interest of $100, at 7 per cent, for 1 year, is------$7.
for 2 years,---------14.
for 3 years,---------21.
Four years' annuity, at $100 per year is $100×4=400.
Ans. 442.

Dem.—It is plain, as 100 dollars is forborne 4 years, which is due at the end of each year, that the 100 dollars must be multiplied by 4 for the annuity, without the interest, and the 100 dollars, which becomes due at the end of the first year, runs on interest three years, consequently the interest on that must be 21 dollars, and the 100 dollars which becomes due at the end of the second year, runs on interest two years, which gives 14 dollars interest, and the $100, which becomes due at the end of the third year, runs on interest one year, which gives 7 dollars interest, and the 100 dollars which becomes due at the end of the fourth year, does not run on interest, because it is paid or reckoned when it becomes due, consequently we must always reckon interest for one year less than the given time; and it is plain, if these several annuities be added to the interest due on them, the sum must be the amount of the annuity.

2. What is the amount of an annuity of $400, forborne 5 years, simple interest, computed at 7 per ct.? Ans. $2280.

3. A man let a house upon a lease of 6 years, at $300 per annum, and the rent being in arrear for the whole term; what sum must he demand at the end of the term, simple interest being allowed at 7 per cent? Ans. $2115.

NOTE.—Pensions in arrears are reckoned the same as in the preceding examples.

CASE II.—*To find the present worth of an annuity at Simple Interest.*

RULE.—First, find the present worth of each year by itself, discounting from the time it becomes due; then, the sum of all these will be the present worth.

NOTE.—This rule depends on the principles of discount. The annuity for each year, may be considered as a debt due 1, 2, 3, 4, or 5 years hence, and so on, of which the present worth is to be found; hence the sum of all these will be the present worth.

EXAMPLES.

1. What is the present worth of 100 dollars per annum, to continue 4 years, at 7 per cent?

$	$	$	$ *cts. m.*	
107	: 100 :	: 100 :	93,45,7	The present worth for 1 year.
114			87,71,9	2 years.
121			82,64,4	3 years.
128			78,12,5	4 years.

Ans. $341,94,5 present worth required.

2. What is the present worth of an annuity of 400 dollars, continued five years, at 6 per cent? Ans. $1703,75cts. 5m.

QUESTIONS ON ANNUITIES.

What is an annuity? A. A sum of money payable at regular periods, for a certain time, or for ever. What are annuities said to be, when they depend on some contingency, as the life or death of a person? A. The annuity is then said to be *contingent.* How do you distinguish annuities, when they do not commence till some future period? A. They are then said to be in *reversion.* When is an annuity said to be in *arrears?* A. When the debtor keeps it beyond the time of payment. What is the *present worth* of an annuity? A. It is such a sum as being now put out at interest, would exactly pay the annuity as it becomes due. What is the amount of an annuity? A. The sum of the annuities for the time, with the interest due on each. How do you find the amount of an annuity at simple interest? A. First find the interest of the given annuity for one year, then for two years, three years, and so on, up to the given number of years, less one; then multiply the annuity by the given number of years, and add the product to the whole interest, and the sum will be the amount of the annuity. How do you find the present worth of an annuity at simple interest? A. First find the present worth of each yearly payment by itself, discounting from the time it becomes due; then the sum of all these will be the present worth.

ALLIGATION,

Teaches how to compound or mix together several simples of different qualities or prices, so that the composition may be of some intermediate quality or price. It is commonly distinguished into two kinds, *Alligation Medial*, and *Alligation Alternate.*

ALLIGATION MEDIAL,

Teaches to find the price or quality of the composition, from having the quantities and prices or qualities of the several simples given.

Case I.—*To find the mean price or quality of any part of the composition, when the several quantities and their prices or qualities are given.*

RULE.—First, multiply the quantity of each ingredient by its price or quality; then add all the products together, and add also all the quantities together, into another sum; then say—as the whole composition is to the sum of the products, so is any part of the composition to its mean value or quality.

NOTE.—Alligation means to mingle, tie, or mix together, two or more simples; and *Medial* means the middle or mean rate between the extremes.

EXAMPLES.

1. A merchant mixes 20 gallons of brandy at 10 shillings per gallon, with 36 gallons of rum at 6 shillings per gallon, and 40 gallons of gin at 4 shillings per gallon; what is a gallon of the mixture worth? Ans. 6s.

gal. *s.* *s.* *gal.* *s.* *gal.*

20×10=200 as 96 : 576 :: 1

36× 6=216 1

40× 4=160 96)576(6s. Ans.

96 gal. 576s. 576

0

gal. *s.* *gal.*

Proof as 1 : 6 :: 96

6

1)576

576s.

Dem.—It is evident, as the whole quantity of liquor is to the money which it cost, so one gallon is to the cost of one gallon.

Note. The student will now perceive, that Alligation is nothing more than the Rule of Three applied to mixing different ingredients.

2. A grocer mixes 60lb. of sugar at 8d. per pound with 20lb. worth 12d. per pound; what is the value of 1 pound of the mixture? Ans. 9d.

3. A farmer mixes ten bushels of wheat at 5s. a bushel, with 18 bushels of rye at 3 shillings a bushel, and 20 bushels of barley at 2 shillings per bushel; how much is a bushel of the mixture worth? Ans. 3s.

4. A refiner melted together 8 ounces of gold, of 22 carats fine, 10 ounces of 20 carats fine, 12 ounces of 16 carats fine, 8 ounces of 18 carats fine; will you make out the fineness of the composition? Ans. $18\frac{14}{19}$ carats fine.

5. Of what fineness is that composition, which is made by mixing 4lb. of silver of 8oz. fine, with 2lb. 4oz. of 9oz. fine, and 8oz. of alloy? Ans. $7\frac{4}{7}$oz. fine.

Note.—An ounce of pure gold being reduced into 24 equal parts, these parts are called carats; but when gold is mixed with baser metal, the mixture is said to be so many carats fine; thus, if 22 carats of pure gold be mixed with 2 of alloy, it is said to be 22 carats fine; and if 20 carats of pure gold be mixed with 4 of alloy, it is said to be 20 carats fine. A pound of pure silver, losing nothing in trial, is said to be 12oz. fine; but if it lose 1oz. by the fire, or be mixed with 1oz. of alloy, it is said to be 11oz. fine, &c.

ALLIGATION ALTERNATE,

Teaches to find what quantity of any number of simples, whose rates are given, will compose a mixture of a given rate. So that it is the reverse of Alligation Medial, and may be proved by it. ☞ This is called Alligation Alternate, because the same question frequently admits of different answers.

Case I.—*When the prices of the several simples are given,* *to find how much of each, at their respective rates, must be taken, to make a compound or mixture, at any proposed price.*

RULE.—1. Place the rates of the simples in a column under each other, with the mean price at the left hand. 2. Connect each rate which is less than the mean rate, with one or more that is greater. 3. Take the difference between each rate and the mean price, and place it directly opposite to that rate with which it is connected. 4. If only one difference stand against either rate, it will be the quantity required at that rate; but if there be two or more, their sum will be the quantity.

NOTE.—When all the given prices are greater, or less, than the mean rate, they must be linked to a cipher.

EXAMPLES.

1. A grocer has several sorts of wine, some at 60 cents per gallon, some at 50, some at 70, and some at 65; how much of each sort must he mix, that he may sell the mixture at 62 cents per gallon?

$$\begin{array}{lll} & \textit{cts.} & \textit{gal.} \\ \begin{array}{l}\text{Mean}\\ \text{rate,}\\ \text{62cts.}\end{array} & \left\{\begin{array}{l}60\\ 50\\ 70\\ 65\end{array}\right. \quad \text{Ans.} & \left\{\begin{array}{l}3 \text{ at } 60 \text{ cts.}\\ 8 \text{ at } 50 \text{ cts.}\\ 12 \text{ at } 70 \text{ cts.}\\ 2 \text{ at } 65 \text{ cts.}\end{array}\right. \end{array}$$

Or,

$$\begin{array}{llll} & \textit{cts.} & \textit{gal.} & \textit{gal.} \\ \text{62 cts.} & \left\{\begin{array}{l}60\\ 50\\ 70\\ 65\end{array}\right. & \begin{array}{l}3+\\ 8\\ 2+\\ 2\end{array} & \left.\begin{array}{r}8=11\\ =8\\ 12=14\\ =2\end{array}\right\} \text{Ans.} \end{array}$$

Or,

$$\begin{array}{llrr} & \textit{cts.} & \textit{gal.} & \$\ \textit{cts.} \\ \text{62 cts.} & \left\{\begin{array}{l}60\\ 50\\ 70\\ 65\end{array}\right. \quad \text{Ans.} & \begin{array}{r}8\times60\\ 3\times50\\ 2\times70\\ \underline{12\times65}\\ 25\end{array} & \begin{array}{r}=4,80\\ =1,50\\ =1,40\\ =7,80\\ \end{array} \end{array}$$

$$\begin{array}{r} 25 \\ \underline{62} \\ 50 \\ \underline{150} \\ \$15,50 \end{array} \qquad \begin{array}{r} 25)15,50(62 \\ \underline{150} \\ 50 \\ \underline{50} \\ 0 \end{array} \ \text{Proof.}$$

A sum equal to the amount of all the simples at the given prices.

DEM.—It is plain, by connecting a rate which is less than the mean rate, with one that is greater than the mean rate, and setting down the difference between them and the mean rate alternately, or one after the other in turn, that the quantities resulting are such, that there is precisely as much gained by one quantity as is lost by the other, and consequently the gain and loss, upon the whole, are equal; and the whole quantity, at the mean rate, amounts to a sum equal to the price of all the simples, at the rates given. No matter whether the number of simples be great or small, or with how many a simple is yoked, since one, that is less than the mean rate, is always linked with one that is greater, there will be an equal balance of loss and gain between every two; and consequently an equal balance on the whole. It is also plain that it will admit of different answers; because having one answer, we may find as many more as we please, by multiplying or dividing each of the quantities found, by 2, 3, 4, &c. The reason of which is plain; for if two quantities of two simples make a balance of loss or gain with respect to the mean price, so must double or treble, or the half, or the third; and so on, to any degree whatever. This demonstration perhaps will appear more plain after noticing the following example, where only two simple quantities are given:

2. A grocer has brandy worth 10s. per gallon, and rum worth 6s.; how much of each must he mix, that he may sell the mixture at 8s. per gallon?

Ans. 2 gallons of each, or 1qt. 1pt. &c. of each.

```
       s.   gal.  Proof.
8s. { 10)  2×10=20
       6)  2× 6=12
           --   -----
           4......4)32(8s.
                    32
                    --
                     0
```

Dem.—From this example it is evident, that the loss on the one is exactly balanced by the gain on the other; for on the sale of the 2gal. of brandy there is a loss of 4s., but it is balanced by the gain on the rum, which is 4s.; and it is also plain, that 1 gallon of each, or 2 quarts of each, would bear the same proportion.

3. A goldsmith has gold of 20 carats fine, of 18, of 22, and of 16; how much must he take of each to make the mixture 19 carats fine? Ans. 3oz. or lb. of 20 carats fine, 3 of 18 carats fine, 1 of 22 carats fine, and 1 of 16 carats fine; admitting of other answers, *as may be seen by referring to example 1, on page* 195.

Case II.—*When one of the ingredients is limited to a certain quantity, to find the several quantities of the rest, in proportion to the given quantity.*

RULE.—Take the difference between each price and the mean rate, and set them down alternately, as in Case I. Then, as the difference standing against that simple whose quantity is given, is to that quantity, so is each of the other differences, severally, to the several quantities required.

EXAMPLES.

1. A grocer wishes to mix teas at 11s., 9s., and 7s. per pound, with 20 pounds at 5s. per pound; how much of each sort must he take to make the composition worth 8s. per pound?

Ans. { 20 lb. at 5s. $6\frac{2}{3}$ lb. at 9s.
20 lb. at 11s. $6\frac{2}{3}$ lb. at 7s.

$$\begin{array}{ll} & \begin{array}{lc} s. & lb. \end{array} \\ 8s. & \left\{\begin{array}{lc} 11 & 3 \\ 9 & 1 \\ 7 & 1 \\ 5 & 3 \end{array}\right. \end{array}$$

NOTE.—This Case may be proved by the one preceding.

3 *Stands against the given quantity.*

$$\begin{array}{ccccc} lb. & & lb. & & \begin{array}{lcr} lb. & & lb. \end{array} \\ \text{As } 3 & : & 20 & :: & \left\{\begin{array}{lcr} 3 : & & 20 \\ 1 : & \text{Ans.} & 6\frac{2}{3} \\ 1 : & & 6\frac{2}{3} \end{array}\right. \end{array}$$

DEM.—It is evident that the quantities obtained by linking the several rates together, would form a compound equal in value to the mean rate; but since one of the quantities is given, the others must be increased or diminished in proportion. Then it is plain, that the quantity which stands against the rate of the given quantity, is to the given quantity as the quantity standing against any other rate, is to the quantity of that rate.

2. How much wine at 8s., 5s., and at 4s. per gallon must be mixed with 3 gallons at 7s. per gallon, so that the mixture may be worth 6 shillings per gallon?

Ans. 3gal. at 7s., 6 at 8s., 3 at 5s., and 6 at 4s.

CASE III.—*When the whole composition is limited to a certain quantity.*

RULE.—First find an answer as before, by linking; then say, as the sum of the quantities, or differences, thus found, is to the given quantity, so is each ingredient found by linking, to the required quantity of each.

NOTE.—The student will discover, by noticing the different cases in this rule, that they are only applications of the Rule of Three.

EXAMPLES.

1. A grocer has wine at 4s., at 5s., at 5s. 6d., and at 6s. a gallon, and he wishes to make a mixture of 72gal., so that it may be afforded at 5s. 4d. per gallon; how much of each sort must he take?

$$\begin{array}{lllccc} & d. & gal. & gal. & & \\ 64d. & \left\{\begin{array}{l} 48 \\ 60 \\ 66 \\ 72 \end{array}\right. & \begin{array}{r} 2 \\ 8+2 \\ 16+4 \\ 4 \end{array} & \begin{array}{r} =2 \\ =10 \\ =20 \\ =4 \\ \hline 36 \end{array} & \begin{array}{c} gal.\ \ gal. \\ \text{as } 36 : 72 :: \end{array} & \left\{\begin{array}{rl} gal. & gal.\ \text{Answer} \\ 2 : & 4 \text{ at } 4s. \\ 10 : & 20 \text{ at } 5s. \\ 20 : & 40 \text{ at } 5s.\ 6d. \\ 4 : & 8 \text{ at } 6s. \\ \hline 72 & \textit{Proof} \end{array}\right. \text{per gallon.} \end{array}$$

DEM.—It is plain, that the sum of the differences or quantities found by linking, bears the same proportion to the given quantity as the differences, severally, bear to a fourth proportional quantity. So it is evident that the quantities may be increased or diminished and sold at the same mean rate, while they bear the same proportion to each other.

2. How much gold of 15, 17, 18, and 22 carats fine, must be mixed together, to form a mixture of 80oz. of 20 carats fine?

Ans. { 10oz. of 15, of 17, and of 18 carats fine, and 50oz. of 22 carats fine.

3. How many gallons of brandy, at $1,20cts. per gallon, must be mixed with water of no value, so as to fill a cask of 110 gallons, which may be afforded at 80cts. per gallon?

cts. $ *cts.*
80 | 1,20 80
| 0 40
120 gal.

gal. *gal.* *gal.* *gal.*
120 : 110 :: { 80 : Ans. 73⅓ of brandy.
40 : 36⅔ of water.

4. A vintner has 3 kinds of wine; one kind at 24d. per gallon, one at 22d., and one at 18d.; how must he fill a cask of 60 gallons with his three kinds of wine, so that he may sell it at 20d. per gallon?

Ans. { 12gal. at 24d. per gal., 12 at 22d. and 36 at 18d.

QUESTIONS ON ALLIGATION.

What is Alligation? A. It teaches how to mix several simples of different qualities, so that the composition may be of some intermediate quality or rate. How many kinds of Alligation are there? A. Two, Alligation Medial, and Alligation Alternate. Why is alligation medial so called? A. Because it teaches to find a mean or middle rate or quality. Why is alligation alternate so called? A. Because it generally admits of different answers, when the simples are differently linked together. What rule do you observe in linking the simple rates or qualities together? A. Always unite one that is less than the mean rate with one that is greater than the mean rate, or quality. Why should the numbers which stand opposite the simple rates or qualities form a compound equal to the mean rate? A. Because the gain and loss exactly balance each other, there being as much loss on that number which exceeds the mean rate, as there is gain on that which falls short of the mean rate. On what rule does alligation depend? A. On the Rule of Three, from which all its principles are drawn.

THE DOUBLE RULE OF THREE.

The Double Rule of Three, or, as it is sometimes called, Compound Proportion, teaches to resolve by one statement such questions as require two or more statements by the *Single Rule of Three*, and hence the rule has derived its name. In this rule there are always an odd number of terms given, as five, seven, nine, &c. In the questions, generally, under this rule, there are five terms given to find a sixth; the first three terms are a supposition, the last two a demand.

RULE.—In stating questions, place the terms of supposition so that the principal cause of gain, loss, or action, possess the first place; that which expresses the space of time, or the distance of place, the second place; and that which expresses the gain, loss, or action, the third place. Then, place the terms of demand under those of the same kind in the supposition. If the blank place, or term sought, fall under the third term, the proportion is direct; then multiply the first and second terms together for a divisor, and the other three for a dividend, and the quotient will be the answer in the same denomination of the term directly above the blank; but if the blank fall under the first or second term, the proportion is inverse; then, multiply the third and fourth terms together for a divisor, and the other three for a dividend, and the quotient will be the answer.

NOTE.—Observe the same rule here, as in the Single Rule of Three, when the terms of the question are in different denominations; the reduction, however, ought to be performed before the question is stated.

EXAMPLES.

1. If 100 dollars in 12 months, gain 6 dollars; what will 600 dollars gain in 9 months?

```
          $    mo.  $
         100 : 12 :: 6   The terms of supposition or conditional terms.
         600 :  9 . . . . The demanding terms, or those which
   $           9           move the question.
  100        5400
   12           6 $
 12|00)324|00(27 Ans.
       24
        84
        84
         0
```

DEM.—It is plain, from the nature of this question, that this is only a contraction of the Single Rule of Three; because the product of the two terms which compose our divisor here, may be considered as a composite number, the component parts of which must be the two divisors, when the question is solved by two statements, in the single Rule of Three. And it is evident that it can make no difference whether we divide directly by a composite number, or by the component parts which compose that number, as the following work shows.

By two statements in the Single Rule of Three.

```
  $   $     $               mo.   $    mo.
 100 : 6 :: 600             12 : 36 :: 9
             6 $                        9
     100)3600(36              12)324
          300               Ans. $27 same as above.
          600 and no remainder.
```

Thus we see the question solved by the Single Rule of Three in plain and easy statements; for it is evident, if 100 dollars gain 6 dollars in one year, that 600 dollars must gain 36 dollars; and it is also obvious, that the gain must be proportional to the time; for as 12 months are to the gain for that time, so 9 months are to the gain for that time.

Perhaps this rule will appear more simple to the young student, by carrying it still further back, and illustrating it by Multiplication and Division, as follows:

$ $ *cts.*	
100)6,00(06	The gain on one dollar for a year.
600 600	
12)36,00	The gain on $600 for a year.
$3,	The gain on $600 for 1 month.
9	
Ans. $27,	The gain on $600 for 9 months.

The principles of this rule must appear plain to the student when he discovers that it is only an application of the Single Rule of Three, and from that may be reduced back to Multiplication and Division; because by reducing it back, all the difficulty vanishes.

2. If 8 persons expend 200 dollars in 9 months, how much will serve 18 persons 12 months? Ans. $600.

pers. *mos.* $
8 : 9 :: 200
18 : 12
200
8 3600
9 12 $
72)43200(600
432
00

NOTE.—Of the three conditional terms, or terms of supposition, the student will discover, that the eight persons must be the cause of expense, and consequently put for the first term; and 9 months, the time, and therefore put in the second place; and 200 dollars is what was expended in that time, and consequently it is put in the third place; the other terms regularly fall under those of the same name.

3. If 10 men spend 9 dollars in 12 weeks, how much will 20 men spend in 24 weeks? Ans. $36.

4. If $100, in 12 months, gain 7 dollars interest, what will $600 gain in 8 months? Ans. $28.

5. If 20 bushels of oats be sufficient for 18 horses 20 days, how many bushels will serve 60 horses 36 days? Ans. 120 bushels.

6. If $100, in one year, gain 7 dollars interest, what sum will gain $38 50cts. in 15 months? Ans. $440.

$ mo. $ cts.
100 : 12 :: 7,00
15 :: 38,50

Note.—In this question, the proportion is inverse; therefore, the third and fourth terms are multiplied together for a divisor, and the other terms for a dividend.

7. If $100 gain 7 dollars in one year, in what time will 440 dollars gain $38,50cts.? Ans. 15 months.

8. An usurer put out $650, to receive interest for the same; at the end of 6 months he received for principal and interest, $672,75cts.; at what rate per cent did he receive interest? Ans. 7 per cent.

9. If 100 men, in 6 days of 10 hours each, can dig a trench 200 yards long, 3 wide, and 2 deep; in how many days of 8 hours long, will 180 men dig a trench of 360 yards long, 4 wide, and 3 deep? Ans. 15 days.

QUESTIONS ON THE DOUBLE RULE OF THREE.

What does the Double Rule of Three teach? A. It teaches to resolve, by one statement, such questions as require two or more statements, when worked by the Single Rule of Three. What number of terms is generally given in the Double Rule of Three? A. Five.—What rule do you observe, in stating questions, in this rule? A. Write that term which is the principal cause of gain, loss, or action, for the first term; time or distance for the second; and, gain, loss, or action, for the third: and then write the two remaining terms under their corresponding terms, that is, terms of the same name. When the blank falls under the third term, how do you proceed? A. Multiply the first and second terms together for a divisor, and the other three for a dividend, and the quotient will be the answer. But when the blank falls under the first or second terms, multiply the third and 4th terms for a divisor, and the other three for a dividend, and the quotient will be the answer. What term must be of the same kind with the answer? A. The term directly above the blank.

EQUATION OF PAYMENTS,

Teaches how to reduce several stated times, at which money is payable, to one mean or equated time, for the payment of the whole.

RULE.—Multiply each payment by its time, and add the several products together; then divide the sum of the products by the whole debt, and the quotient will be the equated time, or answer.

Proof.—The interest of the whole sum to the equated time, at any given rate, will equal the interest of the several payments, for their respective times, at the same rate.

Note.—This rule is founded on the supposition that what is gained, by keeping some of the debts or payments after they are due, is lost by paying other debts or payments, before they are due.

EXAMPLES.

1. A, owes B $800, to be paid as follows, viz. $400 in

4 months, and $400 in 8 months; what is the equated time for the payment of the whole debt? Ans. 6 mo's.

$400×4=1600
400×8=3200 Ans.
Divisor, 800) 4800(6 *mo's.*
4800

DEM.—It is evident, if B wait on A, two months, after one half of his debt is due, that B should receive the other half, two months before it is due. And it is also evident, that the product of each payment, divided by the payment, gives the time it is due; then the sum of the products divided by the sum of the payments, must give the equated time.

2. B, owes C, $400, of which $200 are to be paid in two months, and $200 in four months; but they agree that the whole shall be paid at one time; at what time must it be paid? Ans. in 3 months.

3. A, owes B, $760, to be paid as follows; $200 in 6 months, 240 in 7 months, and 320 in 10 months; what is the equated time for the payment of the whole debt? Ans. 8 months.

4. A, holds B's note for $1200, which is to be paid in the following manner; $\frac{1}{2}$ in 6 months, $\frac{1}{3}$ in 8 months, and the remainder in 10 months; what is the equated time for the payment of the whole? Ans. 7$\frac{1}{3}$ months.

QUESTIONS ON EQUATION OF PAYMENTS.

What does Equation of Payments teach? A. It teaches to find a mean time for the payment of a debt, which is made payable by instalments. How do you proceed in the work? A. First, multiply each payment by its time, then divide the sum of the products, by the whole debt, and the quotient will be the answer. On what supposition is this rule founded? A. On the supposition that what is gained by putting off some payments after they are due, is lost by paying others, before they are due.

INVOLUTION,

Teaches how to find the *powers* of numbers. A *power* is a product or number produced by multiplying any given number, called the *root*, a certain number of times, continually by itself. Thus, 2 is the root or 1st power of 2; 4 is the 2d power, or square of 2, produced thus, 2×2=4; 8 is the 3d power, or cube of 2, produced thus, 2×2×2=8; 16 is the 4th power, or biquadrate of 2, produced thus, 2×2×2×2=16, and so on.

The power is often denoted by a figure placed at the right hand of the number, and a little above it, which figure is called the *index* or *exponent* of that power. This index or exponent is always one more than the number of multiplications, to produce the power; or it is equal to the number of times the given number is taken as a factor, in producing the power; thus, 3 is used twice to produce the square, or

2d power, $3\times3=3^2$, or 9; and the cube, or 3d power, $3\times3\times3=3^3$, or 27; and so on. Thus the student will perceive, in finding the square of 3, there is only one multiplication, or two factors; in finding the cube there are two multiplications, or three factors, and so on.

Involution is performed by the following

RULE.—Multiply the given number, or first power, continually by itself, till the number of multiplications be one less than the index, or exponent of the power to be found, and the last product will be the power required.

The powers of the nine digits, from the 1st power to the 5th, may be found in the following table

TABLE.

Roots, or 1st Powers.	1	2	3	4	5	6	7	8	9
Squares, or 2d Powers.	1	4	9	16	25	36	49	64	81
Cubes, or 3d Powers.	1	8	27	64	125	216	343	512	729
Biquadrates, or 4th Powers.	1	16	81	256	625	1296	2401	4096	6561
Sursolids, or 5th Powers.	1	32	243	1024	3125	7776	16807	32768	59049

EXAMPLES.

1. What is the 5th power of 4?

4 The 1st power or root.
4
16=The 2d power or square.
4
64=The 3d power or cube.
4
256=The 4th power or biquadrate.
4
1024=The 5th power or sursolid.
Answer.

2. What is the 4th power of 5?

5 The 1st power or root.
5
25=The 2d power or square
5
125=The 3d power or cube.
5
625=The 4th power or biquadrate.
Answer.

3. What is the square or 2d power of 8? Ans. 64.

4. What is the square of 40? Ans. 1600.

5. What is the square of 500? Ans. 250,000.

6. What is the cube or 3d power of 5? Ans. 125.

7. What is the cube of 60? Ans. 216,000.
8. What is the square of 1? Ans. 1.
9. What is the cube of 1? Ans. 1.

NOTE.—A decimal fraction is involved or raised to any power, the same as a whole number, and the same rules are observed in pointing off as in Multiplication of Decimals

A vulgar Fraction is raised to any power by multiplying the numerator of the fraction by itself, and the denominator by itself, till the number of multiplications be one less than the index, or exponent of the power to be found; then the power of the numerator, placed over the power of the denominator, gives the power of the fraction sought. If it be required to raise a mixed number to a certain power, first reduce it to an improper fraction, and then proceed as with a simple fraction. But those who desire it, may first reduce the given fraction to a decimal, and then raise the decimal to the power required.

10. What is the square or 2d power of ,5? Ans. ,25.
11. What is the cube, or 3d power of ,5? Ans. ,125.
12. What is the square of $\frac{2}{3}$? Ans. $\frac{4}{9}$.
13. What is the cube, or 3d power of $\frac{4}{7}$? Ans. $\frac{64}{343}$.
14. What is the square, or 2d power of $2\frac{1}{2}$? Ans. $\frac{25}{4}$.
15. What is the square of $4\frac{1}{4}$? Ans. $\frac{289}{16}=18\frac{1}{16}$.
16. How much is 9^3, that is, the 3d power of 9? Ans. 729.
17. How much is 6^5? Ans. 7776.
18. How much is 10^4? Ans. 10,000.

NOTE.—It will be seen from the preceding examples, that raising a simple fraction, whether vulgar or decimal, to a higher power, diminishes it in the same proportion as a whole number becomes increased.

EVOLUTION,

Is the extracting or finding the roots of any given powers; or it is exactly the reverse of Involution.

The root of any number or power is such a number, as being multiplied into itself a certain number of times, will produce that power. Thus, 2 is the square root, or 2d root of 4, because $2^2=2\times2=4$; and 3 is the cube root, or 3d root of 27, because $3^3=3\times3\times3=27$.

The power of any given number or root may be found exactly by multiplying the number continually into itself. But there are numbers, of which a proposed root can never be exactly found. Yet, by means of decimals, we may approximate or approach towards the root, to any degree of exactness. Those numbers whose roots only approximate towards the true root, are called *surd numbers;* but those whose roots can be exactly found, are called *rational numbers.*

The Roots are sometimes denoted by writing the character √, before the power, with the index of the root against it. Thus, the square root of 25 is expressed $\sqrt{25}$, and the cube root of 64 is expressed $\sqrt[3]{64}$; and the 5th root of 16807, $\sqrt[5]{16807}$. The index to the square root is always omitted; the character only, being placed before it; thus, $\sqrt{16}$, the index 2, being omitted.

When the power is expressed by several numbers, with the sign +, or —, between, a line is drawn from the top of the sign over all the parts of it; thus the square root of 41—5, is $\sqrt{41-5}$, or thus, $\sqrt{}(41-5,)$ enclosing the numbers in a parenthesis.

But all roots are now frequently distinguished by fractional indices; thus, the square root of 8, is $8^{\frac{1}{2}}$, the cube root of 64 is $64^{\frac{1}{3}}$, and the square root of 41—5, is $\overline{41-5}^{\frac{1}{2}}$ or $(41-5)^{\frac{1}{2}}$.

EXTRACTION OF THE SQUARE ROOT.

Extracting the square root of any given number, is finding a number which, multiplied by itself, would produce the given number; consequently multiplying the root into itself, is a proof of the work.

RULE.—Divide the given number into periods of two figures each, by setting a point over the place of units, another over the place of hundreds, and so on, over every second figure, both to the left hand in integers, and to the right hand in decimals.

Find the greatest square in the first period, on the left hand, and set its root, on the right hand of the given number, (after the manner of a quotient in division,) for the first figure of the root, and the square number, under the period, and subtract it therefrom, and to the remainder bring down the two figures of the next following period, for a dividend.

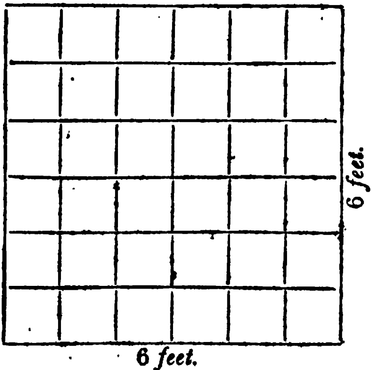
Place twice the root, already found, on the left hand of the dividend for a divisor. Seek how often the divisor is contained in the dividend, (exclusive of the right hand figure,) and place the figure in the root, for the second figure of it, and likewise, on the right hand of the divisor; multiply the divisor, with the last figure annexed, by the last placed in the root, and subtract the product from the dividend; to the remainder, join the next period for a new dividend.

Double the figures already found in the root, for a new divisor, (or bring down your last divisor for a new one, doubling the right hand figure of it,) and from these find the next figure in the root, as last directed, and continue the operation in the same manner, till you have brought down all the periods.

EXAMPLES.

1. What is the square root, or side of a square containing 36 square feet? Ans. 6 feet.

FIG. I.

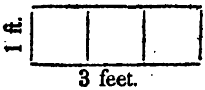


Operation.

36(6 Root.
36 6
36 Proof.

DEM.—The rule for the extraction of the square root, will appear obvious by attending to the process, by which any number is raised to the second power, or square. To find the second power of any number, we multiply the given number, or square root, by itself; therefore, to obtain the root from the power, we must inquire what number multiplied by itself, will produce the given number; we find on inquiring, that 6 multiplied by itself will produce the given number; therefore we place 6 in the quotient, for the root, and multiply it by itself, placing the product directly below the given number; we find that it equals the given number, consequently, 6 feet is the square root or side of a square, containing 36 square feet; which must appear plain to the student on inspecting Fig. I., because he will readily discover, that the figure contains 36 square feet, which is the 2d power of 6 feet, the square root or length of one of its sides.

NOTE.—There is a difference between square rods, square yards, and square feet, &c., and rods square, yards square, feet square, &c., thus: Three square feet may be represented by a Figure, 3 feet long, and one foot wide. And three feet square may be represented by a Figure 3 feet long; and 3 feet square are equal to 9 square feet. ☞ Such a Figure may be easily conceived of, by supposing two tiers of squares precisely like the one annexed, to be added to the bottom, which would give 9 equal squares.

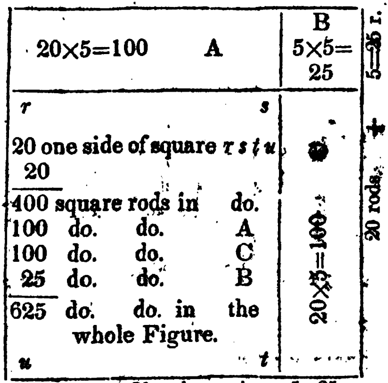


2. A gentleman desirous of fencing into one square lot 625 square rods of land, wishes to know the length of one of the sides.

625(25 rods, the length of one side, or the square root of 625 square rods.
4
45)225
225
0

DEM.—First, as our rule directs, we place a period (.) over the unit figure of our given number, and then one over the second figure beyond it. This we do, because any one figure multiplied by itself, will never produce more than two figures in the product. These periods also show, that our root will consist of as many figures as we have periods over the given number. We next seek the largest square in the left hand period, which we find to be 4, placing it under the first period, and writing its root, 2, in the quotient, for the first figure of the root. We then subtract the 4, the square of the root already found, from 6, the first period; and to the right hand of the remainder, 2, we bring down 25, the next period, making 225 for our next dividend. Now it must appear obvious, on inspecting the annexed Figure, that, at all stages of the work, the quotient expresses the side of a square formed from what has been subtracted from the dividend; therefore our quotient, 2, must express the side of a square made up from what has been subtracted from our given number, 625; but the 2 in the quotient properly stands in the place of tens, because we must have another figure in the quotient, at the right hand of the 2, which gives it the local value of 20; consequently we may call the first figure of the root 20, which expresses one side of the square *r s t u*; therefore 20, the root, multiplied into itself, must give the number of square rods or area contained in the square *r s t u* already formed from what has been subtracted from the first period of our dividend. This will appear plain on inspecting the Figure *r s t u*, each side of which is 20 rods, as expressed by the root already found; then the side of this square multiplied into itself, or the root already found, must give the number of square rods contained in the square *r s t u*, which has been subtracted, because 20 × 20 = 400, the number of square rods in the square *r s t u*, which is just equal to the number subtracted from the dividend; for the 4 under 6, the first period, stands in the place of hundreds, consequently it expresses 400. Now it is evident, that 400 square rods of our dividend are disposed of, in forming the square *r s t u*, each side of which is 20 rods; and we have now left of our dividend 225 square rods, which we must so dispose of, as to keep our Figure in a square form, after the additions are made, and also that our root may express one of the sides of the square after the Figure is completed. And in order to preserve our Figure *r s t u*, in a square form, the additions must be made on two sides; therefore, as our rule directs, we double the root, and place it at the left hand of 225, our dividend, for an imperfect divisor; *imperfect*, because another figure must be placed

FIG. II.



at the right of it, to complete the divisor. We double the root for a divisor, because the additions must be made on two sides of our Figure *r s t u*, to preserve it in a square form, each side being now 20 rods in length, the additions on the two sides must be 40 rods. And after doubling our root for a divisor, it expresses 40 in the divisor, because to complete our divisor another figure must be placed at the right of it; therefore, double the root is just equal to the additions which must be made on the two sides of our Figure *r s t u*. We next inquire how often our divisor 4, is contained in our dividend exclusive of the right hand figure; (we except the right hand figure of the dividend, because we have another figure to place in the divisor,) and we find that 4 is contained in 22 five times; we then write five in the quotient for the second figure of the root, and also place the 5 at the right hand of the 4, in the divisor; we then have 45 for a divisor, which is the whole length of the additions which must be made around the Figure *r s t u*, to preserve it in a perfect square form; because when we made the two first additions, that is, the long squares A and C, which equal in length the two sides of our first square *r s t u*, there was left a vacancy which is supplied by the little square, B; our divisor, 45, is then the whole length of A, B and C, and dividing by the length, our second quotient figure must express the width of the additions. We next multiply 45, our divisor, by 5, the last figure in the root, placing the product under 225, the remaining part of the dividend, which we find it equals, consequently we have no remainder.

Our work is now finished, and it is plain, on inspecting the Figure, that the first figure of the root shows the length of one of the sides of the square *r s t u*, and the second figure of the root shows the width of the additions made to our first square, consequently 25, the root, shows the length of one of the sides of the square after it is completed.

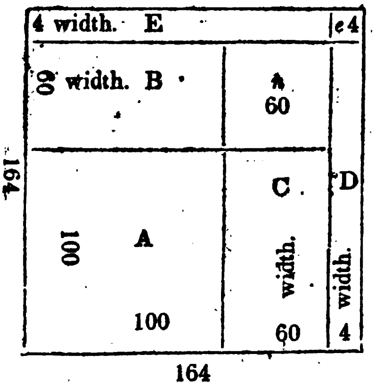
The work may be proved by multiplying the root by itself, that is, one of the sides of the square by itself; or it may be proved by adding in one sum, the area of the different parts of the Figure, and if it be equal to the given sum, the work is right.

rds. Proof by adding the parts.
20
20
400=Area of the square *r s t u*.
20×5=100=Area of the long square A.
20×5=100=Area of the long square C.
5×5= 25=Area of the little square B.
625 Rods, area of the Fig. completed.

Proof by raising the root to the second power or square.
25 *rds.*
25
125
50
625 Rods, a number equal to the given sum.

3. A man has 26896 square rods of meadow ground; he wishes to know the side of a square equal in area to it.

FIG. III.



```
26896(164 R.
1
26)168
   156
324)1296
    1296
Proof as follows:
164
164
656
984
164
26896 Proof.
```

Proof, by adding the parts.

Len.	*Br.*			
100×	100=	10000=	Area of	A.
100 ,	60 ,	6000 ,	do.	B.
100 ,	60 ,	6000 ,	do.	C.
60 ,	60 ,	3600 ,	do.	*n.*
160 ,	4 ,	640 ,	do.	D.
160 ,	4 ,	640 ,	do.	E.
4 ,	4 ,	16 ,	do.	*e.*
		26896	The area of whole Figure.	

NOTE.—We double the right hand figure of our last divisor in forming a new divisor, in order to get the length of the two sides of the little square placed in the corner, because the two additions to be made, must extend on two sides of the little square placed in the corner. The attentive student must now discover, on inspecting this and the two first Figures under this rule, that for every figure in the root, after the first, there must be an addition made around the square, and that the area of the Figure may always be calculated, by adding the area of the different parts of the Figure, giving to the figures representing the different parts their proper local value.

4. What is the square root of 81? Ans. 9.
5. What is the square root of 1296? Ans. 36.
6. What is the square root of 65536? Ans. 256.
7. What is the square root of 427716? Ans. 654.
8. What is the square root of 23059204? Ans. 4802.

NOTE.—If there be a remainder after all the periods are brought down, the operation may be continued, at pleasure, by annexing periods of ciphers.

9. What is the square root of 42? Ans. 6,48.
10. What is the square root of 30? Ans. 5,477.
11. What is the square root of 625,895? Ans. 25,017.

```
 •  • •  •
625,8950(25,017
 4
45)225
   225
5001)  8950
       5001
50027 ) 394900
        350189
         44711 rem.
```

NOTE.—In placing periods in decimals and whole numbers, a period is placed over units, in whole numbers, and so on, to the left over every second figure, and the first period in decimals must be placed over tenths, and so on, to the right, placing a period over every second figure. And if the decimals be odd, a cipher must be joined to the right hand of the last period to complete it; the student will recollect that joining a cipher to the right of a decimal does not alter its value.

12. What is the square root of 262,44? Ans. 16,2.
13. What is the square root of 24,7009? Ans. 4,97.

To extract the square root of a Vulgar Fraction.

RULE.—Reduce the fraction to its lowest terms for this, and all other roots; then extract the root of the numerator for a new numerator, and the root of the denominator for a new denominator.

If the fraction be a surd, reduce it to a decimal, and extract its root; if a mixed number, reduce it to an improper fraction, and then proceed the same as with a simple fraction, or reduce the fraction to a decimal, and proceed as in whole numbers and decimals.

EXAMPLES.

1. What is the square root of $\frac{25}{36}$? Ans. $\frac{5}{6}$.
2. What is the square root of $\frac{27}{147}$? Ans. $\frac{3}{7}$.
3. What is the square root of $\frac{112}{175}$? Ans. $\frac{4}{5}$.

SURDS.—4. What is the square root of $\frac{11}{12}$? Ans. ,9574.
5. What is the square root of $\frac{5}{12}$? Ans. ,64549.
6. What is the square root of $\frac{7}{9}$? Ans. ,88191.

MIXED NUMBERS.—7. What is the square root of $30\frac{1}{4}$? Ans. $5\frac{1}{2}$.
8. What is the square root of $20\frac{1}{4}$? Ans. $4\frac{1}{2}$.
9. What is the square root of $272\frac{1}{4}$? Ans. 16,5.

Application and Use of the Square Root.

PROBLEM I.—*To find the mean proportional between two numbers.*

RULE.—Multiply the given numbers together, and extract the square root of the product: the root will be the mean proportional sought.

EXAMPLES.

1. What is the mean proportional between 24 and 96?

$24 \times 96 = \sqrt{2304} =$ Ans. 48.

NOTE.—When the first is to the second, as the second is to the third, the second is called a mean proportional between the other two, thus, 24 : 48 :: 96.

2. A cheese weighed in one scale of a balance with an unequal beam, 16lb., but weighing it in the other scale of the balance, it weighed 64lb.; what was the true weight of the cheese? Ans. 32lb.

PROBLEM II.—*To find the side of a square equal in area to any given surface or superficies.*

RULE.—First find the area, (if not given,) and the square root of the area is the side of the square sought.

EXAMPLES.

1. If the area of a circle be 1521; what is the side of a square equal in area thereto? Ans. 39.

2. A man has 2 lots, the first contains 2 acres, 1 rood, and 25 poles; the second contains 1 acre and 2 roods; he wishes to exchange the two for one that is exactly square; the length of one of its sides is required? Ans. 25 poles.

3. A certain general has an army of 7056 men; how many must he place in rank and file, to form them into a square? Ans. 84.

PROBLEM III.—*To form a body of soldiers, so that the number in rank may be double, triple, &c. as many as in file.*

RULE.—Extract the square root of ½, ⅓, ¼, &c. of the given number of men, and that will be the number of men in file, which double, triple, quadruple, &c. and the product will be the number in rank.

EXAMPLES.

1. Let 8192 soldiers be so formed, that the number in rank may be double the file.

Ans. 64 in file, and 128 in rank.

2. Suppose a gentleman would set out an orchard of 1200 trees, so that the length shall be to the breadth as 4 to 3; how many trees must there be in length, and how many in breadth? Ans. 40 in length, and 30 in breadth.

PROBLEM IV.—*Any two sides of a right angled triangle being given, to find the third side.*

NOTE.—The square of the hypotenuse is exactly equal to the sum of the squares of the base and perpendicular. The base and perpendicular are sometimes called legs.

When the base and perpendicular are given, to find the hypotenuse.

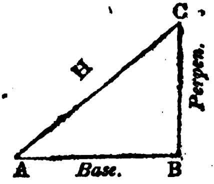
RULE 1.—Add the square of the base to the square of the perpendicular, and the square root of the sum will be the length of the hypotenuse.

When the hypotenuse and one of the legs are given, to find the other leg.

RULE 2.—From the square of the hypotenuse take the square of the given leg, and the square root of the remainder will be the length of the other leg.

EXAMPLES.

1. The base of the right angled triangle, A, B, C, is 40 yards, and the perpendicular 30 yards; what is the length of the hypotenuse? Ans. 50 yards.



NOTE.—A Triangle is a Figure bounded by three sides. A right angled triangle is that which has one right angle, like the angle at B in the annexed Figure. An angle is the opening between two lines meeting in a point. A right angle is that which is made by one line perpendicular to the other, thus, B, C, falling perpendicular to A, B, forms a right angle at B. The line H, is called the Hypotenuse.

2. The length of the hypotenuse is 45 feet, and one leg 36 feet; what is the length of the other leg or side? Ans. 27 feet.

3. The wall of a building is 30 feet high, and the street in front is 40 feet wide; what is the length of a ladder that will reach from the top of the wall to the opposite side of the street? Ans. 50 feet.

NOTE.—A knowledge of this problem will be found very useful in mechanical business, in finding the length of braces, rafters, &c.

PROBLEM. V.—*Having the diameter of a tube or pipe given, which will discharge a given quantity of water in a certain time, to find the diameter of another tube which shall discharge the same or a different quantity, in a given time.*

RULE.—As the given time, is to the square of the given diameter, so is the required time, to the square of the required diameter.

EXAMPLE.

1. Suppose a pipe $2\frac{1}{2}$ inches in diameter, will fill a cistern in 4 hours; what must be the diameter of another pipe which will fill the same cistern in 1 hour?

$2\frac{1}{2}$ inches$=2,5$; and $2,5\times2,5=6,25$. Then, as 4 hours : 6,25 inches, : : 1 hour : 25,00 inversely; and $\sqrt{25,00}=5$ inches. Ans. 5 inches.

PROBLEM VI.—*When time is required, the diameters being given,*

RULE.—As the square of one diameter, is to its time, so is the square of the other diameter inversely, to its time.

EXAMPLE.

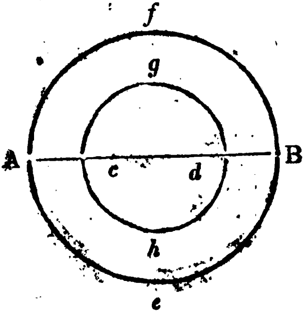
1. Suppose a pipe of 2in. bore, discharge a certain quantity of water in 4 hours; what time will a pipe of 4in. bore, require to discharge the same quantity? Ans. 1 hour.

PROBLEM VII.—*The diameter of a circle being given, to find the diameter of another circle, which shall be 2, 3, 4, &c., times greater or less than the given circle.*

RULE.—Square the given diameter, and if the required circle be greater, multiply the square of the diameter by the given proportion, and the square root of the product, will be the required diameter.—But if the required circle be less, divide the square of the diameter by the given proportion, and the square root of the quotient will be the required diameter.

EXAMPLES.

1. Let the diameter of a given circle be 12 inches; what is the diameter of one 4 times as large? Ans. 24 inches.



NOTE.—The diameter of a circle, is a right line passing through the centre, and terminated both ways by the circumference. A, B, is the diameter of the circle A, *f*, B, *e*; and *c*, *d*, is the diameter, of the smaller circle, *c*, *g*, *d*, *h*. The circumference is the bounding line of a circle. A, *f*, B, *e*, is the circumference of the greater circle, A, *f*, B, *e*; and *c*, *g*, *d*, *h*, is the circumference of the circle *c*, *g*, *d*, *h*. The diameter of the lesser circle is half as long as the diameter of the greater circle, though the greater circle is 4 times as large.

2. The diameter of a circle is 24 inches; what is the diameter of one which is one fourth as large? Ans. 12in.

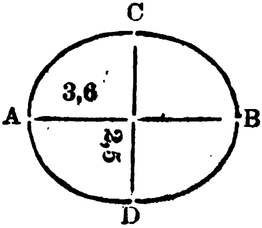
3. Seven men bought a grinding stone of 60 inches diameter, each paying $\frac{1}{7}$ part of the expense; what part of the diameter must each grind down for his share?

Ans. The 1st, 4,4508—2d, 4,8400—3d, 5,3535—4th, 6,0765—5th, 7,2079—6th, 9,3935—7th, 22,6778 inches.

Problem VIII.—*To find the diameter of a circle equal in area to an Ellipsis, whose longest and shortest diameters are given.*

RULE.—Multiply the two diameters of the ellipsis together, and the square root of the product will be the diameter of a circle equal in area to the ellipsis or oval.

EXAMPLE.



1. Let the longest diameter be 3,6 feet, and the shortest 2,5; what is the diameter of a circle equal thereto? Ans. 3 feet.

Note.—The longest and shortest diameters of an ellipsis are sometimes called the *traverse* and *conjugate* diameters.

Problem IX.—*The sum of two numbers being given, and the difference of their squares; to find those numbers.*

RULE.—Divide the difference of their squares by the sum of the numbers, and the quotient will be their difference; then to half the sum of the given numbers, add half the difference for the *greater* number, and from half the sum subtract half the difference for the *less.*

EXAMPLE.

1. The sum of two numbers is 40, and the difference of their squares is 320; what are the numbers?

Ans. The greater is 24, the less 16.

Problem X.—*The difference of two numbers being given, and the difference of their squares; to find those numbers.*

RULE.—Divide the difference of the squares by the difference of the numbers, and the quotient will be their sum; then to half the sum add half the difference for the *greater* number, and from half the sum take half the difference for the *less.*

EXAMPLE.

1. The difference of two numbers is 20, and the difference of their squares is 1200; what are the numbers?

Ans. 40 the greater, 20 the less.

Problem XI.—*Having the sum and product of two numbers given; to find those numbers.*

RULE.—Square half the sum of the two numbers, and from the product, subtract the product of the two numbers; then the square root of the remainder added to half the sum of the two numbers, will give the *greater* number, and subtracted from half the sum, will give the *less.*

EXAMPLE.

1. The sum of two numbers is 52, and their product is 612; what are those two numbers?

Ans. 34 the greater, and 18 the less.

Problem XII.—*Having the sum of the squares of two numbers given, and also the difference of their squares; to find those numbers.*

RULE.—From the sum of the squares, take the difference of the squares, and the square root of half the remainder, will be the *less* number; and half the remainder taken from the sum of the squares, leaves the square of the greater; the square root of which will be the *greater.*

EXAMPLE.

1. The sum of the squares of two numbers is 544, and the difference of their squares 256; what are those numbers?

Ans. 12 the less, and 20 the greater.

Problem XIII.—*The sum of two numbers and the sum of their squares being given; to find those numbers.*

RULE.—Subtract the sum of their squares from the square of their sum; then subtract this remainder from the sum of their squares, and the square root of the difference will be the difference between the two numbers; then to half their sum, add half their difference, and the sum will be the *greater* number; and half their difference taken from half their sum, will give the *less.*

EXAMPLE.

1. The sum of two numbers is 70, and the sum of their squares is 2900; what are the two numbers?

Ans. 50 and 20.

Problem XIV.—*The difference of two numbers, and the sum of their squares being given; to find those numbers.*

RULE.—Subtract the square of their difference from the sum of their squares; add the sum of the squares to the remainder, and the square root of this last sum, will be the sum of the numbers required; then to half the sum of the two numbers, add half the difference for the *greater* number, and from half the sum subtract half the difference for the *less.*

EXAMPLE.

1. A number of crowns are to be divided between A and B, in such a manner that A may have 60 crowns more than B, and that the sum of the square of the respective shares may be 11600; what number must each have?

Ans. A must have 100 crowns, and B 40.

QUESTIONS ON INVOLUTION, EVOLUTION, AND THE SQUARE ROOT.

What is Involution? A. It is finding the *powers* of numbers. What is a power? A. It is the product arising from multiplying any number into itself continually a certain number of times. What is the power arising from multiplying any number into itself? A. the second power or square. How is the third power or cube found? A. By multiplying any number by its square. How is the power generally distinguished? A. By an index or exponent, placed at the right, and a little above the number, and this index or exponent is always one more than the number of multiplications to produce the power. When a simple fraction is raised to a power, does it increase or diminish its value? A. It diminishes its value in the same ratio, that numbers above unity are increased. When the exact root of a power cannot be found, what is the number called? A. It is called a *surd number.*—When the exact root can be found, what is the number called? A. It is called a *rational number.* What is Evolution? A. It is extracting or finding the roots of any given powers. What is the square root of any number? A. It is a number which multiplied into itself, will produce the given number. Why is the given number pointed off into periods of two figures each, or a period placed over units and one over every second figure, counting to the left, to obtain the square root? A. Because any one figure raised to the second power, can never exceed two figures. After the given number is pointed off into periods, how do you proceed to extract the root? A. I find the greatest square in the left hand period, placing it directly below that period, and its root in the quotient; then subtract, and to the remainder bring down the next period, placing double the root at the left for a divisor, and seek how often it is contained in the dividend, except the right hand figure, placing the result in the quotient, and likewise at the right hand of the divisor to complete the divisor; then multiply the divisor by this quotient figure and subtract as before, and to the remainder bring down the next period, and again place double the quotient to the left hand for a divisor, or bring down the old divisor without aliteration except doubling the right hand figure. Why should the quotient be doubled for a divisor? A. Because the root, at all stages of the work, shows the side of a square, and the remaining part of the dividend must be so disposed of, as to preserve the Figure in its square form, or that the root may express one of its sides, and doubling the root gives the length of the two sides, and then inquire how often this length is contained in all except the right hand figure of the dividend, and place the result at the right hand of the divisor to make out the entire length of the addition, and then the quotient figure expresses the breadth of the addition. How is the square root of a vulgar fraction found? A. By reducing the fraction to its lowest terms, and then extracting the root of the numerator for a new numerator, and the root of the denominator for a denominator of the root required; but if the fraction be a *surd*, it should be reduced to a decimal, and its root extracted. What is the use of the square root? A. It is of great use to mechanicks of different kinds, in calculating the length of timbers, such as rafters, braces, &c.; to surveyors in calculating area, &c. &c.

EXTRACTION OF THE CUBE ROOT,

Is the finding of such a number, as being multiplied into its square, will produce the given number.

A *Cube* is a square solid, having six equal sides, and each of the sides an exact square. The *Root* of the cube is the length of one of the sides of the square solid; for the length, breadth, and thickness of a cube or square solid are all alike; consequently, the root or length of one side raised to the third power, gives the solid contents; for raising the root or one side of the cube to the third power, is multiplying the length, breadth, and thickness of such a figure together. Therefore it follows, that extracting the *cube root* of any number of feet, yards, or rods, is finding the length of one side of a cube or solid square, of which the whole contents will be equal to the given number of feet, yards, or rods, &c.

EXAMPLES.

1. How many solid feet are there in a cubick block, each side of which measures 4 feet? Ans. $4^3=4\times4\times4=64$ft.

2. How many solid feet in a cubick block, each side measuring 3 feet? Ans. 27 feet.

3. How many feet in length is the side of a cubick block which contains 8 solid feet? Ans. 2 feet.

DEM.—It is plain, in finding the cube root of 8, we have only to think of such a number for a root as will, when raised to the third power, equal, or, at least, not exceed 8; and the root must always be the greatest number possible that will not exceed the given number when raised to the third power, and 8 is the third power or cube of 2, which may be seen in the table of powers, or by raising 2, the root or one side of the cube, to the third power, thus $2\times2\times2=8$, the given number.

4. What is the side of a cube or solid square containing 64 solid feet? Ans. 4 feet.

RULE.—*For extracting the Cube Root.*

1. Separate the given number into periods of three figures each, by putting a point over the unit figure, and every third figure beyond the place of units.

2. Find the greatest cube in the left hand period, and put its root in the quotient.

3. Subtract the cube, thus found, from the said period, and to the remainder bring down the next period, and call this the *dividend.*

4. Multiply the square of the quotient by 300, calling it the *divisor.*

5. Seek how many times the divisor may be had in the dividend, and place the result in the root; then multiply the divisor by this quotient figure, and write the product under the dividend.

6. Multiply the square of this quotient figure by the former *figure* or *figures* of the root, and this product by 30, and place the product under the last; under all, write the cube of this quotient figure, and call their amount the *subtrahend.*

7. Subtract the subtrahend from the dividend, and to the remainder bring down the next period for a new dividend, with which proceed as before; and so on, till the whole is finished.

EXAMPLES.

1. A man has 13824 feet of timber, in separate blocks, each containing one cubick foot, and having a desire to place them in a cubick pile, he wishes to know the length of each side of such a pile. Ans. 24 feet.

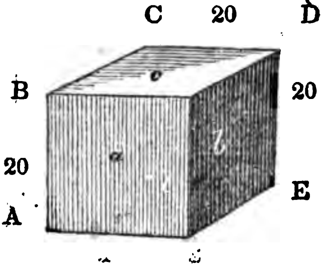
We first ascertain what number of figures we must have in the root, by placing a period over the unit figure, and then one over the third beyond it; these periods show that the root must consist of as many figures as we have periods; they likewise divide the dividend into its proper parts, which must be known before we can proceed in the operation of the work. We place a period over the unit figure, and one over every third counting to the left, because no figure multiplied by its square, or used three times as a factor, will exceed three figures.

DEM.—After placing the periods, as our rule directs, we seek the greatest cube in the left hand period, 13, without regard to its local value, which we find, by the table of powers, to be 8, placing its root, 2, in the quotient for the first figure of the root, and subtracting the 8 from the left hand period. And since we have two periods placed over our given number, our root must consist of two figures; therefore the 2, the first figure of the root, must be 2 tens, then 20 must be the local value of the 2. Now, at all stages of the work, the root, reckoned according to its local value, expresses one side of a cube or square solid, formed from what has been subtracted from the given number. Then let *Figure I.* represent the cube or square solid which has been formed from what has been taken from our dividend. One side of this cube being 20 feet, as our root shows, its solid contents must be $20\times20\times20=8000$ solid feet, which exactly agrees with the number taken from our first period, because our first period expresses thousands, and 8 standing under thousands has a local value of so many thousands, consequently it has taken from our first period 8000, leaving 5 thousand, the remainder of the first period, to the right of which we bring down the next period, 824, leaving for our entire remainder 5824 solid feet, which forms a new dividend. And now the remaining 5824 solid feet, we must so dispose of, in enlarging the cube, as to preserve its cubick form; and in order to do this, it is evident, that the addition to the cube, A, D, must be made on three sides, *a*, *b*, and *c*. And it is plain, if we divide 5824 solid feet, by the area or surface of the three sides, *a*, *b*, and *c*; the quotient will be the thickness of the addition made to each of the sides, *a*, *b*, and *c*. Then the next thing to be done, is to form a divisor, which shall express the area or surface of the three sides, *a*, *b*, and *c*. And to proceed as our rule directs, we multiply the square of the quotient by 300; thus, 2×2=4, the square of the quotient, without regard to its local value; then 4×300=1200 square feet, for the divisor, which is half the area of the cube A, D, or the area of the three sides, *a*, *b*, and *c*. But the root, 2, that is 2 tens, already found, is the length of one of the sides; therefore we might proceed, thus 20×20=400 square feet, the area of one side, which multiplied by 3, the number of sides, gives 400×3=1200 square feet, the area of the three sides as before; but this last method is not so convenient in practice, therefore, we place the ciphers of both factors at the right of our multiplier 3, and call it 300 as in the first operation.

Operation.

```
13824(2
 8
-----
 5824
```

FIG. I.



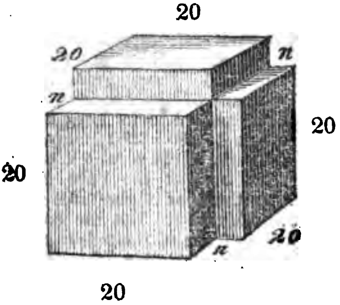
```
  20
  20
 ----
 400
  20
 ----
8000
```

We now find, that our divisor, 1200, is contained in the dividend, 5824, 4 times, consequently the last figure of our root, 4 feet, expresses the thickness of the addition made to each of the sides, *a*, *b*, and *c*; then multiplying 1200 by 4, must give 4800 solid feet, the number contained in the three additions, because it is the product arising from multiplying the length, breadth, and thickness of the three additions together. But on inspecting FIG. II, we find that this addition made of the three sides, falls short of completing the cube; because there are still deficiencies in the three corners, *n*, *n*, *n*. Now it is evident, that the length of the solid required to fill each of these deficiencies, must be the same as the length of the cube before any addition was made; therefore 2, the first figure of the root, must represent the length of each of these deficiencies, because it represents the length of each side of the cube before any addition was made; but this 2 expresses 20, for it stands in the place of tens; then 20 must be the length of each of these deficiencies, and the width and thickness of each must be equal to the last quotient figure, 4; therefore the solid feet necessary to fill the deficiencies, *n*, *n*, *n*, may be found by multiplying 16, the square of the last quotient figure, 4, by the length of all the deficiencies. And it has been shown that the length of each deficiency is 20 feet, then the length of the three deficiencies must be 60 feet; therefore 16, the product of the breadth and thickness of each deficiency, multiplied by 60, the length of the whole, must give the solid contents required to fill the three deficiencies, *n*, *n*, *n*; thus, 16×60=960 solid feet, the solidity required to fill the three deficiencies, *n*, *n*, *n*. Or the solidity required to fill the three deficiencies may be obtained thus, 4×4=16, the product of the breadth and thickness of each deficiency, or the square of the last quotient figure, then 16×2=32, which multiplied by 30, thus, 32×30=960 solid feet, the same result as before. But perhaps it would have rendered the work more plain, if we had multiplied 16, the square of the last quotient figure, or the product of the breadth and thickness of each deficiency, by 20, the length of each deficiency, which would have given the solidity required to fill one deficiency, and then have multiplied that solidity by 3, the number of deficiencies in the corners, *n*, *n*, *n*; thus, 16×20=320, the solidity of one deficiency, then, 320×3=960 solid feet, the solidity in the three deficiencies *n*, *n*, *n*, the same as before. And now having supplied the deficiencies, *n*, *n*, *n*, we find on inspecting Fig. III, that there still remains a deficiency in the corner where the three last pieces meet. Now it is plain, that this last deficiency must be a small cube, because the solid required to fill this

Operation continued.

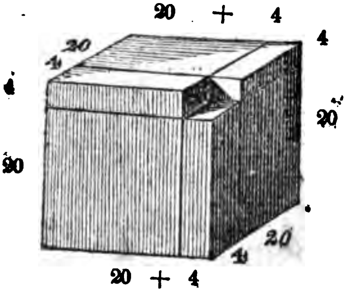
```
                 13824(24 Root.
                  8
Divisor, 1200)5824 Dividend.
              ----
              4800
               960
                64
               ----
Subtrahend,   5824
              ----
              0000
```

FIG. II.



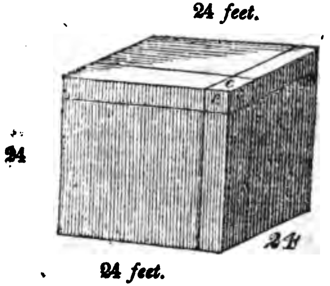
deficiency, must be equal in length, breadth and thickness, to the last quotient figure, 4. Then by cubing this last quotient figure, we will have the solid feet required to fill the vacancy, thus, 4×4×4=64 solid feet, the contents of the corner piece, which will be seen in its place in FIG. IV.

FIG. III.



The number of solid feet in these several additions must be added to compose the subtrahend, thus, 4800+960+64=5824, the number of solid feet in the subtrahend, that is, in all the additions, which, subtracted from the dividend, leaves no remainder, and the work is now finished.

FIG. IV.



Then FIG. IV. represents the cubick pile, each side of which is 24 feet, and contains 13824 solid feet, which may be proved by cubing one of the sides of the cube, that is, the root, thus, 24×24×24=13824 solid feet, the given number, that is, multiplying the length, breadth, and thickness together, to find the solidity; or it may be proved by adding together the solid feet contained in the several parts, thus:

Feet. 8000=contents of the first cube, FIG. I.
4800=first addition to the sides *a*, *b*, *c*, FIG. II.
960=2d addition, to fill deficiencies *n*, *n*, *n*, FIG. III.
64=3d addition to fill the corner *e*, *e*, *e*, FIG. IV.

13824=contents of the whole pile, FIG. IV., and 24 feet, the cube root or length of each side.

2. What is the cube root of 34645976?

34645976(326 Answer.
27
$3^2\times300=2700$)7645 first dividend.
5400 *Carried over.*

Brought over.

5400
$2^2 \times 3 \times 30 = 360$
$2^3 = 8$

5768 first subtrahend.

$32^2 \times 300 = 307200$)1877976 second dividend.

1843200
$6^2 \times 32 \times 30 = 34560$
$6^3 = 216$

1877976 second subtrahend.

DEM.—It has been stated that the root, at all stages of the work, represents the side of a cube, formed from what has been taken from the given number. Now 3, the first figure of the root, having a local value, represents the side of a cube, each side of which must be 300, for if we raise 300 to the third power, the product will exactly agree with the number subtracted from the given number. When we have obtained two figures, 32, in the root, they likewise having a local value, represent the side of a cube formed from what has been subtracted from the dividend or given number; and each side of the cube, at this stage of the work, must be 320. And when we have obtained three figures in the root, as 326, our root, 326, must represent the side of a cube, the solid contents of which are equal to the given number. Then it is evident, that after every subtraction our root represents the side of a new cube, so that for every figure in the root, after the first, there must be additions made similar to that which was made in obtaining the second figure of the root under example 1st; the reason of which is evident from the demonstration there given.

3. What is the cube root of 729? Ans. 9.
4. What is the cube root of 12167? Ans. 23.
5. What is the cube root of 4826809? Ans. 169.
6. What is the cube root of 9129329? Ans. 209.
7. What is the cube root of 15625000? Ans. 250.
8. What is the cube root of 329939371? Ans. 691.
9. What is the cube root of 997002999? Ans. 999.
10. What is the cube root of 729000000? Ans. 900.

NOTE.—To extract the cube root of a Vulgar Fraction, first extract the cube root of the numerator for a new numerator, and then extract the cube root of the denominator for a new denominator. Or reduce the fraction to a decimal, and extract the cube root of the decimal.

11. What is the cube root of $\frac{8}{27}$? Ans. $\frac{2}{3}$.
12. What is the cube root of $\frac{125}{343}$? Ans. $\frac{5}{7}$.
13. What is the cube root of $\frac{1}{125}$? Ans. $\frac{1}{5}$.

When there are decimals in the given number, point off the whole numbers the same as if there were no decimals belonging to the given number, then place a period over tenths in decimals, and one over every third figure beyond it, counting to the right, and if the right hand period should not be complete, annex ciphers to complete the period; then extract the root the same as in whole numbers. The periods over whole numbers, show that the root must have so many figures in whole numbers; the rest will be decimals.

14.	What is the cube root of 41421,736?	Ans. 34,6.
15.	What is the cube root of 85766,121?	Ans. 44,1.
16.	What is the cube root of 117,649?	Ans. 4,9.
17.	What is the cube root of 84,604519?	Ans. 4,39.

NOTE.—If there be a remainder after all the periods are brought down, the operation may be continued, at pleasure, by annexing periods of ciphers.

18.	What is the cube root of 2?	Ans. 1,2599.
19.	What is the cube root of 1?	Ans. 1.
20.	What is the cube root of 3?	Ans. 1,442.

Application and use of the Cube Root.

CASE I.—*To find two mean proportionals between any two given numbers.*

RULE.—Divide the greater extreme by the less, and the cube root of the quotient, multiplied by the less extreme, gives the less mean; multiply the said cube root by the less mean, and the product will be the greater mean proportional.

EXAMPLES.

1. What are the two mean proportionals, between 4 and 256?

Ans. 16 the less, 64 the greater.

NOTE.—Sixteen and 64 are the two mean proportionals between 4 and 256; because 4 : 16 :: 64 : 256, and the product of the extremes is equal to the product of the means. It will be seen that the extremes are the given numbers, and the means, the two numbers in the answer to the question.

2. What are the two mean proportionals between 8 and 4096?

Ans. 64 less mean, 512 greater mean.

CASE II.—*To find the side of a cube that shall be equal in solidity to any given solid, as a globe, cylinder, prism, cone, &c.*

RULE.—Extract the cube root of the solid contents of the given body, and the said root will be the side of the cube required.

EXAMPLES.

1. The statute bushel contains 2150,4252 cubick or solid inches; what must be the side of a cubick box, that shall contain the same quantity? Ans. 12,907 inches.

2. There is a certain cistern, 24 feet long, 18 feet wide, and 4 feet high; required the side of a cistern of a cubick form, that shall hold the same quantity. Ans. 12 feet.

NOTE.—The solid contents of similar Figures are in proportion to each other, as the cubes of their similar sides or diameters.

3. If a ball, 2 inches in diameter, weigh 3lb.; what will a ball of similar metal weigh, whose diameter is 4 inches?

2×2×2=8; 4×4×4=64. As 8 : 3lb. :: 64 : 24lb. Ans. 24lb.

4. If a globe of gold 1 inch in diameter be worth $100; what is the value of another globe 4 inches in diameter?
Ans. $6400.

CASE III.—*The side of a cube being given, to find the side of another cube which shall be double, triple, &c. in quantity to the given cube: Or one fourth, one fifth, &c. of the given cube.*

RULE.—Cube the given side, and multiply it by the given proportion, and the cube root of the product will be the side sought. Or if required to be less; divide the cube of the given side, by the given proportion, and extract the cube root of the quotient; the root will be the side of the cube sought.

EXAMPLES.

1. There is a cubical vessel whose side is 3 feet; required the side of another cubical vessel which shall contain 8 times as much. Ans. 6 feet.

2. There is a cubical vessel whose side is 6 feet; required the side of one that shall contain only one eighth as much.
Ans. 3 feet.

Extraction of the Biquadrate Root,

Is finding a number, which being multiplied by its cube, or involved four times into itself, will be equal to the given number.

RULE.—You may first extract the square root of the given number, and then the square root of that root, and the last root will be the required biquadrate root.

EXAMPLES.

1. What is the biquadrate root of 33362176? Ans. 76.
2. What is the biquadrate root of 5719140625?
Ans. 275.

A general rule for extracting the roots of all powers.

1. Prepare the given number for extracting, by pointing off from the unit's place, as the required root directs, that is, a period over every second figure, for the square root, and one over every third, for the cube root; one over every fourth, for the biquadrate, and one over every fifth, for the fifth root, and so on.

2. Find the first figure of the root by the table of powers, or by trial; subtract its power from the left hand period, and to the remainder bring down the first figure in the next period for an imperfect dividend.

3. Involve the root to the next inferiour power to that which is given, and multiply it by the number denoting the given power, for a divisor; by which find a second figure of the root.

4. Involve the whole ascertained root to the given power; and sub-

tract it from the first and second periods. Bring down the first figure of the next period to the remainder, for a new dividend; to which find a new divisor as before, and so proceed till the work is finished.

NOTE.—The roots of the 4th, 6th, 8th, 9th, and 12th powers may be obtained more readily, thus:

For the 4th root, extract the square root of the square root.
For the 6th root, take the square root of the cube root.
For the 8th root, take the square root of the 4th root.
For the 9th root, take the cube root of the cube root.
For the 12th root, take the cube root of the 4th root.

EXAMPLES.

1. What is the fifth root of 130691232?

130691232(42 Answer.
1024 — 4×4×4×4×4=1024.
1280)2829* — 4×4×4×4×5=1280.
130691232
130691232 — 42×42×42×42×42=130691232.

* We call 2829 an imperfect dividend, because we make no other use of it than determining the next figure in the root, after which we make use of the periods in our given number.

NOTE.—When a number is given to find its root, the given number may be considered as the power of the root required; thus, when the square root is required, the given number is the second power of the required root; when the cube root is required, the given number is the third power, and when the biquadrate root is required, the given number is the 4th power of the required root, and so on.

2. What is the biquadrate root of 5308416?

Ans. 48, the root.

QUESTIONS ON THE CUBE ROOT.

What is the extraction of the cube root? A. It is finding a number which being multiplied by its square, will produce the given number. How is a number prepared for extracting the root? A. By pointing it off into periods of three figures each, beginning at the place of units. Why is the given number pointed off into periods of three figures each? A. Because the cube of one figure never exceeds three figures. How is the first figure of the root found? A. By the table of powers or by trial; the greatest cube that shall not exceed the left hand period, is placed directly below it, and its root in the quotient. What does the first figure of the root show, taken in its local value? A. It expresses the side of a cube formed from what has been subtracted from the given number. What does the first divisor express? A. It expresses, like all other divisors, the area or surface of three sides of the cube which has already been formed. What is the product when this divisor is multiplied by the last quotient figure? A. It expresses the solidity contained in the three pieces, or additions, made to the three sides of the cube. What do the other parts of the subtrahend express? A. The solid contents of the pieces required to fill the spaces, in order to complete the cube. What does the root show at all stages of the work? A. The length of one side of a cube, formed from what has been subtracted from the given number. How is the cube root proved? A. By cubing the quotient or root, or by adding together the solidity of the several parts of the cube. When the solid contents of a cube are given, how do you find the length of one of the sides? A. By extracting the cube root. When the side of a cube is given, how do you find its solidity? A. By cubing the side.

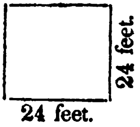
MENSURATION OF SUPERFICIES,

Teaches how to find the area or surface of any square, parallelogram, circle, triangle, &c. Every magnitude is measured by some magnitude of the same kind; thus, a line is measured by a lineal inch, foot, yard, &c.; superficies, or surface, by the square inch, square foot, square yard, &c.; and a solid, by the cubick inch, cubick foot, cubick yard, &c. The superficial inch, foot, yard, &c., is one inch, foot, yard, &c., in length and breadth; and because 12 inches make one foot in long measure, 144 inches make one superficial foot, thus 12×12=144 inches, and 9 feet a superficial yard, thus, 3×3=9.

To find the area or surface of a Square having equal sides.

RULE.—Multiply the side of a square into itself, and the product will be the area or the superficies.

EXAMPLES.



1. How many square feet of boards will lay a floor which is 24 feet square?

24×24=576 Ans. 576 feet.

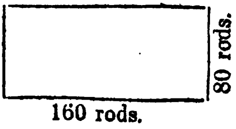
2. How many acres does a square lot contain, measuring 40 rods on each side?

Ans. 10 acres.

To find the area of a parallelogram, or long square.

RULE.—Multiply the length by the breadth, and the product will be the area, or superficial content, in the same name with the length of the sides.

EXAMPLES.



1. How many acres are contained in a piece of land 160 rods long, and 80 rods wide?

160×80=12800÷160=80 acres;

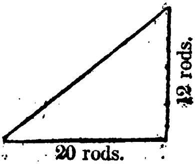
Ans. 80 acres.

2. How many acres are there in a lot of land 320 rods long, and 160 rods wide? Ans. 320 acres.

To find the area of a right angled triangle.

RULE.—Multiply the base of the triangle by half its perpendicular, or multiply half the base by the whole perpendicular, and the product will be the area.

EXAMPLES.



1. Required the area of a triangle, the base of which is 20 rods, and the perpendicular 12 rods.

20×6=120rds. Ans. 120 sq. rds.

DEM.—The reason of multiplying the base by half the perpendicular, is plain, because the triangle is half of a parallelogram, one side of which is 20 rods, and the other 12 rods.

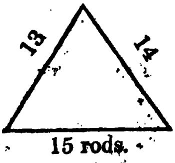
2. How many acres are contained in a triangle whose base is 320 rods, and perpendicular 40 rods? Ans. 40 acres.

To find the area of a triangle having the three sides given.

RULE.—Add the three sides together, then take the half of that sum, and out of it subtract each side severally, and multiply the half of the sum and these remainders, and the square root of this product will be the area of the triangle.

EXAMPLES.

1. Required the area of an oblique triangle, the 3 sides of which are 13, 14, and 15 rods. Ans. 84 square rods.



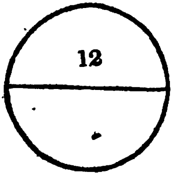
```
  13    21  21  21   21 half the sum.
  14    13  14  15    6
  15    --  --  --  ---
  --     8   7   6  126
2)42 sum.             7
  --                ---
  21 half the sum.  882
                      8
                    ---
                    7056(84 rods the area.
                    64
               164) ---
                    656
                    656
                    ---
                    000
```

2. Required the area of an oblique triangle, the 3 sides of which are 80, 120, and 160 rods. Ans. 29 acres, 7 poles.

To find the area of a circle, the diameter being given.

RULE.—Multiply the square of the diameter by ,7854, and the product will be the area. Or multiply half the diameter by half the circumference, and the product will be the area.

EXAMPLES.



1. What is the area of that circle, whose diameter is 12 rods? 12×12=144×,7854=113,0976 rods. The area of the given circle.

DEM.—We first multiply the diameter by itself, which gives the area of a square whose side is 12 rods, because multiplying the side of a square by itself gives the area of a square whose side is the given number; the area of a square is one foot, one yard, &c., when its side is 1; but the area of a circle whose diameter is 1, is only ,7854, therefore it is evident that we must multiply the square of the diameter by ,7854, or by ,785398, which is nearer the truth, for the area.

2. What is the area of a circle whose diameter is 14 rods? Ans. 153,9384 square rods.

To find the circumference of a circle, the diameter being given: or the circumference given, to find the diameter.

RULE.—Multiply the diameter by 3,141592, and the product will be the circumference. Or, as 7 : is to 22 :: so is the diameter : to the circumference. Or, as 22 : is to 7 :: so is the circumference : to the diameter; or, more exactly, as 113 : is to 355 :: so is the diameter : to the circumference, or, as 355 : is to 113 :: so is the circumference : to the diameter.

EXAMPLES.

1. What is the circumference of a circle whose diameter is 30 rods? As 113 : 355 :: 30 : Ans. 94,248 rods, nearly.

2. What is the diameter of a circle whose circumference is 94,248 rods? As 355 : 113 :: 94,248 : Ans. 30 rods.

The area of a circle being given, to find the diameter.

RULE.—Divide the area by ,7854, and extract the square root of the quotient; the root will be the diameter sought. Or, as 355 : 452 :: so is the area : to the square of the diameter; or, as 1 : 1,273239 :: so is the area : to the square of the diameter; or you may multiply the square root of the area by 1,12837, and the product will be the diameter.

EXAMPLES.

1. What is the diameter of that circle whose area is 1 acre or 160 square rods? Ans. 14,273 rods, nearly.

160 rods÷,7854=√203,7178=14,273 nearly.

2. What length of rope may be tied to a horse's head, and the other end to a stake, to give him the liberty of eating two acres of grass? Ans. 55¼ yards.

NOTE.—The area of circles are to each other, as the squares of their diameters.

To find the area of a globe or ball.

RULE.—Multiply the whole circumference by the whole diameter, and the product will be the area.

NOTE.—The area of a globe or ball is 4 times as much as the area of the circle of the same diameter, hence the rule is obvious.

EXAMPLES.

1. What is the number of square miles on the surface of the earth, allowing the diameter to be 7911 miles, and the circumference 24853 miles? Ans. 196612083 sq. miles.

2. Suppose the ball on the top of St. Paul's Church is 6 feet in diameter; what did the gilding of it cost, at 3½d. per square inch? Ans. £237 10s. 1d.

To find the area of a circle, the circumference and diameter being given.

RULE.—Multiply half the circumference by half the diameter, and the product will be the area. Or multiply the whole circumference by the whole diameter, and one fourth of the product will be the area.

EXAMPLES.

1. What is the area of a circle, whose diameter is 7, and circumference 22? 11×3½=38½, or 22×7=154÷4=38½.
Ans. 38½.

2. What is the area of a circle, the diameter of which is 10 feet 6 inches, and the circumference 31 feet 6 inches?
Ans. 82 feet 8 inches.

MENSURATION OF SOLIDS,

Teaches to find the solidity of bodies that have length, breadth, and thickness.

DEFINITION.—Mensuration signifies to measure, hence measuring surfaces is called mensuration of superficies; and measuring solids is called mensuration of solids, that is, to measure a solid, so as to express its content in solid or cubick inches, feet, yards, &c.

To find the solidity of a cube.

RULE.—Multiply the side of the cube by itself, and that product again by the side, the last product will be the solidity.

EXAMPLES.

1. How many solid or cubick inches, in a cube of marble whose side is 24 inches? 24×24×24=Ans. 13824 sol. in.

2. What is the solidity of a cube, the side of which is 5 feet? Ans. 125 solid feet.

To find the solidity of a parallelopipedon, that is, a solid contained by six quadrilateral planes, every opposite two of which are equal and parallel.

RULE.—Multiply the length by the breadth, and that product again by the thickness or height, and it will give the solidity.

EXAMPLES.

1. What is the solidity of a parallelopipedon, whose length is 12 feet, breadth 4 feet, and height 6 feet?
12×4×6=Ans. 288 solid feet.

2. How many solid feet in a load of wood 8 feet long, 3½ feet wide, and 3½ feet in height? Ans. 98 feet.

3. What number of bricks 8 inches long, 4 inches wide, and 2 inches thick, will it require to build a house 46 feet long, 38 feet wide, and 20 feet high, and the walls to be 1 foot thick? Ans. 88560 bricks.

To find the solidity of a cylinder.

DEFINITION.—A cylinder is a round body whose bases are circles, like a round column or stick of timber, of equal bigness from end to end.

RULE.—Multiply the area of the base by the perpendicular height, and the product will be the solidity.

EXAMPLES.

1. What is the solidity of a cylinder, the height of which is 5 feet, and the diameter of the end 2 feet? Ans. 15,708 ft.

2. One evening I chanc'd with a Tinker to sit,
Whose tongue ran a great deal too fast for his wit;
He talk'd of his art with abundance of mettle;
So I ask'd him to make me a flat-bottomed kettle;
Let the top and the bottom diameters be
In just such proportion as five are to three;
Twelve inches the depth I propos'd, and no more;
And to hold in ale gallons, seven less than a score.
He promis'd to do it, and straight to work went;
But when he had done it he found it too scant.
He alter'd it then, but too big he had made it;
For though it held right, the diameters fail'd it;

Thus altering it often too big and too little,
The Tinker at last had quite spoil'd his kettle;
But he says that he will bring his said promise to pass,
Or else that he'll spoil every ounce of his brass.
Now to keep him from ruin, I pray find it out
The diameter's length, for he'll ne'er do't, I doubt.

Ans. { 14,64 inches, bottom diameter.
24,4 inches, top diameter.

Note.—The kettle is not a cylinder, the top and bottom diameters being unequal, yet there is sufficient given in this and the preceding rules for finding the diameters.

To measure a Sphere or Globe.

Definition.—A sphere or globe is a round, solid body, in the middle of which is a point, from which all lines drawn to the surface are equal.

RULE.—Multiply the cube of the given diameter by ,5236, and the product will be the solid contents.

EXAMPLES.

1. The diameter of a globe is 12 inches; how many cubick or solid inches does it contain? 12×12×12=1728×,5236 equal to Ans. 904,7808 solid inches.

Dem.—By cubing the diameter, the product, 1728, is the solidity of a cube whose side is 12 inches, we then multiply by the decimal, ,5236, because ,5236 is the solidity of a globe whose diameter is 1.

Note.—A cube whose side is one inch, contains one cubick or solid inch. A globe whose diameter is one inch, contains ,5236 of an inch.

2. Suppose the diameter of the earth is 7911 miles; how many solid or cubick miles does it contain?

Ans. 259,235,092,532$\frac{6316}{10000}$ solid miles.

QUESTIONS ON MENSURATION.

What is Mensuration of Superficies? A. It teaches how to measure surfaces or area. How do you find the surface or area of a square? A. Multiply the side of the square into itself, and the product will be the area. How do you find the area of a parallelogram or long square? A. Multiply the length by the breadth, and the product will be the area. How do you find the area of a right angled triangle? A. By multiplying the base by one half the perpendicular, the product will be the area. How do you find the area of a circle? A. Multiply the square of the diameter by ,7854, the product will be the area. Why multiply by ,7854? A. Because the area of a circle is ,7854 when the diameter is one. How do you find the area of a globe or ball? A. Multiply the whole circumference by the whole diameter, and the product will be the area. What does mensuration of solids teach? A. It teaches how to measure solids. How do you find the solidity of a cube? A. Cube one side, and the product will be the solidity. How would you find the number of cubick feet in a load of wood that is 8 feet long, 3 feet wide, and 4 feet high? A. Multiply the length, breadth, and height together, the product will be the solidity. How do you find the solidity of a cylinder? A. Multiply the area of the base by the perpendicular height, and the product will be the solidity. How do you find the solidity of a globe? A. Multiply the cube of the diameter by ,5236, the product will be the solidity. Why multiply by ,5236? A. Because ,5236 is the solidity of a globe whose diameter is 1.

DUODECIMALS,

Is a rule much used by workmen and artificers, in computing contents of their work. The rule has derived its name from the Latin word *duodecim*, which signifies *twelve*.

A foot, which is called an integer, is divided *duodecimally*, that is, into twelve parts, called inches or primes; an inch or prime is divided into twelve parts, called seconds; a second is divided into twelve parts, called thirds, and so on. But *dimensions* are usually taken in feet, inches, and quarters; the parts smaller than these are generally neglected, being of little or no consequence.

RULE.—1st. Set down the two given dimensions, i. e. length and breadth, one under the other, so that feet may stand under feet, inches under inches, &c.

2d. Multiply each term in the multiplicand, beginning at the lowest, by the feet in the multiplier, and set the result of each directly under its corresponding term, observing to carry 1 for every 12, from the inches to the feet.

3d. In like manner, multiply all the multiplicand by the inches and parts of the multiplier, and set the result of each term one place removed to the right hand of those in the multiplicand; omitting, however, what is below the parts of inches, only carrying to these the proper number of units from the lowest denomination.

Or, instead of multiplying by the inches, take such parts of the multiplicand as there are like parts of a foot in the inches. Then add the products together, as in Compound Addition, carrying 1 to the feet for every 12 inches; the result will be the answer, or area, in square feet and inches.

EXAMPLES.

1. How many square feet in a board, 14 feet 9 inches long, and 2 feet 6 inches wide?

Ans. 36 feet, 10½ inches.

	ft.	*in.*			*ft.*
½)	14	9			14,75
	2	6			2,5
	29	6	*or thus,*		7375
	7	4½			2950
Ans.	36	10½		Ans.	36,875
					12
				Inches,	10,500

Dem.—It is plain, in the first operation, that multiplying the multiplicand, 14 feet 9 inches, by the feet in the multiplier, is repeating the multiplicand by 2 units, the same as in Compound Multiplication; hence this rule is sometimes called *Cross Multiplication*, because we first multiply by the units or left hand denomination of the multipliers. Next, it is evident that we may take half of the multiplicand, for the product of the multiplicand multiplied by 6 inches, or one half of a unit, because that part of the multiplier is one half of a unit, or foot; and multiplying by one half, is taking one half of the multiplicand; hence the reason of our rule is obvious.

2. Multiply 48 feet 6 inches by 10 feet 9 inches.

in.	*ft.*	*in.*
6=½)	48	6
	10	9
	485	0
3=½)	24	3
	12	1½
Ans.	521ft.	4½in.

3. Multiply 14 feet 9 inches by 4 feet 3 inches.

	ft.	*in.*
3=¼)	14	9
	4	3
	59	0
	3	8¼
Ans.	62ft.	8¼in.

4. Multiply 4 feet 7 inches by 9ft. 6in. Ans. 43ft. 6½in.

5. What is a marble slab worth, the length of which is 5 feet 6 inches, and the breadth 1 foot 9 inches, at 75 cents per square foot? Ans. $7,21 cents 8 mills.

6. What will the painting of a floor come to, at 10 cents per square yard, allowing the floor to be 21 feet 8 inches long, and the breadth 14 feet 10 inches? Ans. $3,57cts.

Note.—Divide square feet by 9, and the quotient will be sq. yards.

7. Required the solid of cubick feet in a wall that is 53 feet 6 inches in length, 12 feet 3in. in height, and its thickness 2 feet.

	ft.	*in.*	
3=¼)	53	6	length.
	12	3	height.
	642	0	
	13	4½	
	655	4½	
	2		thickness.
Ans.	1310ft.	9in.	

Dem.—It is plain, that we must multiply the three dimensions together for the solidity; we then multiply together the two first dimensions, as in the preceding examples, and then the product multiplied by the other dimension, must evidently give the solidity, for it is multiplying length, breadth, and thickness together.

8. How many cubick feet in a load of wood, 8 feet long, 3 feet wide, and 2 feet 8 inches high? Ans. 64 feet.

9. How many solid feet of timber in a stick which is 40 feet 8 inches long, 2 feet 6 inches wide, and 2 feet 4 inches thick? Ans. 237 feet $2\frac{2}{3}$ inches.

QUESTIONS ON DUODECIMALS.

What is the use of Duodecimals? A. It is a rule by which artificers and workmen find the contents of their work. How do you multiply? A. First by the units of the multiplier, and then by the parts of a unit. When length and breadth are given, how do you find the area? A. By multiplying the length by the breadth. When length, breadth, and thickness are given, how do you find the solidity? A. By multiplying the length, breadth, and thickness together.

VULGAR FRACTIONS.

Having briefly treated of Vulgar Fractions immediately after the Compound Rules, and given some general definitions, and a few such problems as were necessary to give a limited idea, only, of Fractions, the learner is, therefore, requested to read again those general definitions on page 119.

Vulgar Fractions are either *proper*, *improper*, *compound*, or *mixed*.

1. A *proper* or *simple fraction* is when the numerator is less than the denominator, as $\frac{1}{2}$, $\frac{3}{4}$, $\frac{5}{8}$, &c.

2. An *improper fraction*, is when the numerator is equal to, or greater than the denominator, as $\frac{5}{4}$, $\frac{8}{8}$, $\frac{12}{4}$, &c.

3. A *compound fraction*, is a fraction of a fraction, connected by the word *of*, as $\frac{2}{3}$ of $\frac{7}{8}$, $\frac{3}{4}$ of $\frac{11}{12}$, of $\frac{6}{8}$, &c.

4. A *mixed number* consists of a whole number and a fraction, as $3\frac{1}{3}$, $24\frac{2}{12}$, $8\frac{7}{9}$, &c.

A whole number may be expressed like a fraction by drawing a line under it, and placing 1 directly below for a denominator, as $9=\frac{9}{1}$, and $15=\frac{15}{1}$, &c.

CASE I.—*To reduce a fraction to its lowest terms.*

RULE.—Divide the terms of the given fraction by any number that will divide them without a remainder; then divide these quotients again in the same manner; and so on, till it appears there is no number greater than 1 which will divide them; then the fraction will be in its lowest terms.

EXAMPLES.

1. Reduce $\frac{48}{96}$ to its lowest terms. Ans. $\frac{1}{2}$

$6)$
$8)\frac{48}{96}=\frac{6}{12}=\frac{1}{2}$ Ans.

DEM.—Dividing the numerator and denominator both by the same number, never alters the value of a fraction; thus, the fraction $\frac{48}{96}$ is equal to $\frac{6}{12}$, and $\frac{6}{12}$ is equal to $\frac{1}{2}$, consequently, $\frac{1}{2}$ is equal to $\frac{48}{96}$; the operation only alters the terms of the fraction, not its value.

2. Reduce $\frac{60}{125}$ to its lowest terms. Ans. $\frac{12}{25}$.
3. Reduce $\frac{195}{780}$ to its lowest terms Ans. $\frac{1}{4}$.
4. Reduce $\frac{23}{46}$ to its lowest terms. Ans. $\frac{1}{2}$.
5. Reduce $\frac{1344}{1536}$ to its lowest terms. Ans. $\frac{7}{8}$.
6. Reduce $\frac{525}{630}$ to its lowest terms. Ans. $\frac{5}{6}$.

CASE II.—*To reduce a mixed number to its equivalent improper fraction.*

RULE.—Multiply the integer, or whole number, by the denominator of the fraction, and to the product add the numerator; then set that sum above the denominator for the fraction required.

EXAMPLES.

1. Reduce $15\frac{5}{8}$ to an improper fraction. Ans. $\frac{125}{8}$.

$15\frac{5}{8}$
8
120
5 the numerator added.
125 new numerator.
8 denominator.

Proof.
$8)125$
$15\frac{5}{8}$

DEM.—It is plain, when we multiply the whole number by the denominator of the fraction and add the numerator, that the sum expresses eighths, and placing the denominator below, we have a new fraction, equal in value to the mixed number, as the proof plainly shows.

2. Reduce $12\frac{7}{9}$ to an improper fraction. Ans. $\frac{115}{9}$.
3. Reduce $54\frac{11}{12}$ to an improper fraction. Ans. $\frac{659}{12}$.
4. Reduce $183\frac{5}{21}$ to an improper fraction. Ans. $\frac{3848}{21}$.
5. Reduce $199\frac{11}{12}$ to an improper or equivalent fraction. Ans. $\frac{2399}{12}$.
6. Reduce $89\frac{7}{8}$ to an improper fraction. Ans. $\frac{719}{8}$.

CASE III.—*To reduce an improper fraction to a whole or mixed number.*

RULE.—Divide the numerator by the denominator, and the quotient will be the answer sought in a whole or mixed number.

EXAMPLES.

1. Reduce $\frac{144}{12}$ to a whole or mixed number. Ans. 12.

$$12)\underline{144}$$
$$12 \text{ Ans.}$$

Dem.—The numerator showing how many parts the fraction contains, and the denominator showing how many of those parts it requires to make a unit, the rule is obvious from the principles of Division. This being the reverse of the preceding Case, they prove each other.

2. Reduce $\frac{24}{7}$ to its equivalent number. Ans. $3\frac{3}{7}$.
3. Reduce $\frac{749}{17}$ to its equivalent number. Ans. $44\frac{1}{17}$.
4. Reduce $\frac{3848}{21}$ to its equivalent number. Ans. $183\frac{5}{21}$.
5. Reduce $\frac{2399}{12}$ to its equivalent whole or mixed number. Ans. $199\frac{11}{12}$.

Case IV.—*To reduce a whole number to an equivalent fraction, having a given denominator.*

RULE.—Multiply the whole number by the given denominator; then set the product over the said denominator, and it will be the fraction required.

EXAMPLES.

1. Reduce 8 to a fraction whose denominator shall be 6.

Here $8 \times 6 = 48$; then $\frac{48}{6}$ is the Ans. for $48 \div 6 = 8$, Proof.

Dem.—Multiplication and Division, being equally used, the result must evidently be the fraction proposed; and that the fraction $\frac{48}{6}$ is equal to 8, is also evident from the proof.

2. Reduce 14 to a fraction whose denominator shall be 12. Ans. $\frac{168}{12}$.
3. Reduce 125 to a fraction whose denominator shall be 7. Ans. $\frac{875}{7}$.
4. Reduce 29 to a fraction whose denominator shall be 15. Ans. $\frac{435}{15}$.

Case V.—*To reduce a compound fraction to a simple or improper fraction.*

RULE.—Multiply all the numerators together for a new numerator, and all the denominators together for a new denominator, and they will form the fraction required.

Note.—When part of a compound fraction is a whole or mixed number, it must first be reduced to an equivalent or improper fraction by one of the preceding cases.

EXAMPLES.

1. Reduce $\frac{1}{4}$ of $\frac{1}{2}$ to a simple fraction. Ans. $\frac{1}{8}$.

$\frac{1}{4} \times \frac{1}{2} = \frac{1}{8}$ the Ans.

Dem.—The truth of this rule is evident, because $\frac{1}{4}$ of 1 is $\frac{1}{4}$, then $\frac{1}{4}$ of $\frac{1}{2}$ must be $\frac{1}{8}$ of 1.

When a compound fraction consists of more than two single ones, having reduced two of them as above, the resulting fraction and a third will be the same as a compound fraction of two parts; and so on, to the last of all.

2. Reduce $\frac{4}{5}$ of $\frac{5}{4}$ of $\frac{7}{9}$ to a single fraction. Ans. $\frac{140}{300}=\frac{7}{15}$.

3. Reduce $\frac{3}{12}$ of $\frac{14}{15}$ of $\frac{8}{17}$ of 20 to a single fraction.
Ans. $\frac{672}{306}=2\frac{10}{51}$.

4. Reduce $\frac{1}{2}$ of $\frac{12}{13}$ of $33\frac{1}{3}$ to a single fraction.
Ans. $15\frac{5}{13}$.

5. Reduce $\frac{2}{5}$ of $\frac{5}{8}$ of $3\frac{1}{2}$ to a single fraction. Ans. $\frac{7}{8}$.

CASE VI.—*To reduce fractions of different denominators, to equivalent fractions having a common denominator.*

RULE.—Multiply each numerator by all the denominators except its own, for the new numerator; and multiply all the denominators together for a common denominator.

NOTE.—When any part of the given sum consists of mixed numbers or compound fractions, they must be first reduced to improper or single fractions.

EXAMPLES.

1. Reduce $\frac{1}{2}$, $\frac{3}{4}$ and $\frac{7}{8}$ to equivalent fractions having a common denominator.

$1\times4\times8=32$ the new numerator for $\frac{1}{2}$
$3\times2\times8=48$ ditto. for $\frac{3}{4}$
$7\times2\times4=56$ ditto. for $\frac{7}{8}$
$2\times4\times8=64$ the common denominator.

Therefore, the new equivalent fractions are $\frac{32}{64}$, $\frac{48}{64}$, and $\frac{56}{64}$, the answer sought.

DEM.—The reason of this rule is plain, because it is evidently multiplying each numerator and its denominator by the same numbers, and consequently the value of the fraction is not altered, for by reducing our new fractions to their lowest terms, we would again have our given fractions.

2. Reduce $\frac{2}{7}$ and $\frac{5}{9}$ to fractions of a common denominator.
Ans. $\frac{18}{63}$, $\frac{35}{63}$.

3. Reduce $\frac{5}{6}$, $2\frac{3}{5}$, and 4, to a common denominator.
Ans. $\frac{25}{30}$, $\frac{78}{30}$, $\frac{120}{30}$.

4. Reduce $\frac{7}{8}$, $\frac{2}{3}$, $\frac{6}{7}$ and $\frac{1}{5}$ to a common denominator.
Ans. $\frac{735}{840}$, $\frac{560}{840}$, $\frac{720}{840}$, and $\frac{168}{840}$.

5. Reduce $\frac{1}{5}$, $\frac{5}{7}$, $\frac{9}{10}$ and $\frac{1}{2}$ to a common denominator.
Ans. $\frac{28}{140}$, $\frac{100}{140}$, $\frac{126}{140}$, $\frac{70}{140}$.

NOTE.—It will often be found more convenient, to reduce fractions to their lowest terms, before reducing them to a common denominator.

CASE VII.—*To find the value of a fraction in the inferiour denominations of the integer.*

RULE.—Multiply the numerator by the parts in the next inferiour denomination, and divide the product by the denominator. Then if any thing remains, multiply it by the parts in the next inferiour denomination, and divide by the denominator as before, and so on, as far as necessary; then the quotients, placed in order, will be the value of the fraction required.

EXAMPLES.

1. Find the value of $\frac{7}{40}$ of a pound sterling. Ans. 3s. 6d.

```
      7
     20
 40)140(3s. 6d. Ans.
    120
    ---
     20
     12
    ---
    240
    240
    ---
      0
```

DEM.—The numerator of a fraction being considered as a remainder, in Division, and the denominator as the divisor, it is plain, that the operation here, is only continuing the division in the inferiour denominations of the integer.

2. What is the value of $\frac{3}{8}$ of a shilling? Ans. 4½d.

3. What is the value of $\frac{4}{5}$ of a pound Troy? Ans. 9oz. 12pwt.

4. Find the value of $\frac{3}{10}$ of a day. Ans. 7h. 12m.

5. What is the value of $\frac{7}{8}$ of an acre? Ans. 3 roods 20 poles.

6. Find the value of $\frac{3}{5}$ of a dollar. Ans. 60 cents.

7. Reduce $\frac{4}{7}$ of a mile to its proper quantity. Ans. 4fur. 22rds. 4yds. 2ft. 1in. $2\frac{1}{7}$b. c.

CASE VIII.—*To reduce a fraction from one denomination to another, retaining the same value.*

RULE.—Consider how many of the less denomination make one of the greater; then multiply the numerator by that number, if the reduction be to an inferiour denomination, but multiply the denominator, if to a superiour denomination.

EXAMPLES.

1. Reduce $\frac{3}{12}$ of a pound to the fraction of a penny. Ans. $\frac{60}{1}$.

12)

$\frac{3}{12}\times\frac{20}{1}\times\frac{12}{1}=\frac{720}{12}=\frac{60}{1}$, the answer.

DEM.—The reason of this rule is obvious, for it is the same as the rule of Reduction in whole numbers from one denomination to another.

2. Reduce $\frac{4}{5}$ of a penny to the fraction of a pound.
Ans. $\frac{4}{1200}=\frac{1}{300}$.

$\frac{4}{5}\times\frac{1}{12}\times\frac{1}{20}=\frac{4}{1200}=\frac{1}{300}$, the answer.

3. Reduce $\frac{4}{1200}$ of a pound to the fraction of a penny.
Ans. $\frac{4}{5}$.

4. Reduce 2s. 6d. to the fraction of a pound. Ans. $\frac{1}{8}$.

5. Reduce $\frac{1}{126}$ of a cwt. to the fraction of a pound.
Ans. $\frac{8}{9}$.

6. Reduce $\frac{3}{4}$ of a yard to the fraction of an Ell English.
Ans. $\frac{3}{5}$.

It is often better to set the sum down as a compound fraction, and obtain the answer by comparing, thus:

$\frac{3}{4}$ of $\frac{4}{1}$ of $\frac{1}{5}=\frac{12}{20}=\frac{3}{5}$ the answer.

7. Reduce $\frac{3}{5}$ of an Ell English to the fraction of a yard.

$\frac{3}{5}$ of $\frac{5}{1}$ of $\frac{1}{4}=\frac{15}{20}=\frac{3}{4}$yd. the answer.

8. Reduce $\frac{1}{4}$ of a day to the fraction of a year.

$\frac{1}{4}$ of $\frac{1}{365}=\frac{1}{1460}$ Ans.

9. Reduce $4\frac{1}{2}$ furlongs to the fraction of a mile. Ans. $\frac{9}{16}$.

ADDITION OF VULGAR FRACTIONS,

Is collecting in one sum, broken numbers.

RULE.—Reduce compound fractions to single ones, mixed numbers to improper fractions; fractions of different integers to those of the same; and all of them to a common denominator; then the sum of the numerators written over the common denominator will be the sum of the fractions required.

EXAMPLE.

1. Add $\frac{5}{9}$ and $\frac{5}{6}$ together. Ans. $\frac{75}{54}=1\frac{21}{54}$.

5×6=30, numerator of $\frac{5}{9}$.
5×9=45, numerator of $\frac{5}{6}$.
75, sum of the numerators.
9×6=54, common denominator.
Then $\frac{30}{54}+\frac{45}{54}=\frac{75}{54}$.

DEM.—Before fractions are reduced to a common denominator, they are quite dissimilar, because in the first fraction, $\frac{5}{9}$, a unit is divided into 9 parts, and in the second fraction, $\frac{5}{6}$, a unit is divided into 6 parts; consequently the parts are unequal until reduced to a common denominator, then we have $\frac{30}{54}$ and $\frac{45}{54}$, which make $\frac{75}{54}$. That we do not alter the value of the fraction by the operation is evident, because we multiply the numerator and denominator by the same number; thus,

$$\frac{5\times6}{9\times6}=\frac{30}{54} \text{ and } \frac{5\times9}{6\times9}=\frac{45}{54}$$

Now it is plain, that a unit in each fraction is divided into

54 parts, and consequently the numerators may be added, because they are parts of one common integer.

2. Add $\frac{3}{4}$ and $\frac{8}{9}$ together. Ans. $1\frac{23}{36}$.
3. Add $\frac{6}{9}$, $\frac{5}{6}$ and $\frac{3}{4}$ together. Ans. $2\frac{2}{45}$.
4. Add $4\frac{3}{8}$ and $9\frac{5}{8}$ together. Ans. 14.
5. What is the value of $\frac{3}{4}$ of a shilling and $\frac{7}{8}$ of a pound? Ans. 18s. 3d.
6. What is the value of $\frac{2}{3}$ of a pound, and of a shilling? Ans. $\frac{125}{9}$s. or 13s. 10d. $2\frac{2}{3}$ qr.
7. Add $\frac{1}{3}$ of a week, $\frac{1}{4}$ of a day, and $\frac{1}{2}$ of an hour together. Ans. 2d. 14h. 30min.
8. Add $\frac{1}{7}$ of a pound, $\frac{2}{9}$ of a shilling, and $\frac{5}{12}$ of a penny. Ans. 3s. 1d. $1\frac{10}{21}$qr.
9. Add $\frac{7}{10}$ of a furlong to $\frac{3}{4}$ of a mile. Ans. 6 furlongs 28 poles.
10. Add $\frac{3}{4}$ of a shilling to $\frac{4}{15}$ of a penny. Ans. 9d. $1\frac{1}{15}$qr.

SUBTRACTION OF VULGAR FRACTIONS,

Is taking one broken number from another.

RULE.—Prepare the fractions the same as in Addition; then subtract one numerator from the other, and set the remainder over the common denominator, for the difference of the fractions sought.

NOTE.—When the given fractions have a common denominator, they may be added and subtracted without any further reducing.

EXAMPLES.

1. What is the difference between $\frac{5}{6}$ and $\frac{2}{6}$?

Here, $\frac{5}{6}-\frac{2}{6}=\frac{3}{6}=\frac{1}{2}$, the answer.

DEM.—It is evident, that the difference between $\frac{5}{6}$ and $\frac{2}{6}$ is $\frac{3}{6}$ or $\frac{1}{2}$, because the fractions have a common denominator; that is, a unit in each fraction is divided into six equal parts, therefore the difference of the numerators placed over the common denominator, must express the difference of the fractions.

2. What is the difference between $\frac{7}{8}$ and $\frac{2}{3}$?

$7\times3=21$ } $\frac{7}{8}=\frac{21}{24}$, and $\frac{2}{3}=\frac{16}{24}$, therefore,
$2\times8=16$ } $\frac{21}{24}-\frac{16}{24}=\frac{5}{24}$, the answer.
$8\times3=24$ com. denom. } DEM.—The same reason for the reduction of the fractions, might be offered here, as in Addition, hence the operation of the work is manifest.

3. From $13\frac{1}{3}$ take $\frac{3}{4}$ of 15. Ans. $2\frac{1}{12}$.
4. From $\frac{7}{12}$ of a pound take $\frac{2}{3}$ of a shilling Ans. 11s.
5. From $\frac{1}{2}$ of a shilling take $\frac{3}{4}$ of a penny. Ans. $5\frac{1}{4}$d.
6. From $\frac{5}{9}$ of a pound take $\frac{2}{3}$ of $\frac{3}{4}$ of a shilling.
Ans. 10s. 7d. $1\frac{1}{3}$qr.
7. From $\frac{2}{7}$ of $5\frac{1}{6}$ of a pound, take $\frac{3}{5}$ of a shilling.
Ans. £1 8s. $11\frac{3}{35}$d.
8. From $\frac{2}{3}$ of a league, take $\frac{7}{10}$ of a mile.
Ans. 1m. 2fur. 16pol.
9. From 35 days, take $19\frac{4}{5}$ days. Ans. 15d. 4h. 48min.
10. From $\frac{1}{4}$ of a pound, take $\frac{9}{10}$ of a shilling.
Ans. 4s. $1\frac{1}{5}$d.

MULTIPLICATION OF VULGAR FRACTIONS,

Is repeating a whole or broken number, by a part or the parts of an integer.

RULE.—Reduce mixed numbers, if there be any, to equivalent fractions; then multiply all the numerators together for a new numerator, and all the denominators together for a new denominator, which will give the product required.

EXAMPLES.

1. Multiply $\frac{1}{2}$ by $\frac{3}{4}$. $\frac{1}{2}\times\frac{3}{4}=\frac{3}{8}$, the Answer.

DEM.—It is evident, that as many times as the numerator of a fraction is increased, so many times the value of the fraction is increased; then when we multiply the numerator of the fraction, $\frac{1}{2}$, by 3, the fraction is increased 3 times, but we do not wish to increase the value of the fraction 3 times, but as much less as the denominator 4, indicates; then when we multiply the denominator of the fraction by 4, it makes the value of the fraction 4 times less, because it takes 4 times the number of parts to make a unit. Multiplying the denominator of a fraction by any number, is the same as dividing the numerator by the same number.

2. What is the product of $\frac{2}{7}$ and $\frac{5}{8}$? Ans. $\frac{10}{56}=\frac{5}{28}$.
3. Required the product of $\frac{5}{7}$ and $\frac{9}{11}$. Ans. $\frac{45}{77}$.
4. Required the product of $12\frac{1}{3}$ and $16\frac{1}{7}$. Ans. $199\frac{2}{21}$.
5. What is the product of 40 multiplied by $\frac{1}{4}$? Ans. 10.
6. Multiply 6 by $\frac{2}{3}$ of 5. Ans. 20.
7. Multiply 12 by $\frac{3}{4}$ of $\frac{4}{5}$. Ans. $7\frac{1}{5}$.
8. Required the product of $\frac{1}{2}$, $\frac{2}{3}$, and 3. Ans. 1.
9. What is the product of $3\frac{2}{7}$ and $4\frac{14}{33}$? Ans. $14\frac{124}{231}$.
10. Required the product of $12\frac{3}{5}$ multiplied by 7.
Ans. $88\frac{1}{5}$.

DIVISION OF VULGAR FRACTIONS,

Is finding how often a part or the parts of an integer is contained in a given sum.

RULE.—Prepare the fractions as in Multiplication; then invert the divisor, and proceed exactly as in Multiplication; the products will be the quotient required.

EXAMPLES.

1. Divide 20 by $\frac{1}{2}$. Ans. 40.

$\frac{20}{1}+\frac{1}{2}$. Invert the divisor, and it stands thus, $\frac{20}{1}\times\frac{2}{1}=\frac{40}{1}=40$, the Answer.

DEM.—It is evident, that division of fractions might be performed by dividing the numerator of the dividend by the numerator of the divisor, and the denominator of the dividend by the denominator of the divisor; but as it would often give fractional results for a new numerator and denominator, it is not employed, since we can adopt an easier method, going on the principle, that Division is exactly the reverse of Multiplication; therefore we invert the terms of the divisor and proceed exactly as in Multiplication; and to prove the truth of the rule, we need only invert the reasoning in Multiplication, because in Multiplication the work is nothing more than multiplying the multiplicand by the numerator of the multiplier, and dividing the product by the denominator of the multiplier; and in Division, the work is nothing more than multiplying the dividend by the denominator of the divisor, and dividing the product by the numerator of the divisor.

2. Divide $\frac{4}{7}$ by $\frac{2}{5}$. Ans. $\frac{6}{7}$.
3. Divide 9 by $\frac{6}{7}$. Ans. $10\frac{1}{2}$.
4. Divide $5\frac{1}{8}$ by $7\frac{3}{4}$. Ans. $\frac{41}{62}$.
5. Divide $\frac{3}{5}$ by 2. Ans. $\frac{3}{10}$.
6. Divide 5 by $\frac{7}{10}$. Ans. $7\frac{1}{7}$.
7. Divide $\frac{3}{4}$ of $\frac{7}{8}$ by $\frac{5}{8}$ of 2. Ans. $\frac{21}{40}$.
8. Divide $7\frac{1}{3}$ by $9\frac{5}{9}$. Ans. $\frac{33}{43}$.
9. Divide $\frac{3}{5}$ by 2. Ans. $\frac{3}{10}$.
10. Divide $\frac{5}{9}$ of 4 by $4\frac{5}{9}$. Ans. $\frac{20}{41}$.

Rule of Three Direct in Vulgar Fractions.

RULE.—Prepare the given terms, as in Multiplication, and state the question the same as in whole numbers; then multiply the second and third terms together, and divide the product by the first: Or, invert the first term, and multiply the three together, as in Multiplication; the last product will be the answer, in the same name of the second term.

EXAMPLES.

1. If $\frac{1}{4}$ of a yard cost $\frac{3}{9}$ of a pound; what will $\frac{7}{8}$ of a yard cost? $\frac{1}{4}$yd. : £$\frac{3}{9}$ :: $\frac{7}{8}$: to the Answer. The better way, however, is to invert the first term, thus $\frac{4}{1}$yd. × : $\frac{3}{9}$ × :: $\frac{7}{8}$: = £$\frac{84}{72}$ = £1 3s. 4d., the Answer.

DEM.—Inverting the first term is exactly the same as multiplying the second and third terms together and dividing by the first, as plainly appears from Division of Vulgar Fractions, therefore the same reasoning might be offered *here* as in the Rule of Three Direct in whole numbers, so that the reason of our rule is obvious.

2. If $\frac{9}{15}$ of a ship cost 150 pounds; what are $\frac{2}{28}$ of her worth? Ans. £17 17s. 1d. $2\frac{6}{7}$qrs.

3. If 1 yard cost $\frac{5}{6}$ of a pound; what will 42 yards cost? Ans. £35.

4. If $\frac{1}{4}$ of a yard cost $\frac{2}{3}$ of a shilling; what will $\frac{7}{16}$ of a yard cost? Ans. 1s. 2d.

5. If $\frac{3}{8}$ of a yard cost $\frac{4}{6}$ of a dollar; what will $44\frac{1}{2}$ yards cost? Ans. $79,11 cents $1\frac{1}{9}$ mills.

6. If 3 yards cost $2\frac{4}{5}$ pounds; what will $14\frac{3}{4}$ yards cost? Ans. £13 15s. 4d.

Rule of Three Inverse in Vulgar Fractions.

RULE.—Prepare the fractions and state the question as in the RULE OF THREE DIRECT; then invert the third term, and multiply all the three terms together; the product will be the answer in the same name of the second term.

EXAMPLES.

1. How much cloth $\frac{3}{4}$ of a yard wide, will line 15 yards of cloth, $1\frac{1}{2}$ yard wide? Ans. 30 yards.

$\frac{3}{2}$yd. : $\frac{15}{1}$yds. :: $\frac{3}{4}$yd. Invert the third term thus, as $\frac{3}{2}$yd : $\frac{15}{1}$yds. :: $\frac{4}{3}$yd. : 30 yds. the Answer.

DEM.—Inverting the third term is the same as multiplying the first and second terms together and dividing the product by the third; hence the reason of the rule is obvious.

2. What length of board $7\frac{3}{4}$ inches wide, will make a square foot, or as much as another piece of 12 inches long and 12 broad? Ans. $18\frac{18}{31}$ inches.

3. A regiment of soldiers, consisting of 980 men, are to be clothed anew; each coat to contain $2\frac{1}{2}$ yards of cloth that is $1\frac{5}{3}$ yard wide, and lined with shalloon $\frac{7}{8}$ yard wide; how many yards of shalloon will it take to line them? Ans. 4550 yards.

4. If a suit of clothes can be made of $4\frac{1}{8}$ yards of cloth, $1\frac{3}{8}$ yard wide; how many yards of coating $\frac{7}{8}$ of a yard wide, will it require for the same person? Ans. 6yds. 1qr. $3\frac{5}{7}$n.

5. If 3 men finish a piece of work in $4\frac{1}{2}$ hours, in how many hours will 10 men do the same work? Ans. $1\frac{7}{20}$h.

6. How much in length, of a piece of land that is $11\frac{11}{12}$ poles broad, will make an acre? Ans. $13\frac{61}{143}$ poles.

QUESTIONS ON VULGAR FRACTIONS.

Note.—In asking the questions on Vulgar Fractions, the teacher should commence with those following the introduction to Vulgar Fractions, page 123, and end with the following questions:

How do you reduce fractions to a common denominator? A. By multiplying each numerator into all the denominators except its own, for the new numerators, and multiplying all the denominators together for a common denominator. Why does not that operation alter the value of the fractions? A. Because it is multiplying each numerator and its denominator by the same numbers, and consequently the value of each fraction remains the same. What is the use of reducing fractions to a common denominator? A. In order to prepare fractions for adding or subtracting, as the case may be. Until the fractions are reduced to a common denominator, they cannot be added or subtracted, because an integer is unequally divided, as our denominators show, consequently the unequal parts cannot be placed in one column for adding or subtracting. What is the effect, if you multiply the numerator of a fraction by any number, the denominator remaining unchanged? A. It increases the value of a fraction as many times as the multiplier expresses a unit. If you multiply the denominator of a fraction by any number, the numerator remaining unchanged, what is the effect? A. It diminishes the value of the fraction as many times as the multiplier expresses a unit. Multiplying or dividing the numerator and denominator of the fraction by the same number; what is the effect? A. It alters the terms of the fraction, not its value. If you divide the numerator of a fraction by any number, the denominator remaining unchanged; what is the effect? A. It lessens the value of the fraction as many times as the divisor expresses a unit. If you divide the denominator of a fraction by any number, the numerator remaining unchanged; what is the effect? A. It increases the value of the fraction as many times as the divisor expresses a unit.

ARITHMETICAL PROGRESSION,

Is any rank or series of numbers, more than two, increasing or decreasing by a common difference.

When the numbers increase by a continual *addition* of the common difference, they form an *ascending series;* but when they decrease by a continual *subtraction* of the common differenee, they form a *descending series.*

Thus, { 2, 4, 6, 8, 10, 12, &c. is an ascending series.
12, 10, 8, 6, 4, 2, &c. is a descending series.

The numbers which form the series are called the TERMS of the progression. THE FIRST and LAST terms are the EXTREMES, and the other terms are called the MEANS.

In Arithmetical Progression there are five terms, three of which are always given in the question.

1st. The first term. 3d. The number of terms.

2d. The last term. 4th. The common difference. 5th the sum of all the terms. Any three of which being given, the other two may be found.

CASE I.—*The first term, common difference, and number of terms given, to find the last term.*

RULE.—Multiply the number of terms, less 1, by the common difference, and add the first term to the product, the sum will then be the last term, or answer sought.

EXAMPLES.

1. A merchant sold 200 yards of cloth; for the first yard he received 4 cents, 7 cents for the second, 10 cents for the third, and so on, with the common difference of three cents; what did he receive for the last yard?

Ans. 601 cents=$6,01 cent.

199 the number of terms less 1.
3 the common difference.
597
4 the first term added.
Ans. 601 cents, he received for the last yard.

DEM.—It is plain, that each term exceeds that preceding it by the common difference; it is then evident, that the last term exceeds the first by as many times the common difference, as there are terms after the first; therefore the last term must equal the first, and the number of terms, less one, repeated by the common difference, as our example plainly shows.

2. A man put out $100, at 7 per cent, simple interest, which amounted to $107 in a year, $114, in 2 years, and so on, in arithmetical progression, with the common difference of $7; what was the amount due at the expiration of 50 years? Ans. $450.

3. John owes William a certain sum, to be paid in arithmetical progression; the first payment is 6 pence, the number of payments 52, and the common difference of the payments is 12 pence; what is the last payment?

Ans. £2 11s. 6d.

CASE II.—*The first term, common difference, and the number of terms given, to find the sum of all the terms.*

RULE.—Multiply half the sum of the extremes by the number of terms, and the product will be the answer or sum of all the terms.

EXAMPLES.

1. A person bought 16 yards of cloth; for the first yard he gave 5 pence, for the second 9 pence, and so on, in arithmetical progression; what did the last yard cost him, and what did he pay for the whole?

Ans. 65 pence the cost of the last yard, obtained by the preceding case.
5 the cost of the first yard.
2)70 the sum of the extremes.
35 half the sum of the extremes.
16 the number of terms.
210
35
12)560 the sum of all the terms.
2|0)4|6 8d.
A. £2 6s. 8d. the cost of 16 yards.

Dem.—The price of each succeeding yard being increased by a constant excess, it is plain that the *average* price is as much less than the price of the *last* yard, as it is *greater* than the price of the *first* yard; hence it is evident, that half the sum of the *first* and *last* is the average price, and it is also obvious, that the average price multiplied by the whole number of yards, that is, terms, gives the price of the whole number of yards.

2. A man bought 17 yards of Irish linen; for the first yard he gave 2 shillings, for the last, 10 shillings, the price of each yard increasing in arithmetical progression; how much did the whole amount to? Ans. £5 2s.

3. How many times does the hammer of a clock strike in 12 hours? Ans. 78 times.

Note.—It is supposed that every scholar knows the first term and common difference of the hammer of a clock's striking.

Case III.—*The extremes and number of terms being given, to find the common difference.*

RULE.—Divide the difference of the extremes by the number of terms, less 1, and the quotient will be the common difference.

EXAMPLES.

1. A merchant sold 200 yards of cloth; for the first yard he received 4 cents, for the last, 601 cents; the common difference is required. Ans. 3, common difference.

200 601 price of the last yard.
1 4 price of the first yard.
199) 597(3 common difference.
597

Dem.—It is plain, that the difference between the price of the first yard, and the price of the last yard, that is, the difference of the extremes, is equal to the additions which were made to the price of the first yard till it became equal to the last yard, and these additions were one less than the number of terms, therefore it is evident, that the sum of these additions, that is, the difference of the extremes, divided by the number of terms, less one, that is, the number of additions, gives the common difference.

2. The first term is 3, the last term 48, and the number of terms 10; what is the common difference? Ans. 5.

3. A gentleman has 7 sons, whose several ages differ alike; the youngest is 5 years old, the eldest 47; what is the common difference of their ages? Ans. 7 years.

QUESTIONS ON ARITHMETICAL PROGRESSION.

What is Arithmetical Progression? A. It is any rank or series of numbers, increasing or decreasing by a common difference. What are numbers called when they increase by a continual addition? A. An ascending series. What are numbers called when they decrease by a continual subtraction? A. A descending series. When the first term, the common difference, and the number of terms are given, how do you find the last term? A. By multiplying the number of terms, less one, by the common difference, and adding the first term to the product for the last term. When the first term, common difference, and number of terms are given, how do you find the sum of all the terms? A. Multiplying one half the sum of the extremes by the number of terms, gives the answer or sum of all the terms. How do you find the common difference, when the extremes and number of terms are given? A. Dividing the difference of the extremes by the number of terms, less one, gives the common difference.

GEOMETRICAL PROGRESSION,

Is any rank or series of numbers increasing by one common multiplier as 2, 4, 8, 16, 32, &c. or decreasing by a common divisor; as 32, 16, 8, 4, 2, &c. First, the multiplier, 2, by which the series is increased, is called the ratio; secondly, the divisor, 2, by which the series is diminished, is called the ratio.

When any number of terms is continued in Geometrical Progression, the product of the two extremes will be equal to the product of any two means equally distant from the extremes, or when the terms are odd, equal to the square of the middle term; thus, 2, 4, 8, 16, 32; 2×32=64, 4×16=64, 8×8=64.

There are five terms given in Geometrical Progression, the same as in Arithmetical Progression, viz.:

1. The first term.
2. The last term.
3. The number of terms.
4. The ratio or common difference.
5. The sum of all the terms.

NOTE.—The first and last terms are called the extremes. Any two terms equally distant from the extremes, are called the means.

Any three of these being given, the other two may be found.

The powers of the ratio are frequently denoted by figures placed over them, called *indices:*

Thus,	1	2	3	4	5, &c.	Indices or Arithmetical series
	2,	4,	8,	16,	32, &c.	Geometrical series.

CASE I.—*The first term, ratio, and number of terms being given, to find the last term.*

RULE.—Raise the ratio to a power whose index shall be one less than the number of terms; multiply that power by the first term, and the product will be the last term or answer. Or, write down a few leading powers of the ratio, with their indices over them. Add the most convenient indices, to make an index one less than the number of the term sought. Multiply together the powers belonging to those indices, and their product multiplied by the first term, will be the answer, or term sought.

EXAMPLES.

1. A merchant bought 9 yards of broadcloth, and by agreement was to give what the last yard would come to, reckoning 4 cents for the first yard, 12 cents for the second, 36 cents for the third, and so on, to the last; what did the cloth cost him, paying only the price of the ninth yard?

Ans. $262,44 cents.

$3\times3\times3\times3\times3\times3\times3\times3\times4=26244$. Ans.

Or thus:

	1st power,	2d p.	3d p.	4th p.
Ratio,	3,	9,	27,	81
				81
				81
				648
8th power,				6561
				4
Ans. or 9th term,				26244

DEM.—The reason of the rule is evident from the manner in which a geometrical series is formed, because in the geometrical series, 2, 4, 8, 16, 32, &c. which has already been given, the ratio being 2, it is plain, that the second term is formed by multiplying the first term by the ratio, the third term, by multiplying the second term by the ratio, and so on; hence it is obvious, that the ratio raised to a power one less than the number of terms, and that power multiplied by the first term, must give the last term, because the same multipliers are used, and whether the first term be first or last used as a factor, can make no difference.

2. A sum of money was divided among 8 persons, the first received $5, the second $15, the third $45, and so on, in three fold ratio; the share of the eighth person is required?

Ans. $10,935, the share of the last.

3. A farmer bargains for 16 sheep, to pay only the price of the last, reckoning one cent for the first, two cents for the second, four for the third, and so on, doubling the price to the last; how much must he pay for them? Ans. $327,68cts.

CASE II.—*The first term, the last term, and the ratio being given, to find the sum of all the terms or series.*

RULE.—Divide the *difference* of the extremes, that is, the difference of the *first* and *last* terms, by the ratio, *less* 1, and the quotient, increased by the *greater term*, will give the *answer* or *sum* of all the terms.

EXAMPLES.

1. If the first term be 5, the last term 3645, and the ratio 3; what is the sum of all the terms? Ans. 5465.

3645 the last term.
5 the first term.
2)3640
1820
3645
Ans. 5465 sum of all the terms.

DEM.—The reason of this rule is obvious from the nature of a geometrical series; thus, if the ratio of any geometrical series be 2, the *difference* of the *greatest* and *least* terms is *equal* to the *sum* of all the terms except the *greatest*; let 2, 4, 8, 16, 32, be a geometrical series whose ratio is 2, and whose sum is 2+4+8+16+32=62, and the difference of the extremes 32—2=30, and it is plain that the difference of the extremes is equal to the sum of all the terms except the last, for 2+4+8+16=30; the last term added to the difference of the extremes makes 32—2=30+32=62: Here dividing the difference of the extremes by the ratio, less one, could make no difference, because it would be dividing by 1; hence when the ratio is 3, the *difference* of the extremes is double the sum of all the terms except the greatest; and when the ratio is 4, the *difference* of the extremes is triple the sum of all the terms except the last, and so on.

2. The daughter of a wealthy gentleman, marrying on a new year's day, he gave her a guinea, promising to triple it on the first day of each month in the year for her portion; what was the daughter's portion? Ans. 265720 guineas.

3. What debt can be paid in a year, by paying one cent the first month, ten cents the second, and so on, increasing each month in a tenfold proportion?

Ans $1111111111,11 cents.

NOTE.—First find the last term, as in Case I. then find the sum of the series.

QUESTIONS ON GEOMETRICAL PROGRESSION.

What is Geometrical Progression? A. It is either increasing series of numbers by one common multiplier, or diminishing a number by one common divisor. When the first term, the ratio, and number of terms are given, how do you find the last term? A. By raising the ratio to a power one less than the number of terms, and multiplying that power by the first term, the product will be the last term. When the first term, the last term, and the ratio are given, how do you find the sum of all the terms? A. By dividing the difference of the extremes by the ratio, less one, and that quotient increased by the greater term, will be the sum of all the terms.

POSITION,

Is a rule for performing certain questions which cannot be worked by the common direct rules, until certain operations are performed, which are required to be done in the conditions of the questions.—This rule is sometimes called False Position, or False Supposition, because there is a supposition of some false numbers made, to work with, the same as if they were the true ones, and by means of these suppositions, the true numbers are discovered. It is sometimes called Trial and Errour, because we proceed by trial of false numbers, and by them find out true ones by a comparison of the errours. Position is divided into two parts, Single and Double.

SINGLE POSITION,

Teaches to resolve such questions as require only one supposition.—The result in this rule is always proportional to the supposition.—

RULE.—Take or assume any number for that which is required, and perform the same operations with it, as are described or performed in the question. Then say, as the result of the operation, is to the position, or number assumed, so is the result in the question, to a fourth term, which will be the answer to the question, or number sought. *Proved*, by adding the several parts of the sum together, and if it agrees with the given sum, the work is right.

EXAMPLES.

1. What number is that, which being multiplied by 7, and the product divided by 6, the quotient may be 42? Ans. 36.

84 the supposed number.
7
6)588
98 : 84 : : 42
42
168
336
98)3528(36 Ans.
294
588
588 no rem.

PROOF.
36
7
6)252
42

DEM.—It is evident, that this rule is nothing more than the Rule of Three, after the operation is performed which is required to be performed in the conditions of the question, because it is obvious, that as the result of the operation, 98, is to the supposed number, 84, which produced it, so is 42 to the true number which has produced 42 by the same operation, which is evident from the proof. It is obvious that we here have a plain statement in the Rule of Three, for as the result of the supposed number, is to the supposed number, so is the result which is given in the question, to the number that produced it; hence the reason of our rule is evident.

2. A, B, and C, buy a quantity of cloth for $340; of which A pays three times more than B, and B four times more than C; what did each pay?

Ans. A paid $240,—B, $80,—and C, $20.

3. A general, after sending out a foraging $\frac{1}{2}$ and $\frac{1}{3}$ of his men, had one thousand remaining; what number of men had he at first? Ans. 6000.

4. A gentleman being asked his age, said, if $\frac{3}{5}$ of the years I have lived, be multiplied by 7, and $\frac{2}{3}$ of them be added to the product, the sum will be 219; what was his age?

Ans. 45 years.

5. A cistern containing 600 gallons of cider, has three unequal pipes; the greatest pipe when opened, will empty the cistern in one hour, the second will empty it in two hours, and the third will empty it, in three hours; in what time will the three empty it all running at the same time? Ans. $32\frac{8}{11}$ min.

6. What sum of money, at 6 per cent per annum, will amount to $860 in ten years? Ans. $537,50 cents.

7. A gentleman bought a horse, chaise, and harness for $540, the harness cost half as much as the horse, and the horse and harness half as much as the chaise; what did each cost? Ans. Harness $60, horse $120, and chaise $360

8. A gentleman being asked how much money he had on hand, said that $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, and $\frac{1}{6}$ of his money amounted to $114; what amount of money had he? Ans. $120.

QUESTIONS ON SINGLE POSITION.

What is Single Position? A. It teaches to resolve such questions as require only one supposition. How do you proceed in the work? A. Assume any number, and with it perform the operations which are required to be performed in the conditions of the question; then say, as the result of the operation, is to the assumed number, so is the result in the question to the answer. Wherein does this rule differ from the Single Rule of Three? A. In nothing except the operation to be performed according to the conditions of the question, previous to the statement.

DOUBLE POSITION,

Is a rule for working certain questions by means of two suppositions of false numbers. The questions belonging to this rule, have results which are not proportional to their suppositions or numbers assumed; such are those, in which the number sought is increased or diminished by some given number, which is no known part of the number required.

RULE I.—1. Take or assume any two convenient numbers, and proceed with each of them separately according to the conditions of the question, as in Single Position.—2. Place the results or errours, against the positions or supposed numbers, thus, and if the errour be too great, mark it with +; and if too small, with —.

Pos.		Er.
6		30
	X	
8		20

3d. Multiply them crosswise; that is, the first position by the last errour, and last position by the first errour.—4. If the errours be alike, that is, both too small, or both too great, divide the difference of the products by the difference of the errours, and the quotient will be the answer.—5. If the errours be unlike, that is, one too small, and the other too great, divide the sum of the products by the sum of the errours, and the quotient will be the answer. Or, instead of the preceding rule, it would perhaps be better to use Rule II. see below:

NOTE.—This Rule is founded on this supposition: That the first errour is to the second, as the difference between the true and first supposed number, is to the difference between the true and second supposed number; when that is not the case, the exact answer cannot be found by this Rule.

RULE II.—As the difference of the errours, if alike, or their sum, if unlike, is to either errour, so is the difference of the positions, to a fourth number, which being added to, or subtracted from, the position producing the errour, will give the answer, or number sought.

NOTE.—The student will readily determine whether the position producing the errour, is to be added or subtracted from the fourth proportional number, by considering whether the position should be added or subtracted to produce the desired effect in the errour.

1. A person bought 30 quarts of liquor for 50s., part whiskey, at 6d. per quart, and brandy at 3s. per quart; how much did he buy of each? Ans. whiskey, 16qts., brandy, 14qts.

1st Position.—8 quarts whiskey at 6d. per quart is	4s.
Then 22 quarts of brandy at 3s. per quart is	66s.
	70
The given sum..	50
First Errour,..	20+

2d Position.—6qts. whiskey at 6d. per qt. is	3s.
Then, 24qts. of brandy at 3s. per qt. is	72s.
	75s.

First Position,	8qts.	The given sum,	50
Second Position,	6qts.	The 2d errour,	25+
Dif. of Pos.	2	The 1st errour,	20
		Dif. of errours,	5

By Rule I.	8		20+
		×	
	6		25+
	20		8
	120		200
			120
			5)80
			16

PROOF

Whiskey,	16 at 6d.=	8s.
Brandy,	14 at 3s.=	42s.
		50s

By Proportion, Rule II.

dif. err. dif.
As 5 : 20 :: 2
2
5)40
8
Add the Position, 8
Ans. 16

Or,

dif. err. dif.
As 5 : 25 :: 2
2
5)50
10
Add the Position, 6
Ans. 16

DEM.—It is easy to perceive, that the fourth number here must be added to the position, because by increasing the quantity of whiskey, the quantity of brandy is lessened, and consequently lessens the result, which is too great. The reason of the assumed proportion will appear evident by considering that the result varies with the position; and that the errour of the result is ever proportional to the errour of its position; and the difference of any result is to the errour of its position; so that the errour of any two results has the same proportion to the difference of the position from which they flow, as the difference of any other two results has to the difference of their position. But the difference of the errours is equal to the difference of the results, therefore, as the difference of the errours is to either errour, so must the difference of the positions be to a fourth number, that is, the errour of the position.

2. A labourer was hired for 20 days, at 6 shillings per day, for every day he worked; but with this condition, that for every day he played, he should forfeit 2 shillings; at the expiration of the time he received for his services 4 pounds 8 shillings; what number of days did he work?

Ans. 16 days.

3. Two gentlemen, A and B, have both the same income. A saves $\frac{1}{5}$ of his; but B, by expending $50 a year more than A, at the end of 4 years finds himself $100 in debt; what does each receive and spend per annum?

Ans. Each receives $125 per annum; A spends $100 a year, and B spends $150.

4. There was a fish caught in Black River Bay, near Sackets Harbour, in 1831, whose head was 15$\frac{1}{2}$ inches long, its tail was as long as the head and half the body, and the body was exactly the length of both head and tail; what was the length of the whole fish? Ans. 10 feet 4 inches.

5. What number is that, which being increased by its $\frac{1}{2}$, its $\frac{1}{4}$, and 5 more, will be just doubled? Ans. 20.

6. A gentleman has 2 fine horses, and a carriage worth $400; now if the first horse be harnessed in it, he and the

carriage will be triple the value of the second; but if the second be put in, he and the carriage will be worth 7 times the value of the first horse; what is the value of each horse?

Ans. $80 the value of the first, $160, the second.

7. Friend John, who had in credit liv'd,
Though now reduc'd, a sum receiv'd—
This lucky hit 's no sooner found,
Than clam'rous duns came swarming round;
To th' landlord—baker—many more,
John paid, in all, pounds ninety-four.
Half what remain'd, a friend he lent,
On Joan and 'self one fifth he spent;
And when of all these sums bereft,
One tenth o' th' sum received had left;
Now show your skill, you learned youths,
And by your work the sum produce.

Ans. £141.

8. A gentleman dying left his two sons $20,000, (the one 10 years old and the other 15,) to be so divided that the share of each being put out on interest at 5 per cent, would amount to equal sums when they respectively became 21 years of age; what was the share of each?

Ans. { 9122\frac{46}{57}$, the younger son's share.
Ans. { 10877\frac{11}{57}$, the share of the elder son.

QUESTIONS ON DOUBLE POSITION.

What does Double Position teach? A. It teaches to resolve certain questions by means of two suppositions and false numbers. How do you obtain the errours? A. By assuming any two convenient numbers, and proceeding with each of them separately, according to the conditions of the question, as in Single Position. After obtaining the errours, how do you proceed to obtain the answer by *Proportion?* A. By stating the question thus: as the difference of the errours, if alike, or their sum if unlike, is to either errour, so is the difference of the positions, to a fourth proportional number, which being added to, or subtracted from, the position producing the errour, gives the number required.

PRACTICAL QUESTIONS.

1. Three brothers on counting their money, found that Joseph had $20, George five dollars more than Joseph, and John as many as the other two; what number of dollars had the three? Ans. $90.

2. The Arabian or Indian method of notation was first known in England about the year 1150; how long is it since to the present year, 1834? Ans. 684 years.

3. There are two numbers, the greater of them is 73 times 109; and their difference 17 times 28; what is their sum and product? Ans. { 15438 their sum. 59526317 their product.

4. Suppose nine thousand men march in a column of seven hundred and fifty deep; how many march abreast? Ans. 12.

5. A merchant would ship 360 bushels of corn in barrels which hold 3 bushels and 3 pecks each; how many barrels does he need? Ans. 96 barrels.

6. A man can perform a certain journey in $35\frac{1}{2}$ days, travelling $13\frac{5}{8}$ hours a day; how long will it require to perform the same journey when he travels only $11\frac{2}{10}$ hours a day. Ans. $40\frac{615}{952}$ days.

7. A man bought 180 oranges at the rate of 2 for a penny, and 180 more at the rate of 3 for a penny; after which, he sold them out at the rate of 5 for 2 pence; did he gain or lose by the bargain? Ans. He lost 6 pence.

8. Two men, A and B, being on opposite sides of a wood, which is 536 yards about, they begin to go round it, both the same way, at the same instant of time; A goes at the rate of 11 yards per minute, and B at the rate of 34 yards in 3 minutes; what number of times must the wood be gone round before B overtakes A? Ans. 17 times.

9. It is required to divide 600 acres of land among three men, A, B and C; so that B may have 100 acres more than A, and C 64 more than B. Ans. A 112a. B 212a. C 276a.

10. A gentleman divided his fortune among his three sons, giving A £9 as often as B £5, and to C but £3 as often as B £7, and yet C's dividend was £2584; what was the whole estate? Ans. £19466 2s. 8d.

11. The clocks in Italy go on to 24 hours; then how many times does a clock strike, in that country, in performing one complete revolution of the index? Ans. 300 times.

12. What length must be cut off from a board $9\frac{1}{2}$ inches broad, to contain a square foot, or as much as 12 inches in length and 12 in breadth? Ans. $15\frac{3}{19}$ inches.

13. If 14 men can finish a job in 15 days; how many men must be added to complete it in 2 days? Ans. 91 men.

14. The top of a liberty-pole, being broken off by a gust of wind, struck the ground on a horizontal plane, at the distance of 10 feet from the foot of the pole; what was the height of the whole pole, allowing the length of the broken piece to be 26 feet? Ans. 50 feet.

15. *Suppose a pole 90 feet high, to stand on a horizontal plane; at what height must it be cut off so that the top may fall touching the ground 30 feet from the bottom, and the other end of the part cut off may rest on the stump or upright part? Ans. 40 feet.

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* From the square of the length of the pole, (that is, the sum of the parts forming the hypotenuse and perpendicular,) take the square of the base, (that is, the square of the distance between the bottom of the pole and the top resting on the ground,) then divide the remainder by twice the length of the pole, and the quotient will express the height at which the pole must be cut off.

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16. There are three horses belonging to different men, employed to draw a load from Watertown to Albany for \$35; A and B are supposed to do $\frac{3}{11}$ of the work, A and C $\frac{5}{13}$, and B and C $\frac{4}{14}$ of the work. They are to be paid proportional; will you by your work find out what each man is entitled to receive?

Ans. A's share \$$13\frac{374}{472}$.
B's ---- $6\frac{213}{472}$.
C's ---- $14\frac{357}{472}$.

17. A man has a room $19\frac{1}{2}$ feet long, and $12\frac{1}{2}$ feet broad; he wishes to have the floor laid with hard bricks, each 6 inches long, and 3 inches wide; how many bricks will it take? Ans. 1950 bricks.

18. Three towns, A, B, and C, are so situate that B lies 30 miles directly east of A, and C lies 40 miles directly south of A; what is the distance from B to C? Ans. 50 miles.

19. In working a sum in Division, I found the remainder to be 649, the quotient was 113, the divisor was a sum equal to both and 24 more; what was the dividend? Ans. 89467.

20. A circular fish-pond is to be made in a garden, that shall take up just half an acre; what length of cord will it require to reach from the centre to the circumference?

Ans. $27\frac{3}{4}$ yards.

21. How many 3 inch cubes may be cut out of a 12 inch cube? Ans. 64 cubes.

22. A person goes to a tavern with a certain sum of money in his pocket, where he spends 2 shillings; he then borrows as much money as he had left, and going to another tavern, he there spends 2 shillings also; then borrowing again as much money as he had left, he went to a third tavern, where he likewise spent 2 shillings; and thus again repeating the same at a fourth tavern, he then had nothing remaining; what sum had he at first? Ans. 3s. 9d.

23. What is the area of a right angled triangle, the three sides of which are 30, 40 and 50 rods? Ans. 600 rods.

24. A stationer sold quills at 11s. per thousand, by which he cleared $\frac{3}{8}$ of the money, but quills growing scarce, he raised them to 13s. 6d. per thousand; what did he clear per cent by the latter price? Ans. £96 7s. $3\frac{3}{11}$d.

25. When Hymen's golden knot was tied
That made friend Delia thine,
Your age did hers as much exceed
As two times three do three;
But after three and thrice three years
She a bride will have been,
Your age will be to that of hers,
As eleven are to seven.
Now, Joel, vers'd in numbers, tell
Their true ages on the wedding day?

Ans. Delia's age was 16, yours 32 years.

26. A father left his son a fortune, $\frac{1}{4}$ of which he ran through in 8 months; $\frac{3}{7}$ of the remainder lasted him 12 months longer; after which he had only £820 left; what sum did the father bequeath his son? Ans. £1913 6s. 8d.

27. A, leaves Exeter at 10 o'clock in the morning for London, and goes at the rate of 2 miles an hour without intermission; B sets out from London for Exeter, at 6 o'clock the same evening, and rides 3 miles an hour constantly: the

question is, whereabout on the road will they meet, if the distance between the two cities be 130 miles?

Ans, $61\frac{3}{5}$ miles from Exeter.

28. There is a certain mast, $\frac{1}{9}$ of its length stands in the ground, 12 feet of in the water, and $\frac{5}{6}$ of its length in the air, or above water; what is its whole length?

Ans. 216 feet.

29. A lent B one hundred pounds for six months, with a promise from B to do him a like favour whenever he should require it; but when A called on B to requite his kindness, B could only lend him seventy-five pounds; how long may A keep B's money to recompense himself for his courtesy to B?

Ans. 8 months.

30. Seven eighths of a certain number exceed four fifths by 6; what is that number?

Ans. 80.

31. A gentleman lent his friend a sum of money unknown, to receive interest for the same at the rate of 6 per cent, simple interest, and at the end of twelve years, received for principal and interest 860 dollars; what was the sum lent?

Ans. $500.

32. Three men, A, B, and C, bought a grindstone of 36 inches diameter; how much of its diameter must each grind off, to have an equal share of the stone, if A grind first, B next, and then C, making no allowance for the eye?

$$\text{Ans.}\left\{\begin{array}{ll} A & 6{,}607 \\ B & 8{,}609 \\ C & 20{,}784 \end{array}\right\}\text{ inches.}$$

USEFUL FORMS, (WITH REMARKS.)

Bills of Exchange or Drafts, Promissory Notes, Receipts, Orders, &c.

A Bill* of Exchange or Draft is an open letter or request, from one person to another, desiring him to pay on account of the former, a sum of money therein mentioned, to a third person; and is the same which we call, in common language, an order, or draft, for the payment of money. Its form may be thus:

$100 *Watertown, June* 20, 1831.

Three months after date, (*or* "*at sight,*" *or* "*on demand,*" *or* "*at ten days after sight,*") pay James Jackson, or order, (*or* "*bearer,*" *or* "*pay James Jackson,*" *simply omitting* "*order*" *and* "*bearer,*" *if you do not wish to make the Bill negotiable,*) one hundred Dollars, for value received.

RICHARD ROE.

To Mr. John Doe, Merchant, Watertown.

A CHECK, OR ORDER, is only a *shorter Bill,* governed by the same law as a regular Bill of Exchange. Its form may be thus:

Watertown, June 20, 1831.

MR. JOHN DOE,

Pay James Jackson, or bearer, fifty Dollars.

RICHARD ROE.

The one who draws the BILL, *Richard Roe,* is called the *drawer;* the one to *whom* it is *directed, John Doe,* is called the *drawee,* the one to *whom payable, James Jackson,* is called the *payee.*

A Bill must be accepted in order to bind the *drawee.* This may be done by either writing on it, thus:

Accepted, JOHN DOE,

or the *drawee* may accept it by parole, by saying that he accepts it, or promising to accept it, either before or after it is drawn.

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* The definition of the word BILL, as here used, is "a written paper of any kind; an account of money," &c. N. B. The words *Bill, Writ,* and *Document,* apply with propriety to all kinds of communications among men, whether written or printed, and are somewhat similar in their meaning. Custom will generally suggest which word to use with the greatest propriety.

A CHECK on a Bank may be drawn thus:

Watertown, June 20, 1831.

Cashier of the Jefferson County Bank, pay James Jackson, or bearer, one hundred dollars.

$100. RICHARD ROE.

NOTES.

A Promissory Note may be drawn thus:

$100. *Watertown, June* 20, 1831.

I promise to pay James Jackson, or order, (*or "bearer," or you may omit the words "order," or "bearer," if you wish to prevent its being negotiable,*) one hundred dollars, thirty days after date, (*or "on demand," or omitting any mention of time, in which case it is deemed payable the same as if on demand,*) with interest, for value received.

RICHARD ROE.

Here the signer, *Richard Roe,* is called the *maker,* and *James Jackson,* to whom payment is to be made, is called the *payee.*

These instruments, if made payable in any thing besides money, are not BILLS or NOTES, but mere *Contracts,* not negotiable.

No particular form of promise, in a Note, is necessary; any words which import an absolute engagement to pay, are sufficient.

A joint and several Note may be written thus:

Watertown, June 20, 1831.

Ninety days after date, we jointly and severally promise to pay James Jackson, or bearer, one hundred dollars, with interest, for value received.

$100.

JOHN DOE,
RICHARD ROE.

RECEIPTS.

Watertown, June 20, 1831.

Received of James Jackson ten dollars, in full of all accounts.

$10. RICHARD ROE.

RECEIPT FOR MONEY RECEIVED ON A NOTE.

Watertown, June 20, 1831.

Received of James Jackson, (by the hands of John Doe,) fifty dollars twenty-five cents, which is endorsed on his note of April 15, 1831.

$50,25. RICHARD ROE.

RECEIPT FOR MONEY RECEIVED ON ACCOUNT.

Watertown, June 20, 1831.

Received of James Jackson one hundred dollars on account. RICHARD ROE.

NOTE.—There is a distinction between receipts given in full of *all accounts*, and receipts given in full of *all demands*. The former cut off accounts only; the latter cut off not only accounts, but all obligations and right of action; all receipts, however, may be explained by parol or written evidence.

ORDERS.

Watertown, June 20, 1831.

Mr. John Doe,

Please to deliver James Jackson such goods as he may call for, not exceeding the sum of thirty Dollars, and place the same to my account.

$30. RICHARD ROE.

Watertown, May 20, 1831.

Mr. John Doe,

Pay James Jackson, or bearer, fifty Dollars at sight. RICHARD ROE.

NOTE.—An order payable in money, is the same as a Bill of Exchange.

BOOK-KEEPING BY SINGLE ENTRY,

Requires only two Books, a Day-Book and a Leger. With Farmers and Mechanicks these two books are sufficient. Merchants may multiply the number without impropriety; a Blotter, Journal, on which to transfer the Blotter entries, and a Leger, where the various accounts contained in the Journal or Day-Book are collected under their respective *names* or *titles*, for the convenience of inspection or settlement.

It is proper that the owner's name be written at the commencement of his Book.

The *date* should always be given with the entry. It may be kept in the margin, or in the middle of the line between charges.

Erasures should be avoided as much as possible in all Books of Account. In many cases it may be more proper to correct an errour by making a contrary or annulling entry; in which case it should be remembered, that it takes *two* entries to make the matter right—

one to cancel, or make even the erroneous entry, and *one* to show it as was first intended. Where it is thought best to make an erasure, the better way is to draw a line across the objectionable word or words, not so heavy but that they may be read. This answers the double purpose of an erasure, and exhibiting a fairness of intention, to all who may have occasion to examine the Accounts with a view of arriving at their correctness.

In the Day-Book, the town's name should be written across the head of the page, together with the date.

In the Leger, the person's names should be written in a fair hand, across the head of the left hand page, and Contra—Cr. on the right hand page.

The Index or Alphabet refers to the pages of the Leger, and when accounts are carried forward in the Leger, the circumstance may be noted in the index by placing a dot after the previous paging and setting down the new page to which the account is transferred; for example, John Doe, 5. 12.

Persons desirous of keeping a Cash Account are only to personify Cash, or consider it as an individual of flesh and blood; making a charge or debtor entry for every cent added to the heap, and giving credit for every cent taken from it; and to state what use was made of it. Or in other words, to begin a Cash Account, Cash should be charged with the amount you have on hand, and all receipts thereafter should be charged, and all disbursements or paying away of the same should be credited. To balance such an account, just give credit for the amount on hand, and if you have not neglected any entries, the Dr. and Cr. will foot alike.

The following are the principal points of law relating to Book Accounts:

Books of Account are evidence in favour of the party keeping them, under the following restrictions:

1. The charges to be proved, must be such as are *matter* of Book account. A Book, therefore, would not be evidence of money lent, had, and received, or paid, laid out, and expended, for the use of the opposite party.

2. They are not evidence of a *single* charge; because there exists, in such case, no regular dealing between the parties.

3. They are not to be admitted, where there are *several* charges, unless a foundation is first laid for their admission by proving,

1. *That the party had no Clerk.*
2. *That some of the articles charged had been delivered.*
3. *That the Books produced, are the Account Books of the party.*
4. *He must prove by those who have dealt and settled with him, that he keeps fair and honest accounts.*

When these requisitions are complied with, Book Accounts are evidence for the consideration of a Court or Jury.

Dr. **Peter Wheelwright.** **Contra.** **Cr.**

1831.				1831.			
Jan. 1.	To 5 cords wood, at 1,50	7	50	April 6.	By one Plough, ...	10	50
May 10.	To 2 days' work, 1,00 .	2	00	May 20.	By repairing Wagon, .	1	16
June 7.	To 6 bushels wheat, at ,75	4	50	July 4.	By am't. carried to *n|a**	2	50
1831.		$14	00			$14	00
				1831.			
July 4.	To balance from *o|a** . . .	2	50	July 20.	By a Wheelbarrow, . .	7	00
July 12.	To 400 ft. boards, at 500 M.	2	00	Aug. 10.	By Note to balance, . .	4	00
Aug. 6.	To Oak timber, pr. agree't.	6	50				
		11	00			11	00

* This is the balance brought forward, which again places the parties to the account in their proper relative situations. The fictitious Credit is now converted into a reality on the Dr. side.

* When accounts are examined by the parties, it is proper, if they have been standing for any length of time, to strike a balance and carry forward, if not paid at the time of such examination. The entry having the * attached to it is a mere fictitious one, for the purpose of making the Dr. and Cr. foot alike. *See Dr. side.*

THE FOREGOING WORK HAS THE FOLLOWING

RECOMMENDATION.

This is the universal Recommendation, in substance, of teachers who have introduced it into their schools: "That the simplicity of the rules and the observations accompanying, contrast strongly in its favour, when compared with the old arbitrary forms....That matters relating to the science, which have heretofore been treated as though beyond the reach of human investigation, or beneath the dignity of scientifick research, are in this work represented in their true light; freed from all obscurity.,...That its whole tenour is upon the just supposition that absolute rules, or *words*, are soon forgotten, while time is scarcely sufficient to efface *principles* from the memory....That the questions and answers, calculated more fully to illustrate the principles of rules and mathematical operations; to brighten the faculties of pupils, and leave lasting impressions, are of vast importance to teachers: lessening the labour of a heretofore tedious branch of instruction, and giving greater force to oral communications.....That the more than usual attention paid to the elementary rules is of infinite advantage, inasmuch as it renders it difficult for learners to pass on to the more intricate parts *and yet remain ignorant of the first rudiments of the science*; a fault justly chargeable upon nearly every other Arithmetick in use."